CUBIC CURVES AS PFAFFIANS

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Abstract. In this article we find all (decomposable and indecomposable) linear pfaffian representations of a plane cubic curve given by a Weierstrass equation.

1. Introduction

Let k be an algebraically closed field and C an irreducible curve in $\mathbb{P}^2$ defined by a polynomial $F(x, y, z)$ of degree 3. Every smooth cubic can be brought (by a projective change of c0ordinates [7]) into the Weierstrass form

$$F(x, y, z) = yz^2 - x(x - y)(x - \lambda y) = 0,$$

for some $\lambda \neq 0, 1$.

We consider the following question. For given C (and F) find a linear matrix

$$A(x, y, z) = x\, A_x + y\, A_y + z\, A_z$$

such that

$$\det A(x, y, z) = c\, F(x, y, z)^r$$

where $A_x, A_y, A_z \in \mathrm{Mat}_{3r}(k)$ and $c \in k, c \neq 0$. Here $\mathrm{Mat}_{3r}(k)$ is the algebra of all $3r \times 3r$ matrices over k.

We call A *determinantal representation* of C of order r. Two determinantal representations A and A' are *equivalent* if there exist $X, Y \in \mathrm{GL}_{3r}(k)$ such that

$$A' = XAY.$$

Obviously, equivalent determinantal representations define the same curve. *Pfaffian representation* is a representation of order 2 with all 6×6 matrices being skew-symmetric. Study of pfaffian representations is strongly related to and motivated by determinantal representations. Every 3×3 determinantal representation A induces *decomposable pfaffian representation*

$$\begin{bmatrix} 0 & A \\ -A^t & 0 \end{bmatrix}.$$

Note that the equivalence relation is well defined since

$$\begin{bmatrix} 0 & XAY \\ -(XAY)^t & 0 \end{bmatrix} = \begin{bmatrix} X & 0 \\ 0 & Y^t \end{bmatrix} \begin{bmatrix} 0 & A \\ -A^t & 0 \end{bmatrix} \begin{bmatrix} X^t & 0 \\ 0 & Y \end{bmatrix}.$$

2. Determinantal representations

Nonequivalent linear determinantal representations of order 1 are in bijection with line bundles on C and they can be parametrised by the nonexceptional points on the Jacobian variety of C. Vinnikov [14] found an explicit one to one correspondence between the linear determinantal representations (up to equivalence) of C and the points on an affine piece of C:

Lemma 2.1 ([14])**.** *Every smooth cubic can be brought into the Weierstrass form*

$$F(x_0,x_1,x_2) = -x_1x_2^2 + x_0^3 + \alpha x_0x_1^2 + \beta x_1^3.$$

A complete set of determinantal representations of F is

$$x_0 \operatorname{Id} + x_2 \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + x_1 \begin{pmatrix} \frac{s}{2} & l & \alpha + \frac{3}{4}s^2 \\ 0 & -s & -l \\ -1 & 0 & \frac{s}{2} \end{pmatrix},$$

where $l^2 = s^3 + \alpha s + \beta$. Note that the last equation is exactly the affine part $F(s,1,l)$.

Proof. Let $A(x_0,x_1,x_2) = x_0A_0 + x_2A_2 + x_1A_1$ be a representation of $F(x_0,x_1,x_2)$. First we show that it is equivalent to a representation with

$$A_0 = \operatorname{Id} \text{ and } A_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

We remark that this is a Lancaster Rodman canonical form for real matrix pairs [6] . Observe that A_0 is invertible since $\det A(1,0,0) \neq 0$. We can multiply $A(x_0,x_1,x_2)$ by A_0^{-1} to obtain an equivalent representation with $A_0 = \operatorname{Id}$. The characteristic polynomial of A_2 equals $\det A(x_0,0,-1) = x_0^3$ which implies that A_2 is nilpotent. The nonzero term $x_1x_2^2$ in F determines the order of nilpotency. Further GL_3 action from left and right which preserves this canonical form (the first two matrices in the determinantal representation) reduces A_1 to the above. □

3. Moduli

In [3] we prove the following two theorems:

Theorem 3.1. *Let C be a curve defined by a polynomial F of degree d in $\mathbb{P}^2$. There is a one to one correspondence between linear pfaffian representations of F (up to equivalence) and rank 2 bundles (up to isomorphism) on C in the open set*

$$M_C(2, \mathcal{O}_C(d-3)) \setminus \Theta_{2,\mathcal{O}_C(d-3)}.$$

Theorem 3.2. *There is a one to one correspondence between decomposable vector bundles in $M_C(2,\mathcal{O}_C(d-3)) \setminus \Theta_{2,\mathcal{O}_C(d-3)}$ and the open subset of Kummer variety*

$$(JC \setminus W_{g-1}) / \equiv,$$

where $\equiv$ is the involution $\mathcal{L} \mapsto \mathcal{L}^{-1} \otimes \mathcal{O}_C(d-3)$.

In the case of cubics we obtain

Corollary 3.3 (§1 in [2])**.** *On a cubic curve C all linear pfaffian representations can be parametrised by the points on the Kummer variety $\mathcal{K}_C - \{$one point$\}$.*

Proof. Recall that on an elliptic curve $K_C \cong \mathcal{O}_C$. Since $M_C^s(2,0)$ is empty, there are no stable bundles on C. On the other hand, by [2, §4] the non-stable part of $M_C(2,\mathcal{O}_C)$ consists of decomposable vector bundles of the form $\mathcal{L} \oplus \mathcal{L}^{-1}$ for $\mathcal{L}$ in the Jacobian JC. Obviously $\mathcal{L} \oplus \mathcal{L}^{-1}$ and $\mathcal{L}^{-1} \oplus \mathcal{L}$ are equivalent. For $\mathcal{L} \in JC$ the following conditions are equivalent:

- $h^0(C, \mathcal{L} \oplus \mathcal{L}^{-1}) = 0$,
- $h^0(C, \mathcal{L}) = 0$,
- $\mathcal{L} \neq \mathcal{O}_C$.

Therefore

$$M_C(2, \mathcal{O}_C) \setminus \Theta_{2,\mathcal{O}_C} \;=\; \{\mathcal{L} \oplus \mathcal{L}^{-1};\ \mathcal{L} \in JC\} \setminus \{\mathcal{O}_C \oplus \mathcal{O}_C\}.$$

□

Recall that the Jacobian of a cubic curve C with $g = 1$ is the curve itself and $J - \{W_0\}$ is an affine piece of C. In particular, Corollary 3.3 implies that the complete set of pfaffian representations of F (put in the Weierstrass form) equals

$$\begin{bmatrix} 0 & M \\ -M^t & 0 \end{bmatrix},$$

where M are the determinantal representations in Lemma 2.1. Note that M and $-M^t$ are not equivalent determinantal representations, but

$$\begin{bmatrix} 0 & M \\ -M^t & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 & -M^t \\ M & 0 \end{bmatrix}$$

are quivalent pfaffian representations since

$$\begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} 0 & M \\ -M^t & 0 \end{bmatrix} \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} = \begin{bmatrix} 0 & -M^t \\ M & 0 \end{bmatrix}.$$

Remark 3.4. Each point in the the moduli space $M_C(2, \mathcal{O}_C)$ corresponds to a decomposable bundle, thus decomposable Pfaffian. However, this does not imply that all the bundles are decomposable. Moduli space consists of S-equivalence classes rather than bundles, so that the direct sum of $\mathcal{L}$ and $\mathcal{L}$, or the non-trivial extension of $\mathcal{L}$ by $\mathcal{L}$ really represent the same point.

4. Pfaffian representations

Pfaffian representations are equivalent under the action

$$A \;\mapsto\; P \cdot A \cdot P^t,$$

where P is an invertible 6×6 constant matrix. By a suitable P we can reduce the number of parameters in A. In other words, we will reduce the number of equivalent representations in each equivalence class. The proof of Theorem 4.1 outlines an algorithm for finding all pfaffian representations (up to equivalence) of

$$C = \{(x, y, z) \in \mathbb{P}^2 \;:\; yz^2 - x(x-y)(x-\lambda y) = 0\}.$$

Theorem 4.1. *Let C be a smooth cubic in the Weierstrass form*

$$F(x, y, z) = yz^2 - x(x-y)(x-\lambda y).$$

A complete set of pfaffian representations of F consists of three indecomposable representations and for the whole affine curve of decomposable representations:

$$x\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ & 0 & 0 & 0 & 1 & 0 \\ & & 0 & 1 & 0 & 0 \\ & & & 0 & 0 & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix} + z\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ & 0 & 0 & 1 & 0 & 0 \\ & & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix} + y\begin{pmatrix} 0 & 1 & 0 & \frac{3t^2-2t(1+\lambda)-(1-\lambda)^2}{4} & 0 & \frac{t-1-\lambda}{2} \\ & 0 & 0 & 0 & -t & 0 \\ & & 0 & \frac{t-1-\lambda}{2} & 0 & -1 \\ & & & 0 & 0 & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix} \quad \textit{for } t = 0, 1, \lambda$$

and

$$x\begin{pmatrix}0&0&0&0&0&1\\ &0&0&0&1&0\\ &&0&1&0&0\\ &&&0&0&0\\ &&&&0&0\\ &&&&&0\end{pmatrix}+z\begin{pmatrix}0&0&0&0&1&0\\ &0&0&1&0&0\\ &&0&0&0&0\\ &&&0&0&0\\ &&&&0&0\\ &&&&&0\end{pmatrix}+y\begin{pmatrix}0&0&0&\frac{3t^2-2t(1+\lambda)-(1-\lambda)^2}{4}&s&\frac{t-1-\lambda}{2}\\ &0&0&-s&-t&0\\ &&0&\frac{t-1-\lambda}{2}&0&-1\\ &&&0&0&0\\ &&&&0&0\\ &&&&&0\end{pmatrix},$$

where $s^2 = t(t-1)(t-\lambda)$. *Note that the last equation is exactly the affine part* $F(t, 1, s)$.

The proof will be based on Lancaster–Rodman canonical forms of matrix pairs [6]. This generalizes Vinnikov's construction of determinantal representations [14]. Let $A = xA_x + zA_z + yA_y$ be a pfaffian representation of C. Observe that A_x is invertible and A_z nilpotent since C is defined by $\operatorname{Pf} A$ and contains x^3 term and no z^3 term. Moreover, yz^2 determines the order of nilpotency. This determines the unique skew-symmetric canonical form [6, Theorem 5.1] of the first two matrices. In other words, every pfaffian representation of C can be put into the following form

$$x\begin{pmatrix}0&0&0&0&0&1\\ &0&0&0&1&0\\ &&0&1&0&0\\ &&&0&0&0\\ &&&&0&0\\ &&&&&0\end{pmatrix}+z\begin{pmatrix}0&0&0&0&1&0\\ &0&0&1&0&0\\ &&0&0&0&0\\ &&&0&0&0\\ &&&&0&0\\ &&&&&0\end{pmatrix}+y\begin{pmatrix}0&c_{12}&c_{13}&c_{14}&c_{15}&c_{16}\\ &0&c_{23}&c_{24}&c_{25}&c_{26}\\ &&0&c_{34}&c_{35}&c_{36}\\ &&&0&c_{45}&c_{46}\\ &&&&0&c_{56}\\ &&&&&0\end{pmatrix}.$$

Since $\operatorname{Pf} A$ defines the equation of C, we get

$$\begin{aligned}
c_{36} &= -1,\\
c_{26} &= -c_{35},\\
c_{25} &= -1-\lambda-c_{16}-c_{34},\\
c_{14} &= c_{16}+c_{16}^2+c_{34}+c_{16}c_{34}+c_{34}^2+2c_{24}c_{35}+c_{16}c_{35}^2-c_{34}c_{35}^2-\\
&\quad c_{23}c_{45}-c_{13}c_{46}+c_{23}c_{35}c_{46}-c_{12}c_{56}+c_{13}c_{35}c_{56}+\lambda(1+c_{16}+c_{34}),\\
c_{15} &= -c_{24}-c_{16}c_{35}+c_{34}c_{35}-c_{23}c_{46}-c_{13}c_{56}.
\end{aligned}$$

There are $15-5$ parameters c_{ij} left in the representation. Additionally, the coefficient at y^3 equals $(c14c26c35 - c14c25c36 - c13c26c45 + c12c36c45 + c16(c25c34 - c24c35 + c23c45) + c13c25c46 - c12c35c46 - c15(c26c34 - c24c36 + c23c46) + c14c23c56 - c13c24c56 + c12c34c56) = 0$.

Lemma 4.2. *The action* $A \mapsto P \cdot A \cdot P^t$ *preserves the canonical form of the first two matrices in the representation if and only if* P *equals*

$$\begin{bmatrix} P_1 & P_2 \\ P_3 & P_1^{-1} + P_3P_1^{-1}P_2 \end{bmatrix} \text{ or } \begin{bmatrix} P_2 & P_1 \\ -P_1^{-1} + P_3P_1^{-1}P_2 & P_3 \end{bmatrix}$$

where P_1 *is invertible and* P_i *are of the form*

$$\begin{bmatrix} p_{i1} & p_{i2} & p_{i3} \\ 0 & p_{i1} & p_{i2} \\ 0 & 0 & p_{i1} \end{bmatrix}, \quad i = 1, 2, 3.$$

Proof. Denote

$$I = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad N = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

We will need the following obvious observation, which can be proved directly by comparing matrix elements:
Let Y, Y' be 6×6 matrices for which

$$Y.\begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix} = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}.Y' \text{ and } Y.\begin{bmatrix} 0 & N \\ -N & 0 \end{bmatrix} = \begin{bmatrix} 0 & N \\ -N & 0 \end{bmatrix}.Y' \text{ hold.}$$

Then $Y = \begin{bmatrix} Y_1 & Y_2 \\ Y_3 & Y_4 \end{bmatrix}$ and $Y' = \begin{bmatrix} Y_4^t & -Y_3^t \\ -Y_2^t & Y_1^t \end{bmatrix}$, where

$$Y_i = \begin{bmatrix} y_{i1} & y_{i2} & y_{i3} \\ 0 & y_{i1} & y_{i2} \\ 0 & 0 & y_{i1} \end{bmatrix}, \quad i = 1, 2, 3, 4.$$

We call the specific form of the above Toeplitz matrices "$\triangle$ form".

Now we can find all invertible

$$P = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix}$$

that satisfy

$$P.\begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}.P^t = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix} \text{ and } P.\begin{bmatrix} 0 & N \\ -N & 0 \end{bmatrix}.P^t = \begin{bmatrix} 0 & N \\ -N & 0 \end{bmatrix}.$$

By the above observation all P_i's are of $\triangle$ form. Moreover,

$$\begin{aligned} p_{11}p_{41} - p_{21}p_{31} &= 1, \\ p_{22}p_{31} + p_{21}p_{32} - p_{12}p_{41} - p_{11}p_{42} &= 0, \\ p_{23}p_{31} + p_{22}p_{32} + p_{21}p_{33} - p_{13}p_{41} - p_{12}p_{42} - p_{11}p_{43} &= 0. \end{aligned}$$

In other words, if P_1 is invertible then $P_4 = P_1^{-1} + P_3P_1^{-1}P_2$. The same way we see that $P_3 = -P_2^{-1} + P_1P_2^{-1}P_4$ when P_2 is invertible.

Since P is invertible and consists of $\triangle$ blocks, at least one of P_1, P_2 is also invertible. Note that

$$\begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix} \cdot \begin{bmatrix} 0 & -\mathrm{Id} \\ \mathrm{Id} & 0 \end{bmatrix} = \begin{bmatrix} P_2 & -P_1 \\ P_4 & -P_3 \end{bmatrix}$$

exchanges P_1 and P_2 which finishes the proof. □

The action of Lemma 4.2 enables us to reduce the number of parameters c_{ij}. We can choose such P that its action eliminates

(1) $$c_{13} = c_{23} = c_{46} = c_{56} = 0, \ c_{35} = 0 \text{ and } c_{16} = c_{34}.$$

Indeed, if we choose $p_{11} = 1$, the above condtions determine p_{12}, p_{13} and $p_{22}, p_{23}, p_{32}, p_{33}$:

$$\begin{aligned} p_{32} &\to c_{56} - c_{35}p_{31} + c_{56}p_{31}p_{21}, \\ p_{22} &\to -c_{23} + c_{35}p_{21}, \\ p_{33} &\to \frac{1}{2}((c_{16} - c_{34} + c_{35}^2 - c_{23}c_{56})p_{31} + 2c_{46}(1 + p_{31}p_{21})), \\ p_{23} &\to \frac{1}{2}(-2c_{13} + (-c_{16} + c34 + c_{35}^2 - c_{23}c_{56})p_{21}), \\ p_{12} &\to -c_{35} + c_{56}p_{21}, \\ p_{13} &\to \frac{1}{2}(c_{16} - c_{34} + c_{35}^2 - c_{23}c_{56} + 2c_{46}p_{21}). \end{aligned}$$

The relations among c_{ij} then simplify to:

$$
\begin{aligned}
c_{14} &= 3c_{16}^2 + \lambda + 2c_{16}(1+\lambda), \\
c_{24} &= -c_{15}, , \\
0 &= c_{15}^2 - 8c_{16}^3 - c_{12}c_{45} - \lambda - \lambda^2 - 8c_{16}^2(1+\lambda) - 2c_{16}(1+3\lambda+\lambda^2). \qquad (2)
\end{aligned}
$$

which leaves us with 4 parameters $c_{12}, c_{45}, c_{15}, c_{16}$ and equation (2) connecting them:

$$
\begin{pmatrix}
0 & \mathbf{c_{12}} & 0 & c_{14} & \mathbf{c_{15}} & \mathbf{c_{16}} \\
 & 0 & 0 & -c_{15} & -1-\lambda-2c_{16} & 0 \\
 & & 0 & c_{16} & 0 & -1 \\
 & & & 0 & \mathbf{c_{45}} & 0 \\
 & & & & 0 & 0 \\
 & & & & & 0
\end{pmatrix}.
$$

It is easy to check that $A \mapsto P \cdot A \cdot P^t$ from Lemma 4.2 preserves all zeros and -1 in the above matrix if and only if

$$
P_i = \begin{bmatrix} p_{i1} & 0 & 0 \\ 0 & p_{i1} & 0 \\ 0 & 0 & p_{i1} \end{bmatrix} \quad \text{for } i = 1,2,3,4, \quad \text{together with} \quad p_{11}p_{41} - p_{21}p_{31} = 1.
$$

We can use this "diagonal" action to make $c_{45} = 0$ by chosing appropriate p_{41} like in (1). When $c_{15} \neq 0$ we can furthermore make $c_{12} = 0$ by $p_{11} = p_{41} = 1, p_{31} = 0, p_{21} = -c_{12}/2c_{15}$. The only case left to consider is $c_{15} = 0$. The action which keeps $c_{45} = 0$ maps $c_{12} \mapsto c_{12}p_{11}^2$ where $p_{11}p_{41} = 1$ and $p_{31} = 0$. Thus either $c_{12} = 0$ or we can make $c_{12} = 1$.

In order to simplify notations even further, we introduce parameters t and s by $c_{16} = \frac{1}{2}(t-1-\lambda)$ and $c_{15} = is$ (here $i^2 = -1$). When $c_{45} = c_{12} = 0$ the matrix becomes

$$
\begin{pmatrix}
0 & 0 & 0 & \frac{3t^2-2t(1+\lambda)-(1-\lambda)^2}{4} & s & \frac{t-1-\lambda}{2} \\
 & 0 & 0 & -s & -t & 0 \\
 & & 0 & \frac{t-1-\lambda}{2} & 0 & -1 \\
 & & & 0 & 0 & 0 \\
 & & & & 0 & 0 \\
 & & & & & 0
\end{pmatrix}
$$

and relation (2) in the new parameters equals $s^2 - t(t-1)(t-\lambda) = 0$.

Additionally we get

$$
\begin{pmatrix}
0 & 1 & 0 & \frac{3t^2-2t(1+\lambda)-(1-\lambda)^2}{4} & 0 & \frac{t-1-\lambda}{2} \\
 & 0 & 0 & -s & -t & 0 \\
 & & 0 & \frac{t-1-\lambda}{2} & 0 & -1 \\
 & & & 0 & 0 & 0 \\
 & & & & 0 & 0 \\
 & & & & & 0
\end{pmatrix}
$$

where t is one of the three solutions of $0 = -t(t-1)(t-\lambda)$.

Remark 4.3. The representations in Theorem 4.1 are non-equivalent to each other, since they are not connected by the action $A \mapsto P \cdot A \cdot P^t$.

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