Delaunay Triangulations with Predictions



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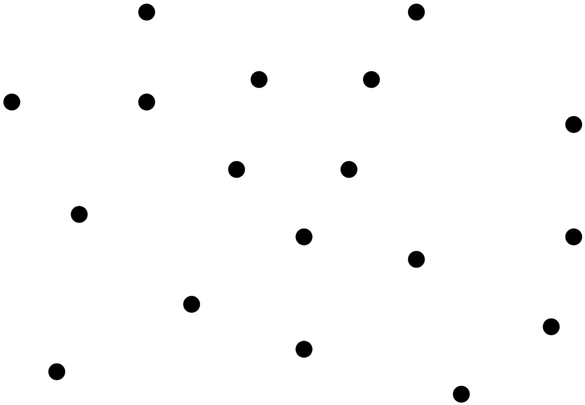
Delaunay triangulations



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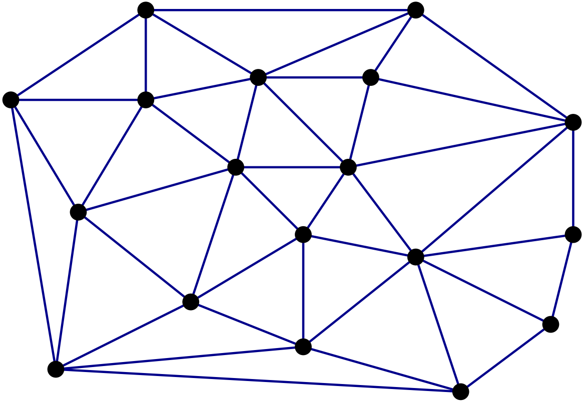

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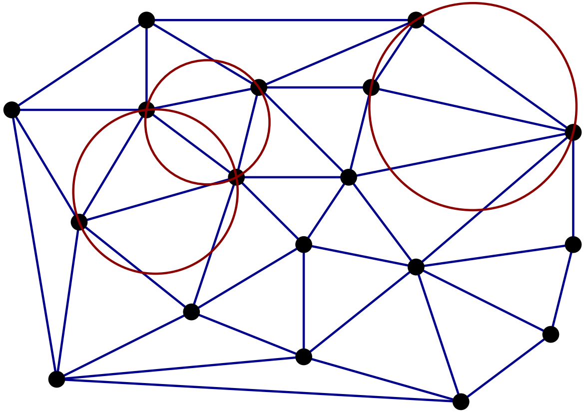

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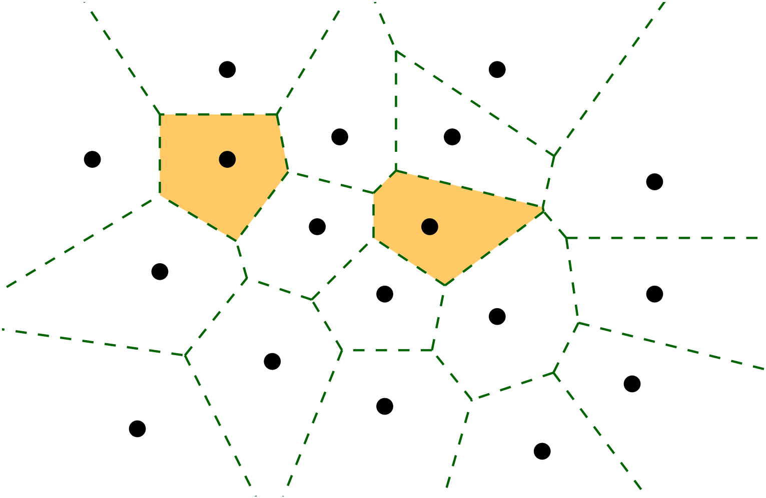

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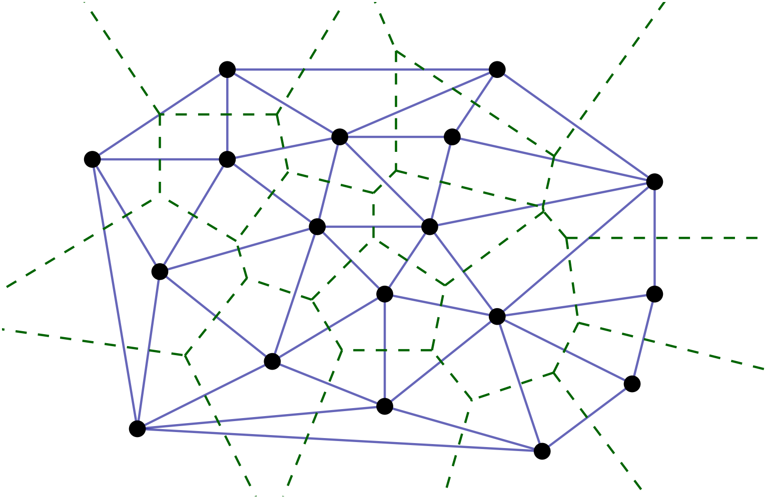
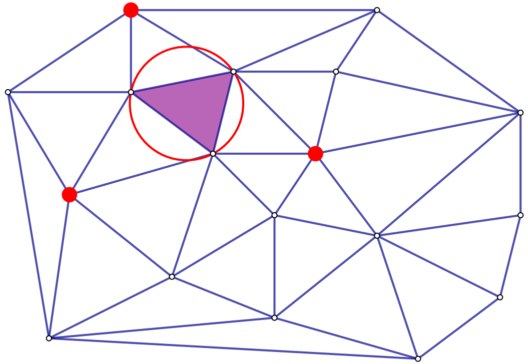


Delaunay triangulations

- triangles with empty circumcircle
- dual to Voronoi diagram
- lexicographically maximizes the vector of non-decreasing angles
- partition of the convex hull
- basic and ubiquotous geometric structure
- computable in $O(n \log n)$, and lower bound of $\Omega(n \log n)$
- several paradigms: incremental, randomized incremental, divide and conquer, via Voronoi diagram, via CH in 3d, etc.
- a 2-dimensional "extension" of sorting

Delaunay triangulations - locality

- testing whether a given triangulation is a Delaunay triangulation
- suffices local test of each triangle with its ≤ 3 neighbor triangles



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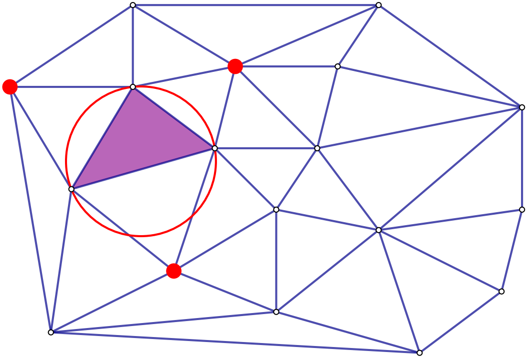

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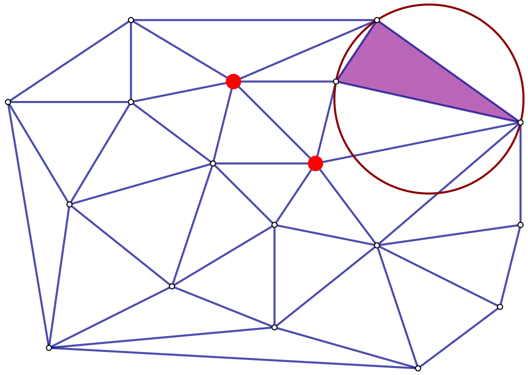

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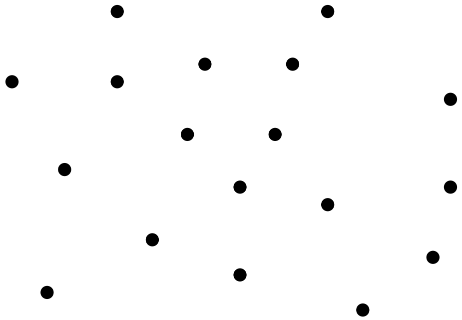
- $\Theta(n \log n)$ to build Delaunay triangulation
- $\Theta(n)$ to test if a given triangulation is Delaunay

Euclidean MST

- Given a set P of points in $\mathbb{R}^2$, compute its Euclidean MST



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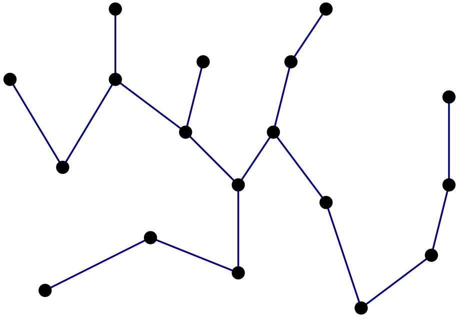

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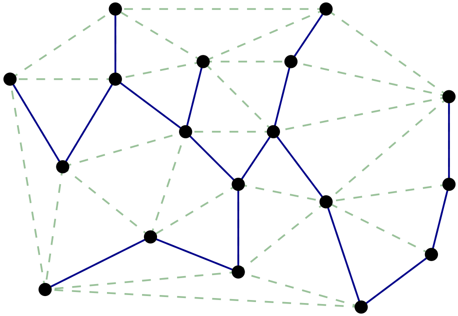

- $\mathrm{EMST}(P) \subset \mathrm{DT}(P)$
- $\mathrm{EMST}(P)$ in $O(n \log n)$ time via $\mathrm{DT}(P)$
- Non-obvious local testing for a candidate $\mathrm{EMST}(P)$
  $O(n \log^* n)$ expected time via $\mathrm{DT}(P)$ [Devillers 1992]

Predictions

- Prediction: additional information, possibly inaccurate or noisy, which may assist an algorithm to be more effective
(= 'better' solution, faster, etc.)
- Analyze the performance of the algorithm parameterized by how good is the prediction

- Easy example: sorting a list of numbers that is nearly sorted
- Prediction: the list itself
- Parameter: a measure on how sorted the input list is

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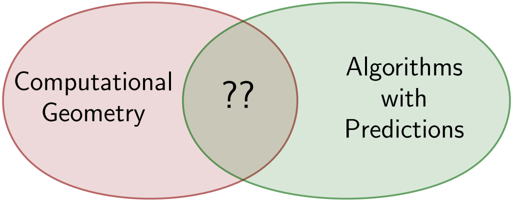
- Algorithms with predictions [Mitzenmacher, Vassilvitskii '20]
- Multiple focused events since 2020
- `https://algorithms-with-predictions.github.io/`
  List of 364 papers as of January 2026

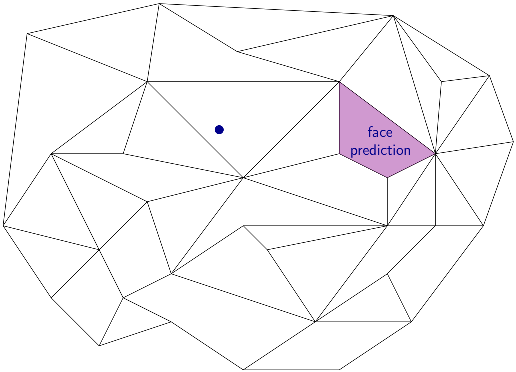
Our aim



- focus in fundamental object: Delaunay triangulation (and EMST)

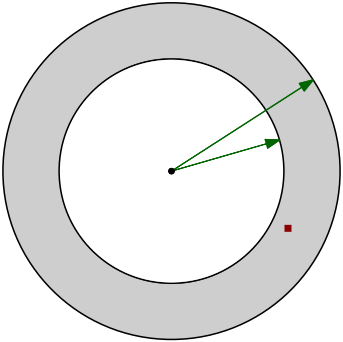
Related work

- Point location with a finger [Iacono, Langerman '03]



Related work

- Point location with a finger [Iacono, Langerman '03]
- Searching a point target in $\mathbb{R}^d$ with an estimate of the distance to the target [Cabello, Giannopoulos '24]



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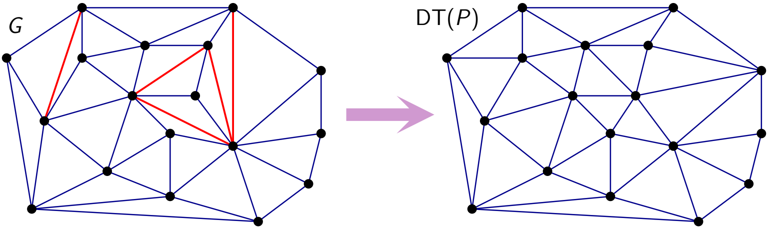
- Computing $DT(P)$ with additional information:
  - DT(P) from EMST(P) [Devillers '92]
  - merging: DT($P \cup Q$) from DT(P) and DT(Q) [Chazelle '92, Chan '16]
  - splitting: DT(P) from DT($P \cup Q$) [Chazelle et al '02; Chazelle, Mulzer '11]
- (Extension of) verification of structures
  - Given a spanning tree of P, is it EMST(P)?

Sample of our results

- Input: a set P of n points and
  a triangulation G of P, hopefully close to $\mathrm{DT}(P)$
- Measure of the prediction:
  (unknown) number D of edges in $E(G) \setminus \mathrm{DT}(P)$



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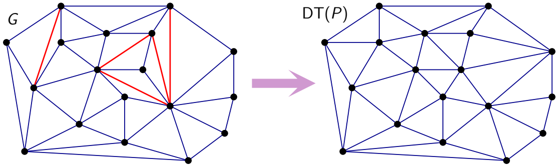
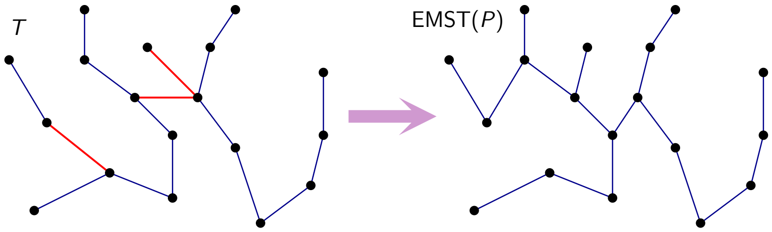


- We can compute $\mathrm{DT}(P)$ in $O(n + D \log n)$ expected time
- Smooth interpolation between $O(n)$ for $D = 0$
  and $O(n \log n)$ for $D = \Theta(n)$.
- Optimal, randomized
- Deterministic in $O(n + D \log^3 n)$

Sample of our results II

- Input: a set P of n points and
  a spanning tree T of P, hopefully similar to EMST(P)
- Measure of the prediction:
  (unknown) number D_{emst} of edges in $E(T) \setminus E(\mathrm{EMST}(P))$



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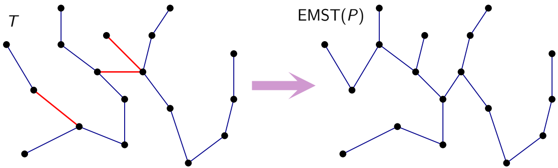
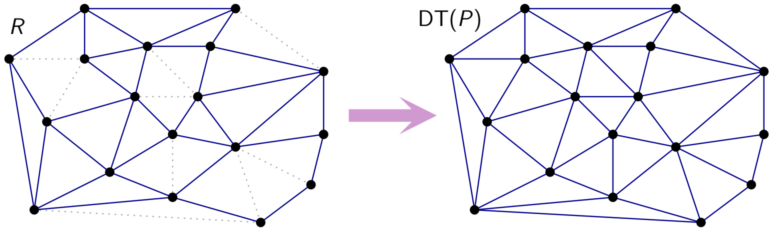


- We can compute $\mathrm{EMST}(P)$ in $O\big((n + D_{\mathrm{emst}} \log n) \log^* n\big)$ expected time
- When $D = 0$, we recover the $O(n \log^* n)$ for testing

Sample of our results III

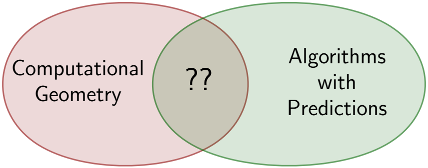
Model with probabilistic noise

- $R \subset_{\text{random}} E(\mathrm{DT}(P))$
  each edge selected independently with probability ρ
- G a triangulation extending R, or R with info about embedding
- Measure of the prediction: (unknown) probability ρ



- We can compute $\mathrm{DT}(P)$ in $O(n \operatorname{loglog} n + n \log(1/\rho))$ whp
- For $\rho = 1/\log n$ a small fraction of $E(DT(P))$ is kept

Conclusions



- New paradigm in Computational Geometry
- Basic premise: checking correctness faster than constructing from scratch
- We also use other measures of how close to DT(P)
- Some of our optimal algorithms are randomized. Deterministic?
- Some other algorithms are (not expected?) to be optimal $\log^* n$ and $\log\log n$ factors?

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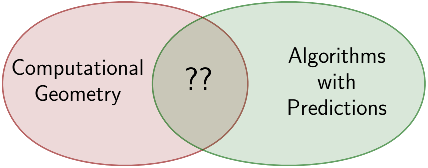

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THANKS for your time!!