Approximation Algorithms for Aligning Points

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GIVE

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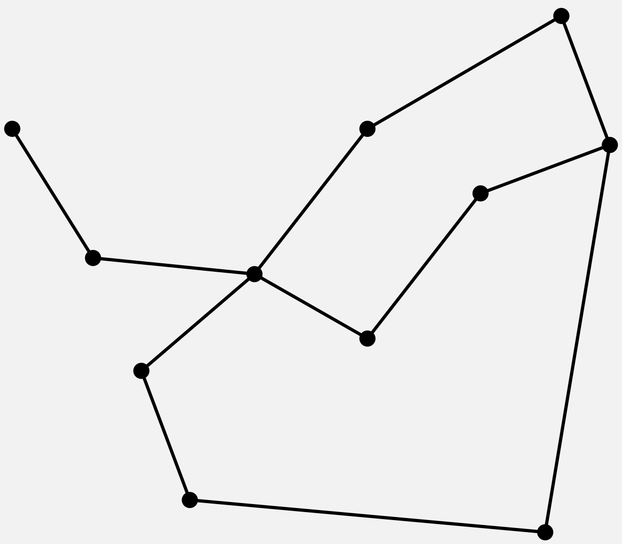
Motivation + Problem statement

Abstraction from the motivation



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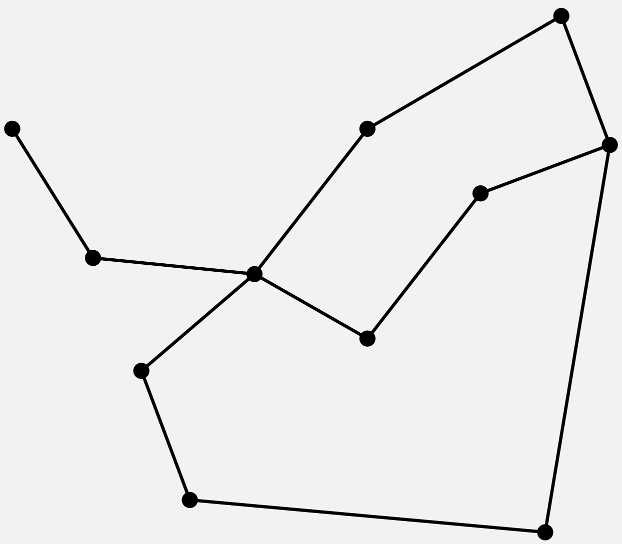

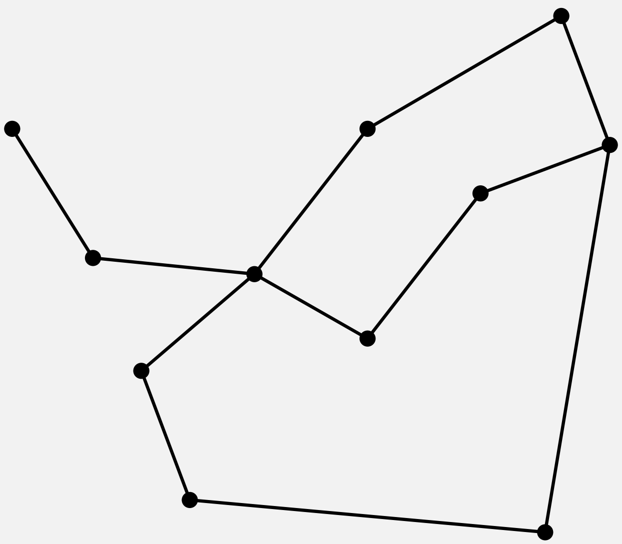

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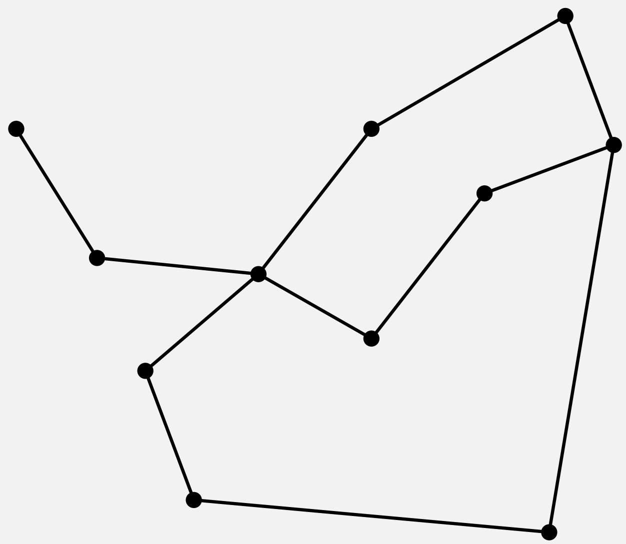

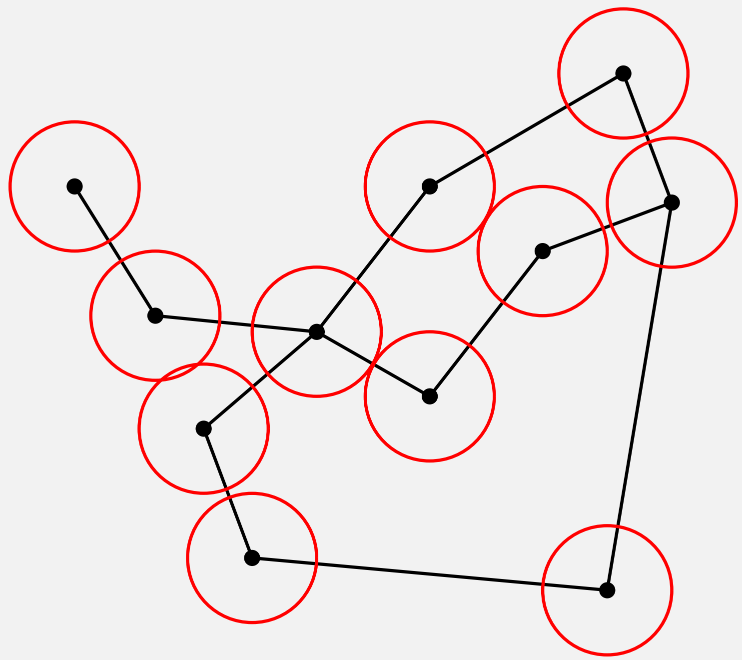

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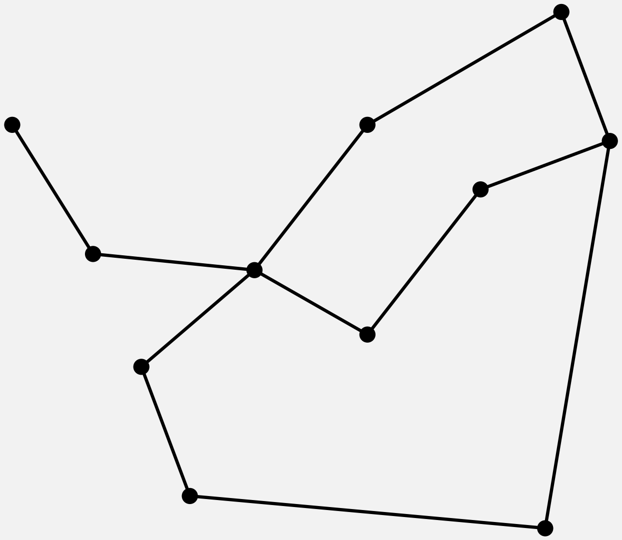

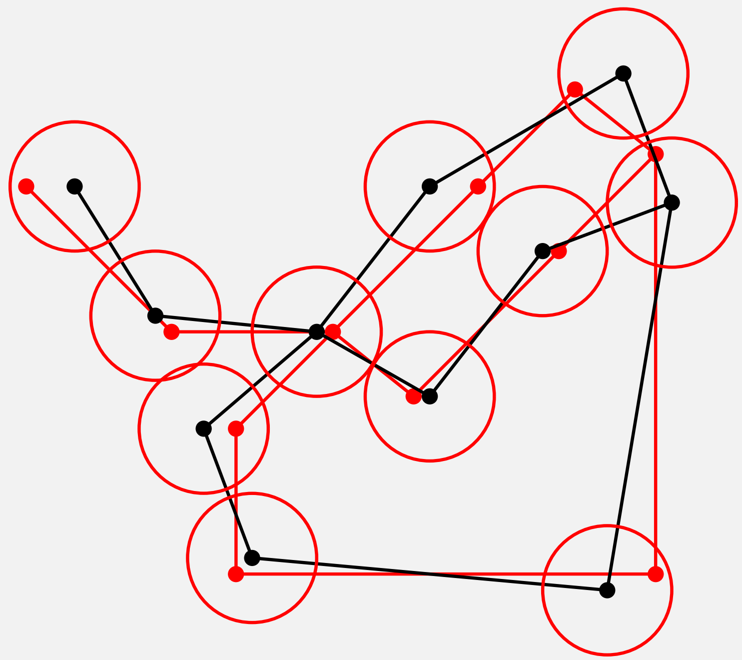

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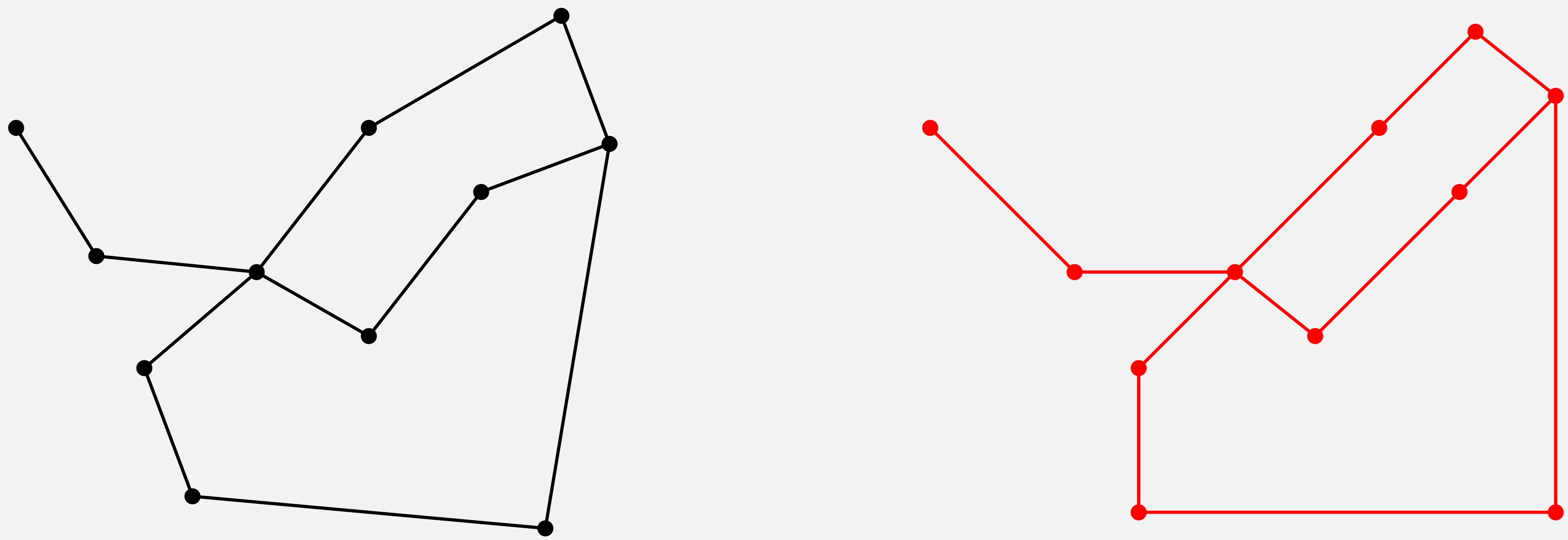
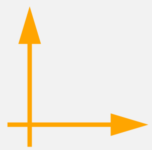


Problem statement

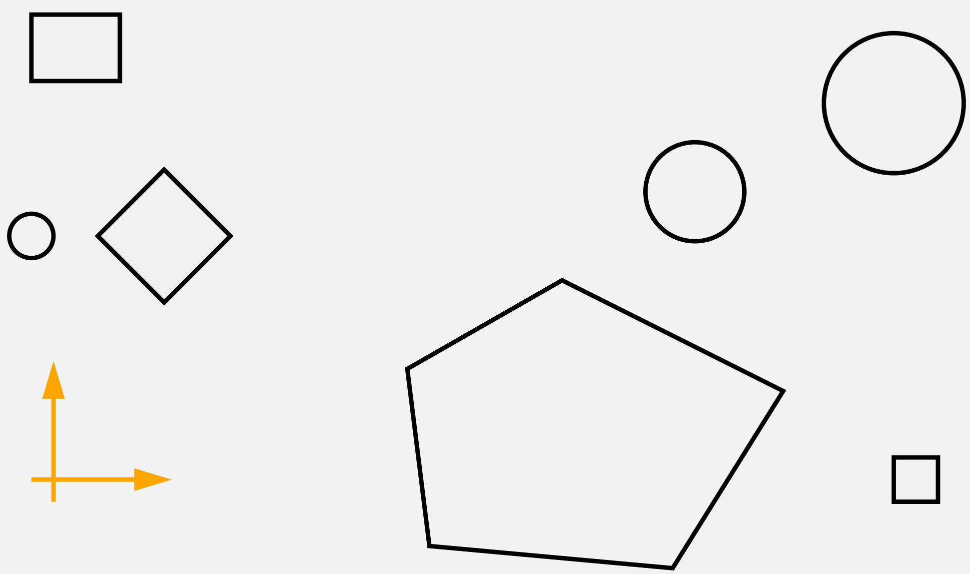
O set of orientations



Problem statement

O set of orientations

$S = \{S_0, \ldots, S_{n-1}\}$ set of n convex regions



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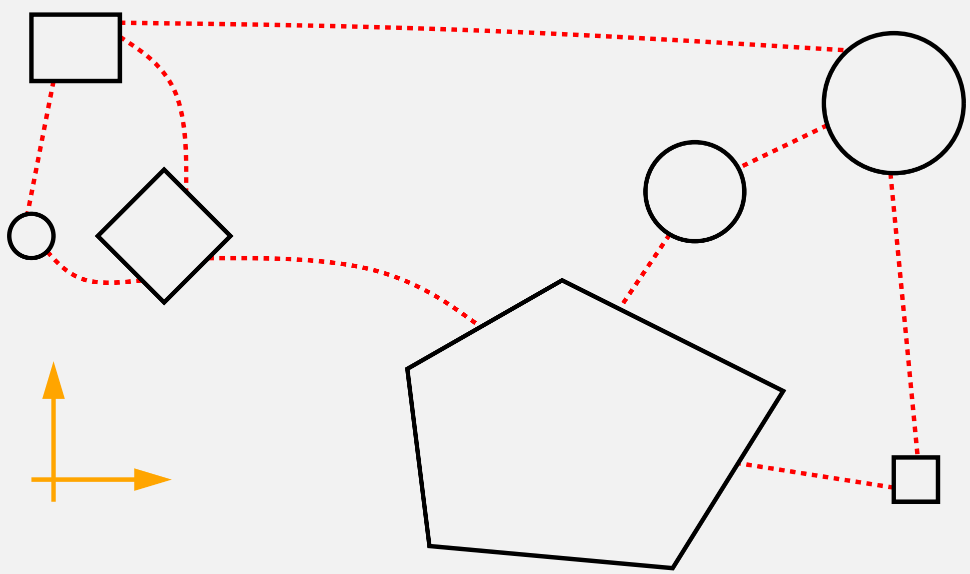

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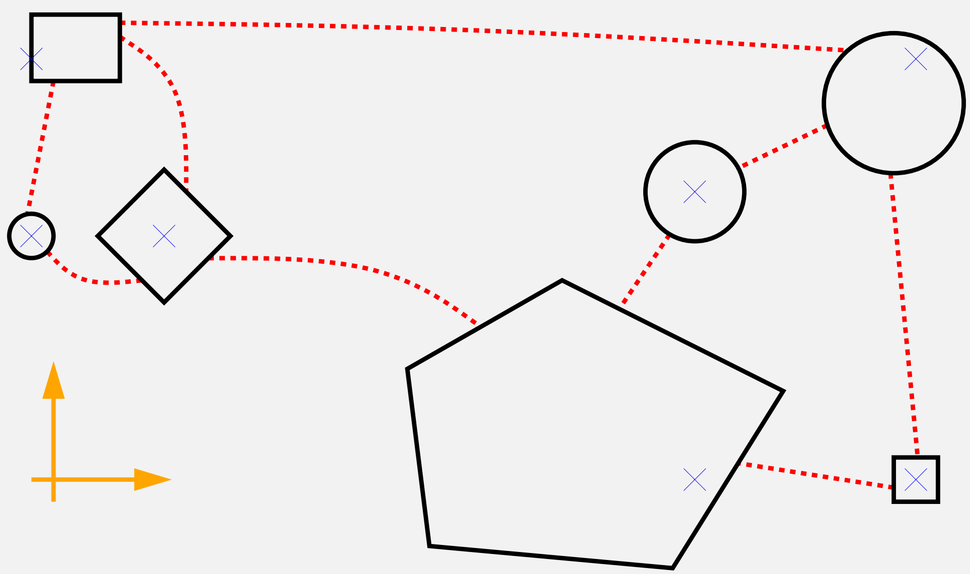
Goal: place $p_0 \in S_0, \ldots, p_{n-1} \in S_{n-1}$



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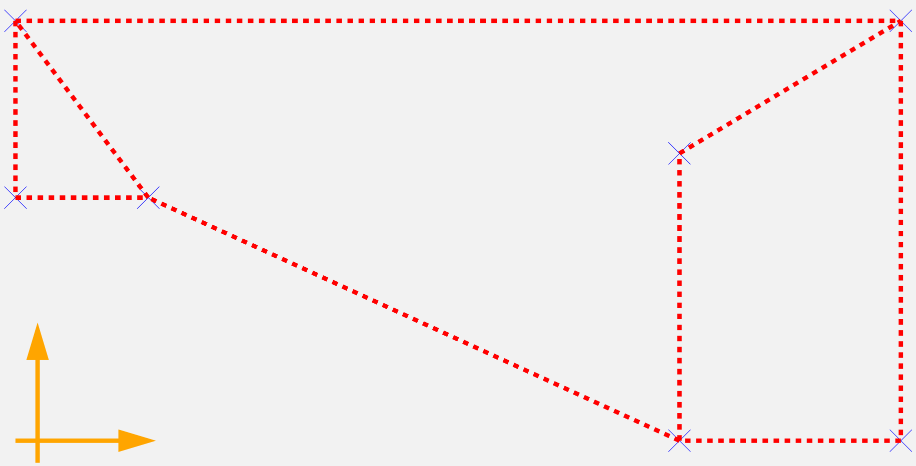
Goal: place $p_0 \in S_0, \ldots, p_{n-1} \in S_{n-1}$
maximizing the edges with orientation in O .

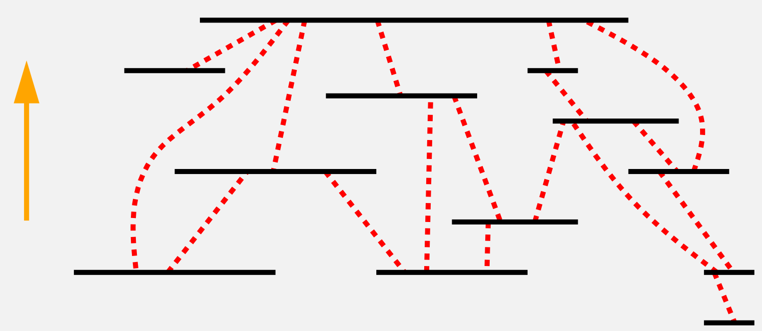


Results I: hardness

O the vertical orientation

$S = \{S_0, \ldots S_{n-1}\}$ horizontal segments

$G = (S, E)$ graph



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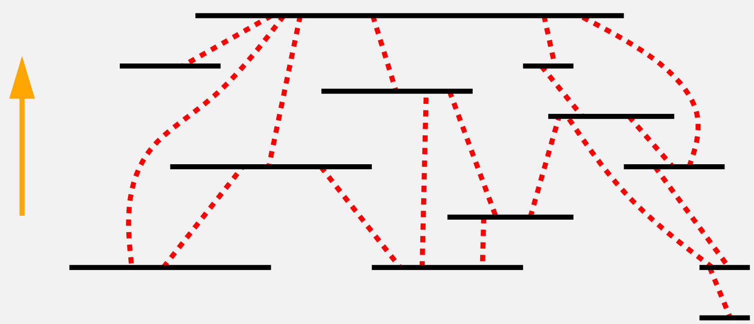

Theorem:

G planar $\Rightarrow$ NP-hard to maximize vertical edges

G non-planar $\Rightarrow$ NP-hard to $\frac{15}{16} + \epsilon$-approximate the maximum

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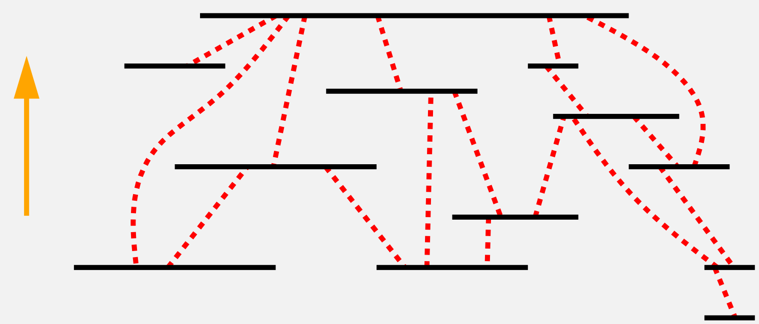

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Proof: reduction from E3-SAT.

Results I: hardness

$x_1, \ldots, x_t$ variables, m clauses

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$x_1 \vee x_i \vee \neg x_k$

Results I: hardness

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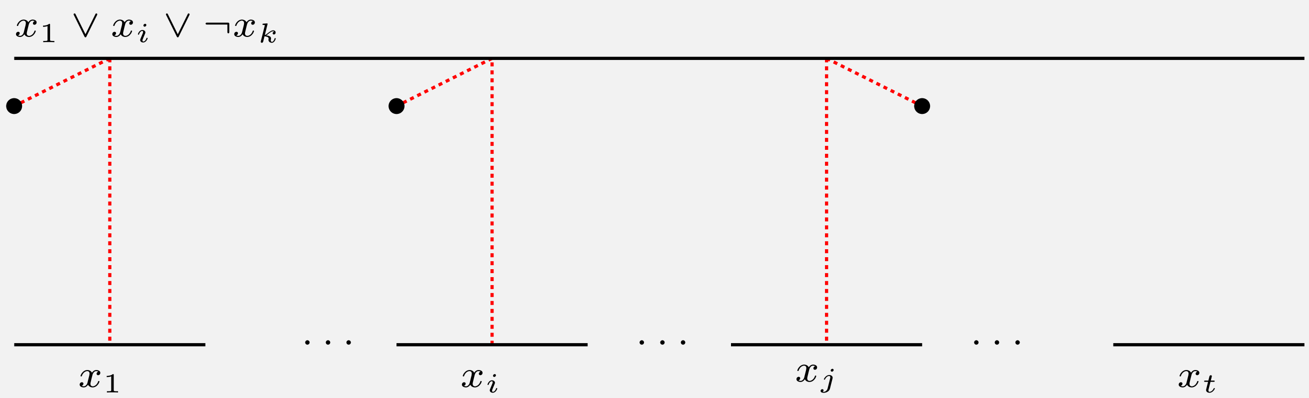

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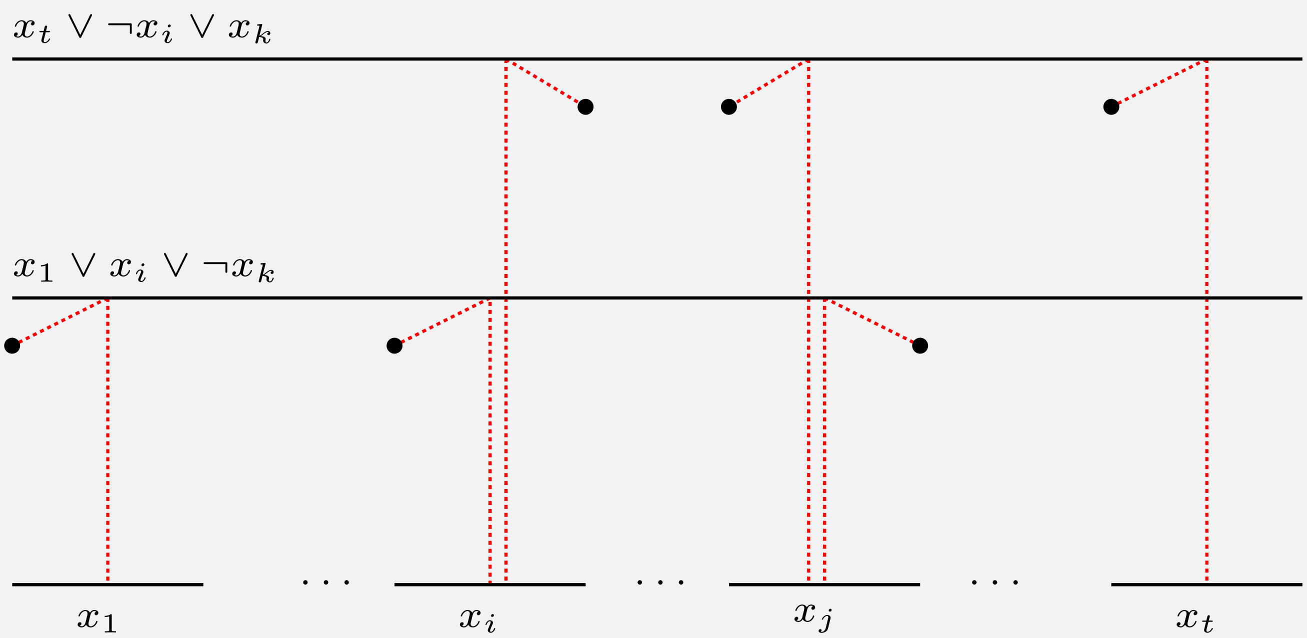
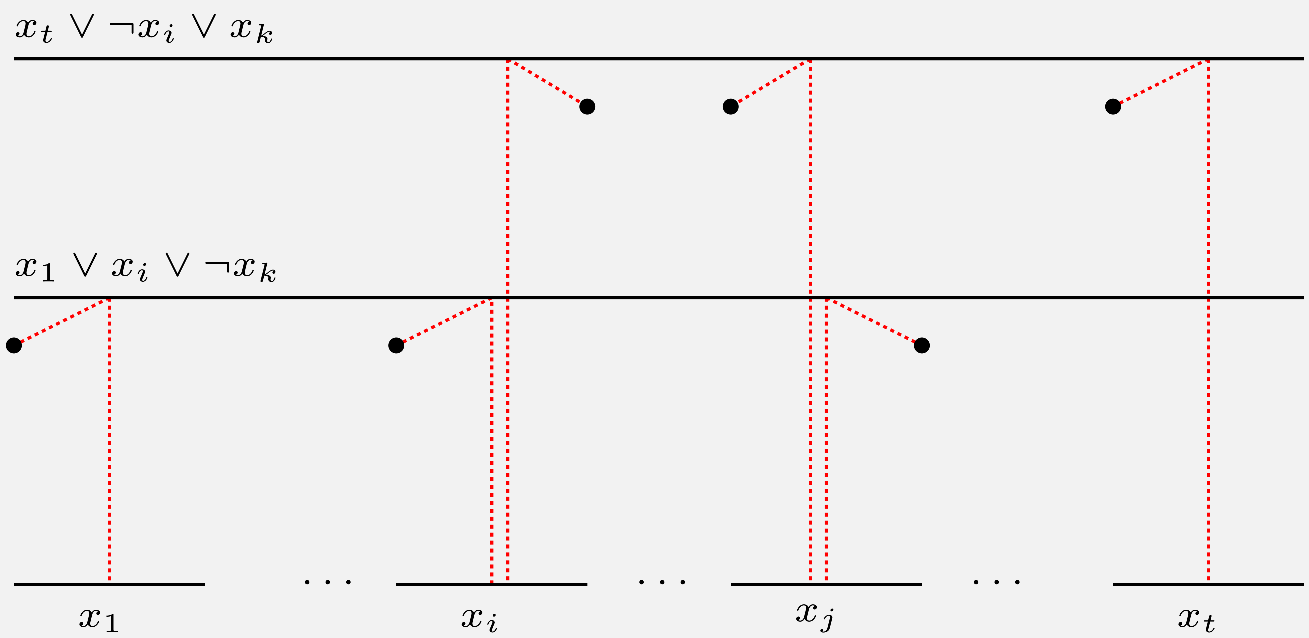


RESULTS I: HARDNESS

$x_1, \ldots, x_t$ variables, m clauses



Satisfiable clause $\Leftrightarrow$ 2 vertical edges

Non-satisfiable clause $\Leftrightarrow$ 1 vertical edge

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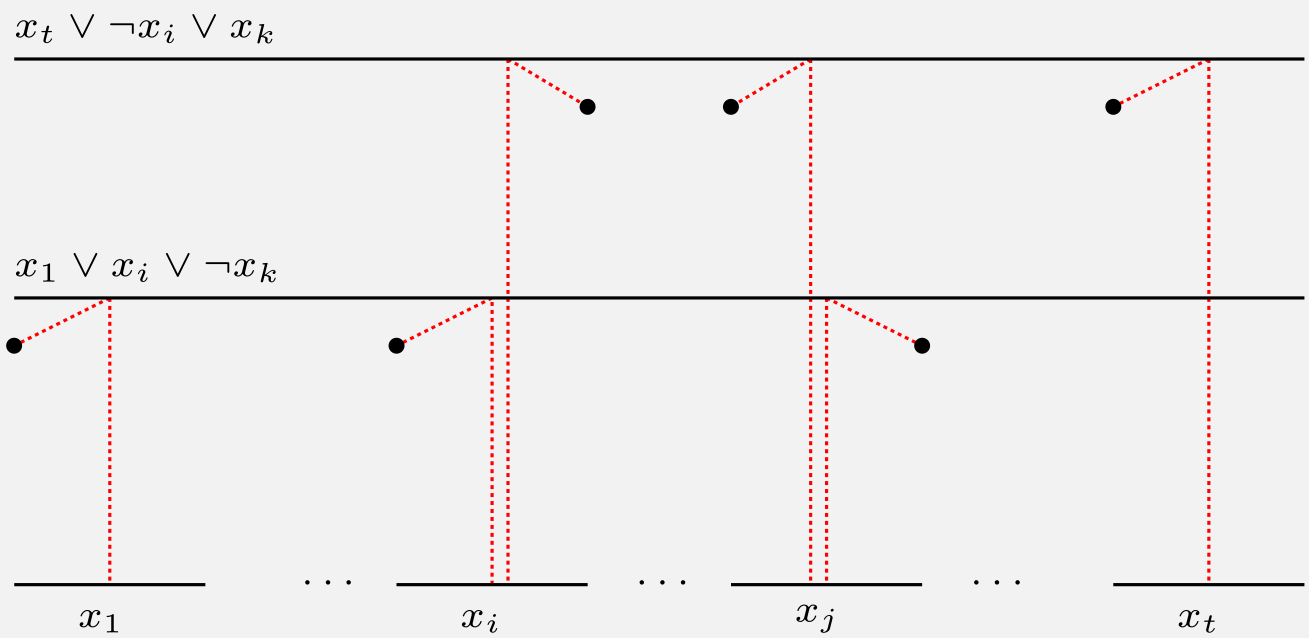


Satisfiable clause $\Leftrightarrow$ 2 vertical edges

Non-satisfiable clause $\Leftrightarrow$ 1 vertical edge

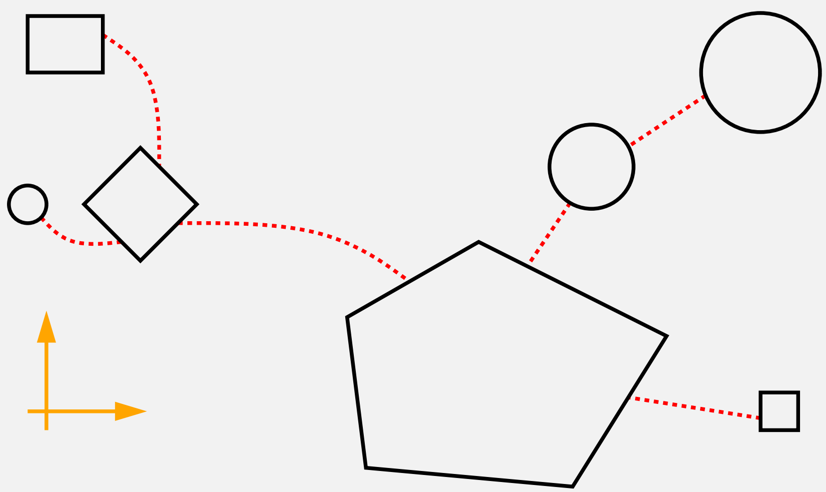
$\Rightarrow$ s clauses satisfiable $\Leftrightarrow$ $m + s$ vertical edges $\diamond$

Results II: PTAS for trees

O with two orientations

$S = \{S_0, \ldots, S_{n-1}\}$ convex regions

$T = (S, E)$ a tree



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$S = \{S_0, \ldots, S_{n-1}\}$ convex regions

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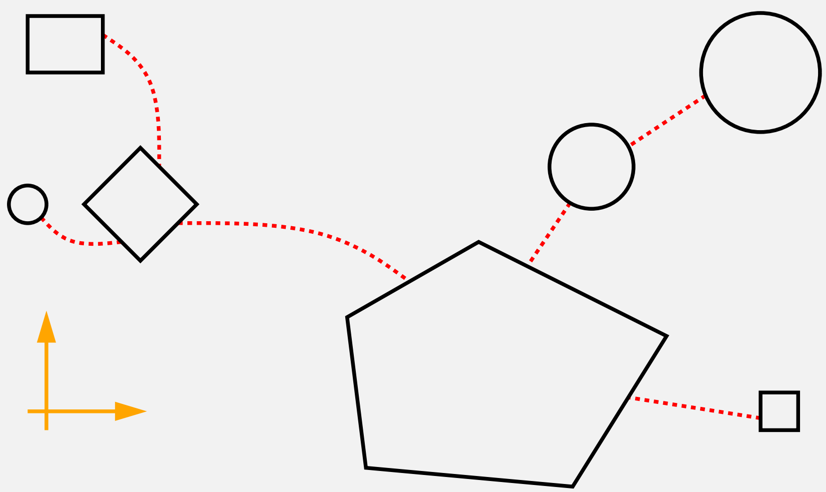
Lemma: If T has height k, we can compute a placement maximizing the alignments in $O(9^k n^2)$ time.



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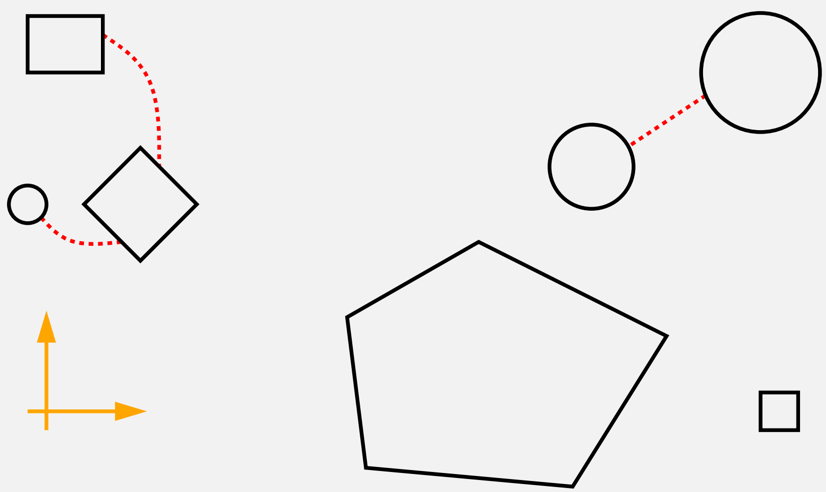

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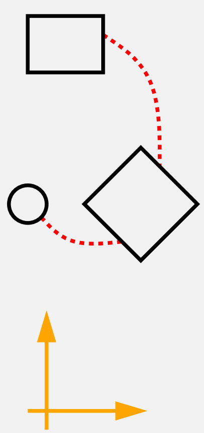

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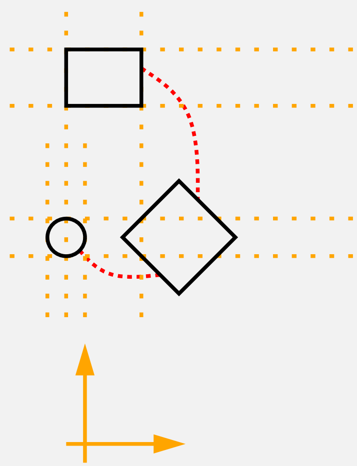

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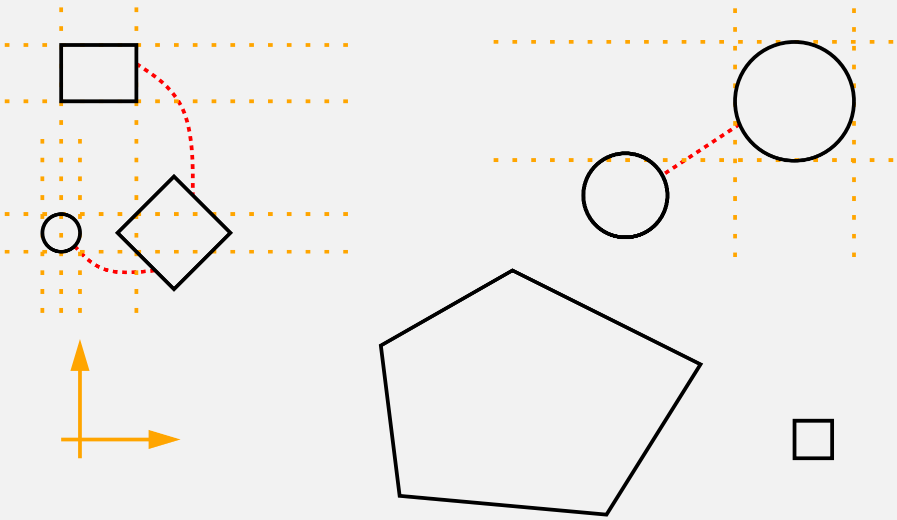

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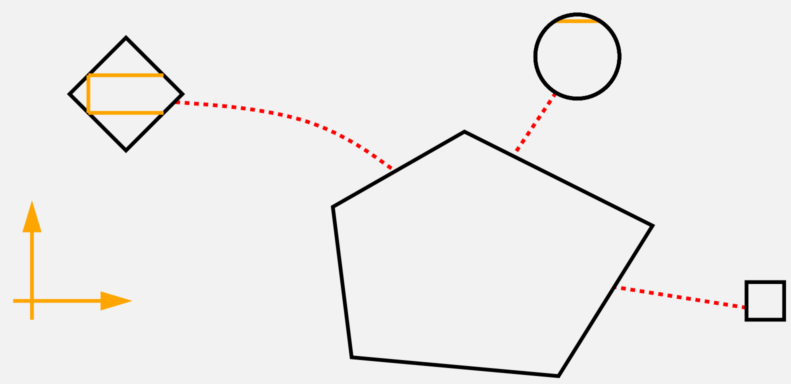

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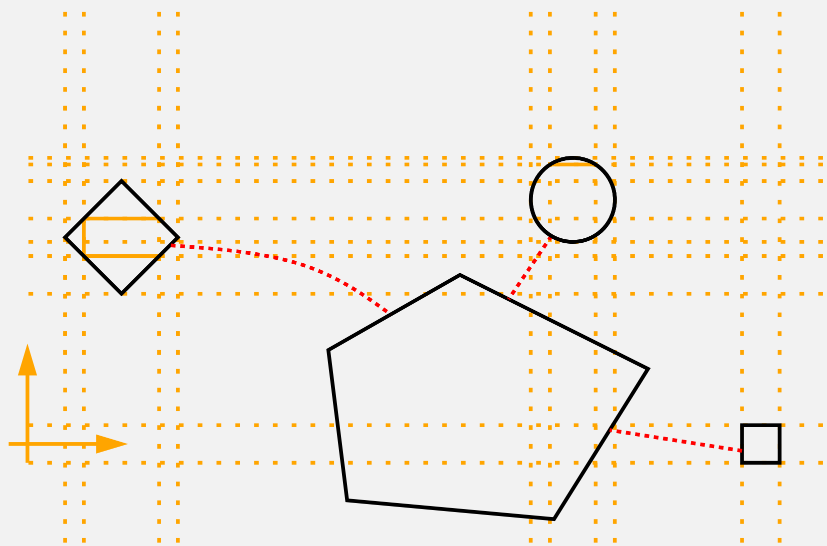

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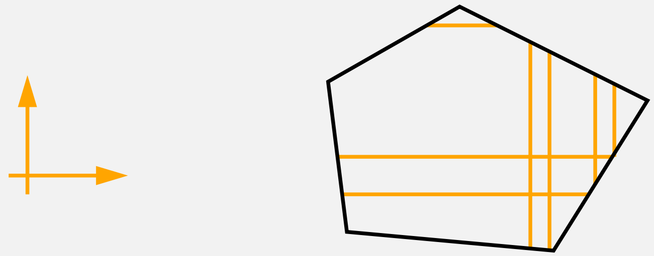

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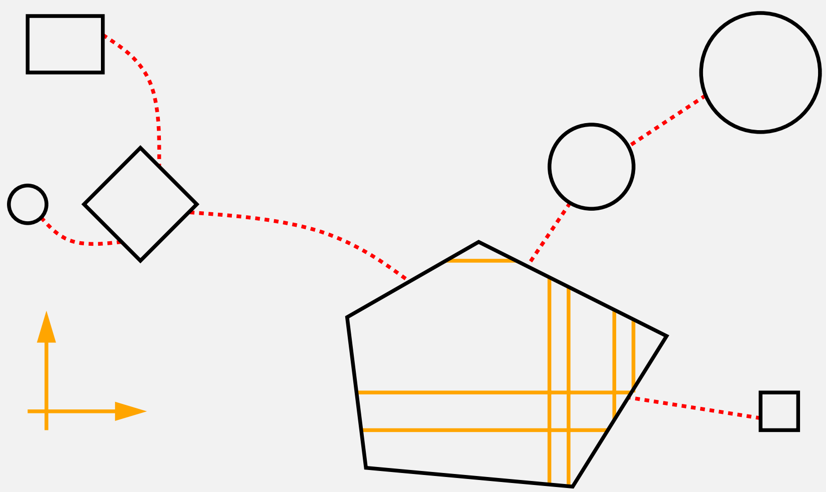

Results II: PTAS for trees

O with two orientations, $S = \{S_0, \dots, S_{n-1}\}$ convex regions
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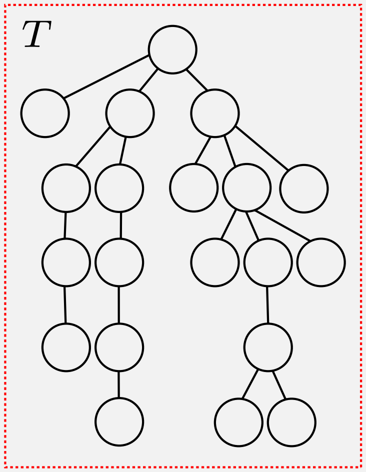
Theorem: Given $k > 0$, we can compute a $\frac{k}{k+1}$-approximation of the best alignment in $O(k9^k n^2)$ time.

Proof: Shifting technique of Hochbaum and Maass. (k=2)



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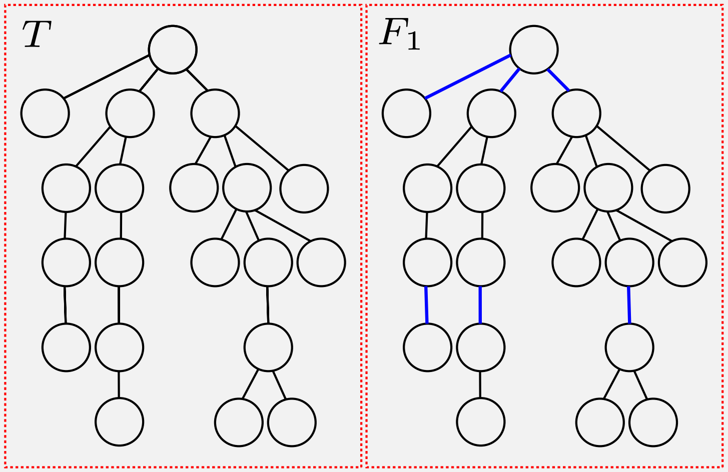

Compute $\max(F_1)$.

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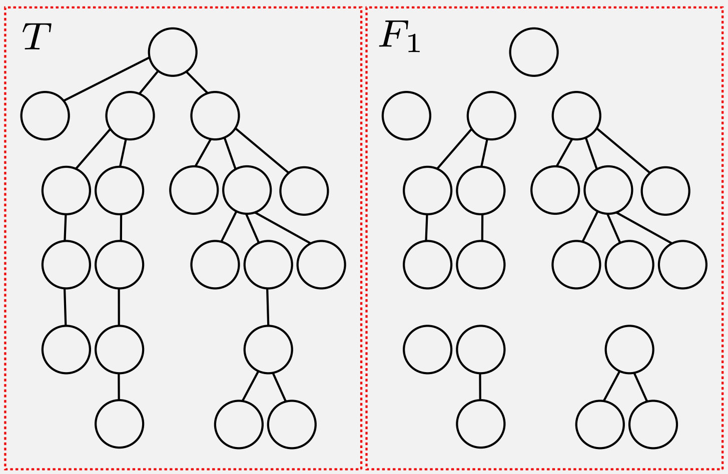

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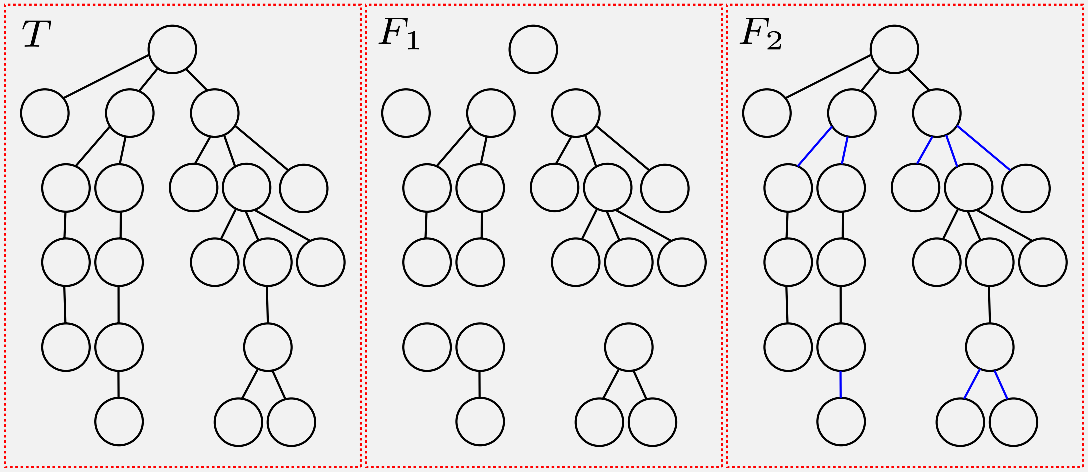

Compute $\max(F_1), \max(F_2)$.

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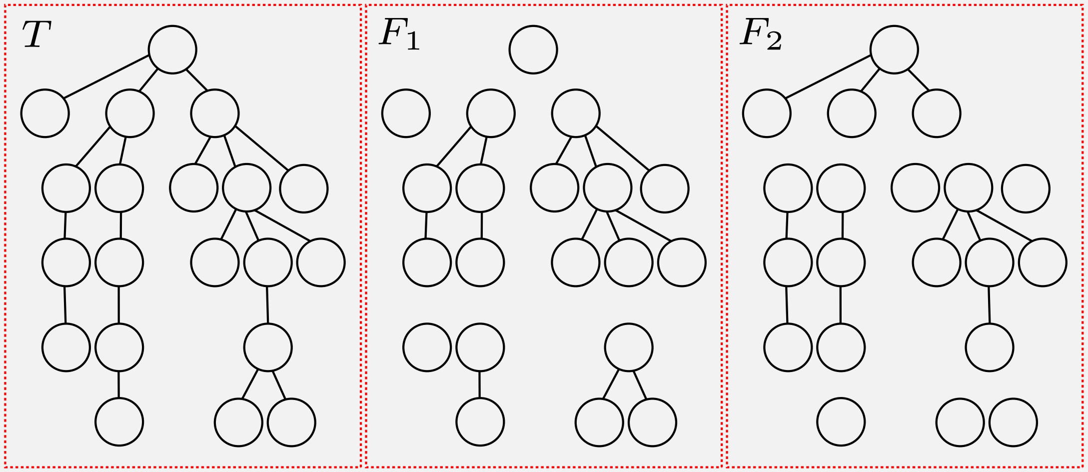

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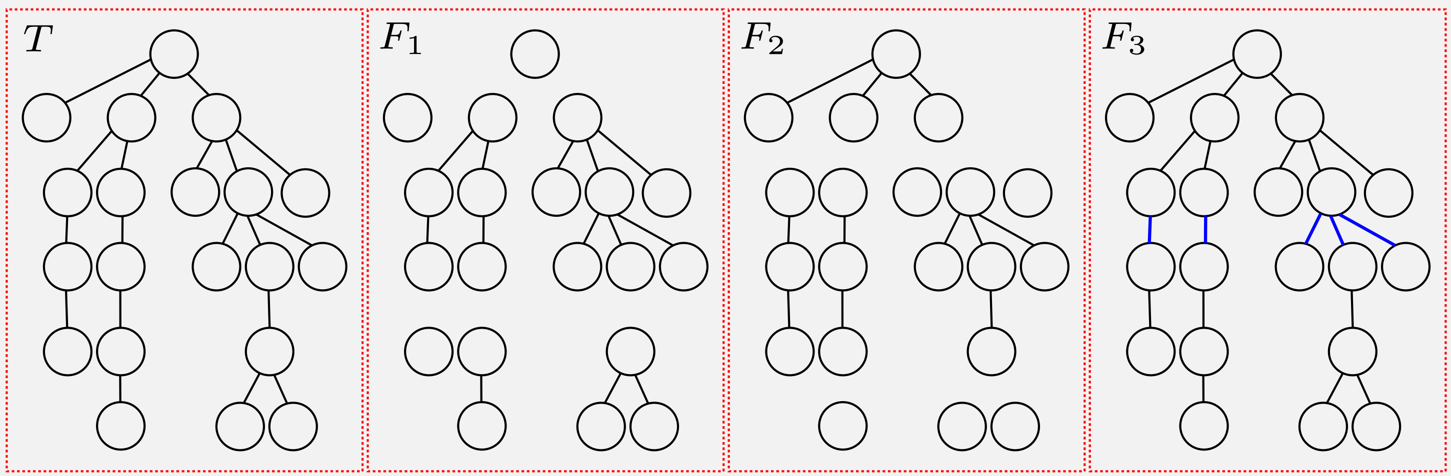

Compute $\max(F_1), M(F_2), \max(F_3)$, and take the maximum.

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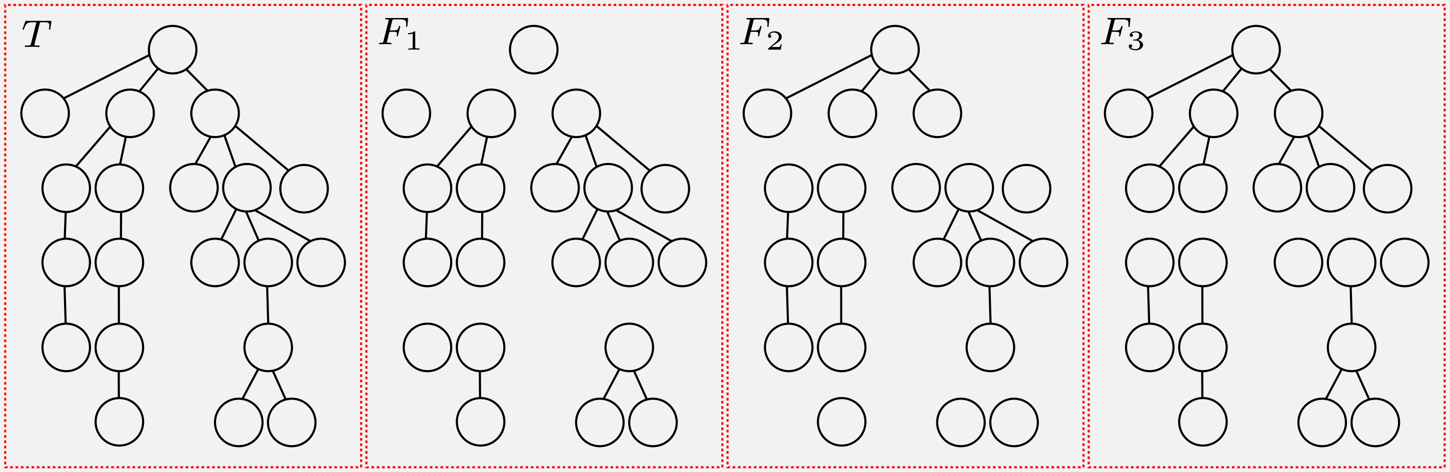

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Results III: planar graphs

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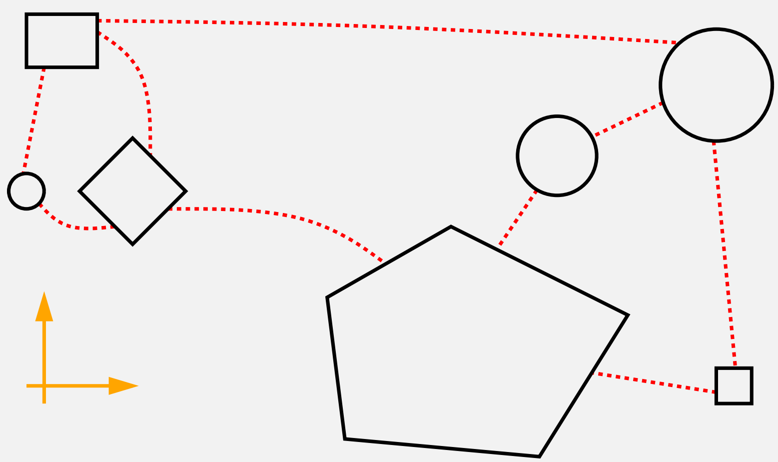

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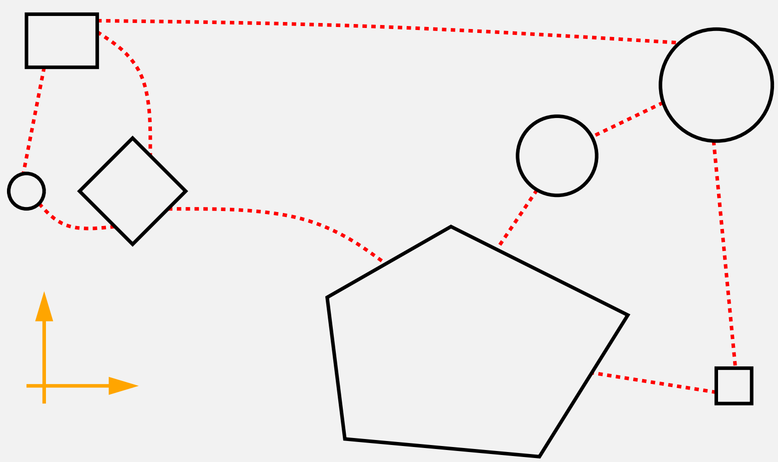
Theorem: Given $k > 0$, we can compute a $\frac{k}{3(k+1)}$-approximation of the best alignment in $O(k9^k n^2)$ time.

Proof: Decompose G into three edge-disjoint forests + Results II



Results IV: PTAS for axis-aligned problems

O orientations of the coordinate axes

$S = \{S_0, \ldots, S_{n-1}\}$ n disjoint, axis-parallel rectangles

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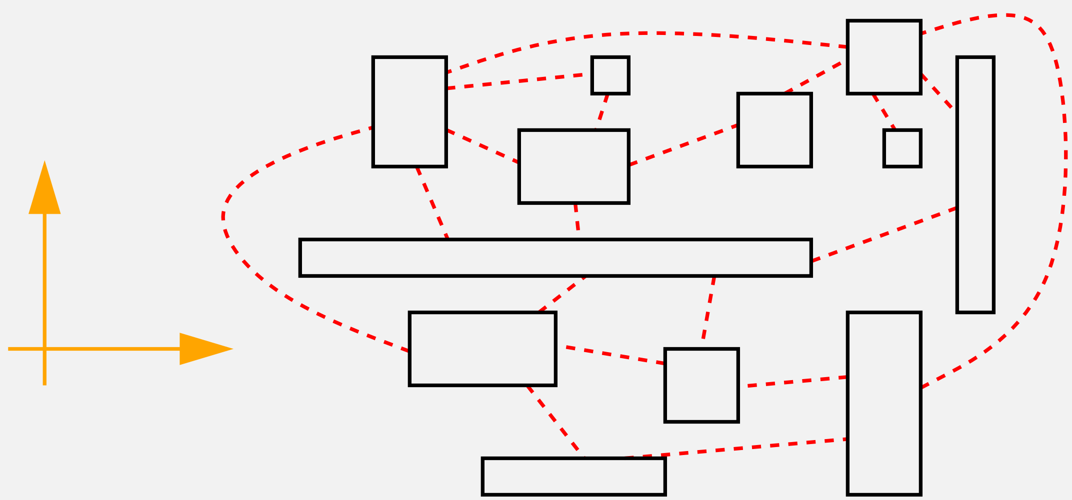

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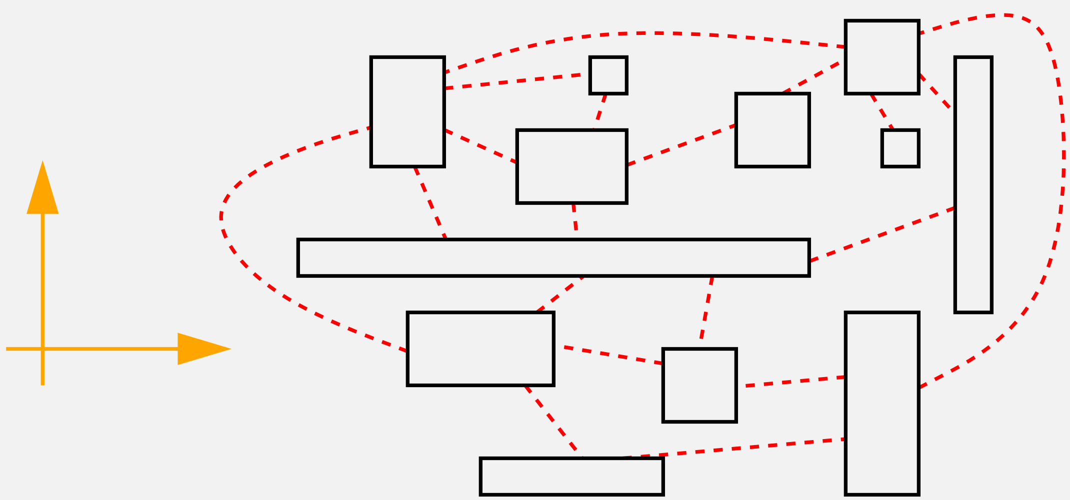

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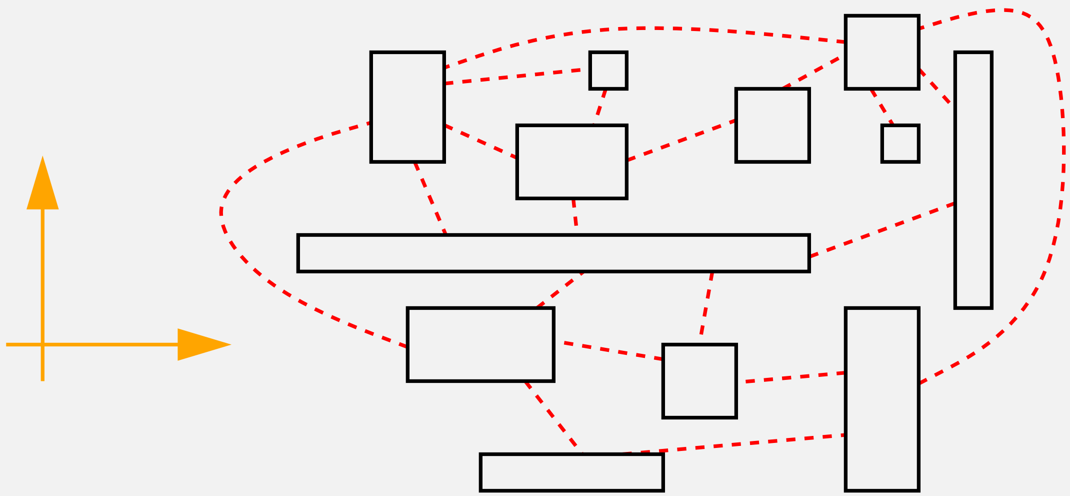
Proof: Baker's slices + shifting technique.



What did I explain?

- Aligning problem: a geometric optimization problem with applications in cartography
- Aligning is hard
- PTAS for trees
- $(\frac{1}{3} - \epsilon)$-approximation for planar graphs
- PTAS for "planar, axis-aligned problems"