APPROXIMATION ALGORITHMS for SPREADING POINTS

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THE ABSTRACT PROBLEM

- (X, d) metric space

  $S_1, \dots, S_n \subseteq X$

- t-distant representatives: [Fiala et. al '02]

  $$\begin{matrix} x_1, & \dots, & x_n \\ \cap & & \cap \\ S_1, & \dots, & S_n \end{matrix} \quad \text{s.t.} \quad d(x_i, x_j) \geq t \quad \forall i \neq j$$

- Optimization problem:

  maximize t that admits t-representatives

OUR PROBLEMS

- $(\mathbb{R}^2, L_\infty)$

  $S_1, \ldots, S_n$ disks (squares in L_∞)

  Choose $p_i \in S_i$ maximizing the distance of the closest pair.

- $(\mathbb{R}^2, L_2)$

  $S_1, \ldots, S_n$ (congruent) disks

  Choose $p_i \in S_i$ maximizing the distance of c.p.

PREVIOUS/RELATED WORK

- Fiala et. al ~~>
- Baur & Fekete ~~>

NP-hard to get PTAS

- Baur & Fekete: Choose K points inside a polygonal region maximizing the distance of c.p.
  NP-hard to get a PTAS. $(3/2)$- approximation in L_∞
- Packing problems – Map labelling problems.

OUR RESULTS

SPACE	REGIONS	RESULT
$(\mathbb{R}^2, L_\infty)$	disks	2-approximation, $O(n\sqrt{n}\log^2 n)$ *
$(\mathbb{R}^2, L_2)$	disks	$\left(\frac{8}{3}\right)$-approximation, $O(n^2)$
	congruent disks	(~ 2.23)-approximation, $O(n^2)$

* today's aim

OUTLINE

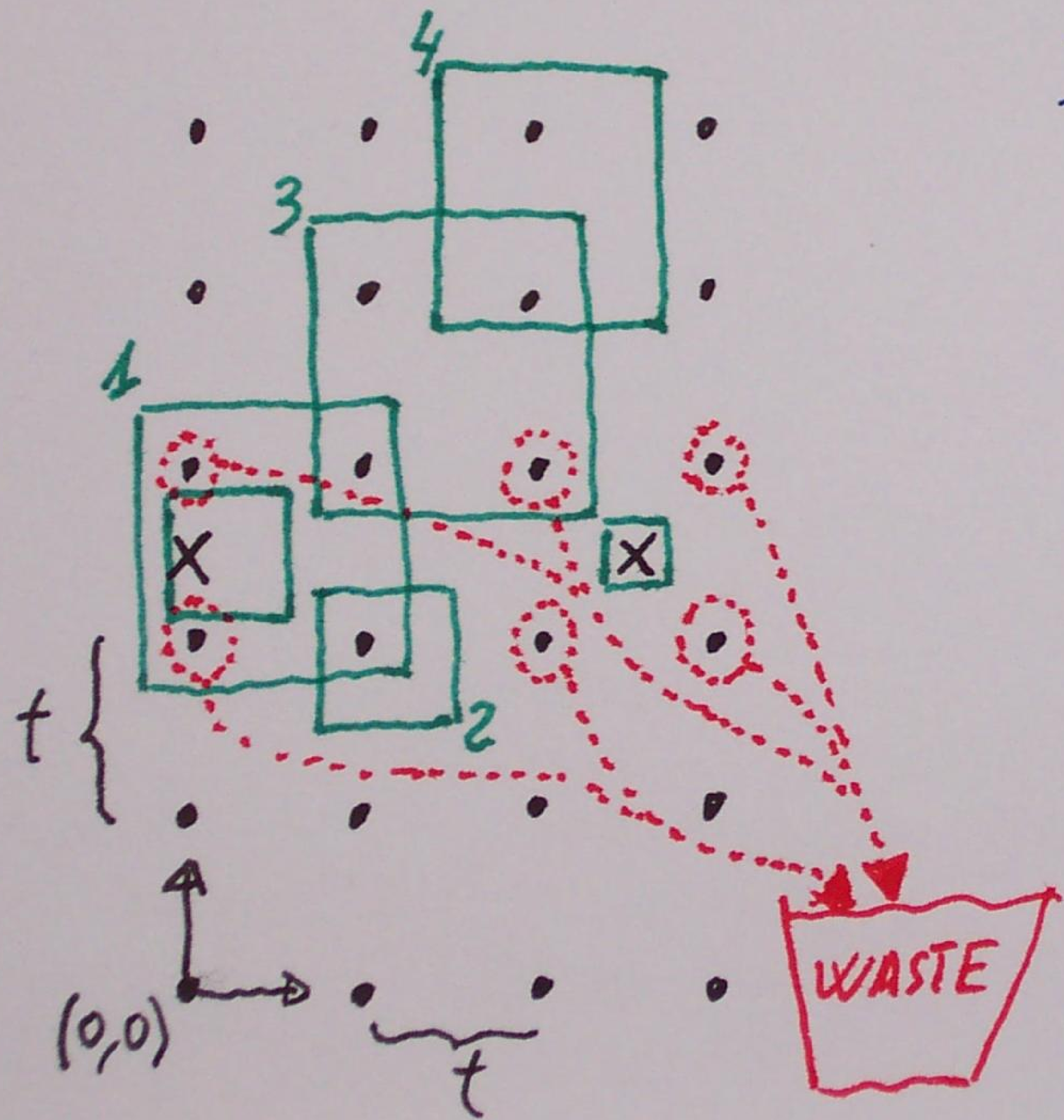
- ✓ The abstract problem / Our problem
- ✓ Previous/Related work
- ✓ Our results
- Approximate placement algorithm
- Efficiency of approximate placement
- Decision vs Optimization

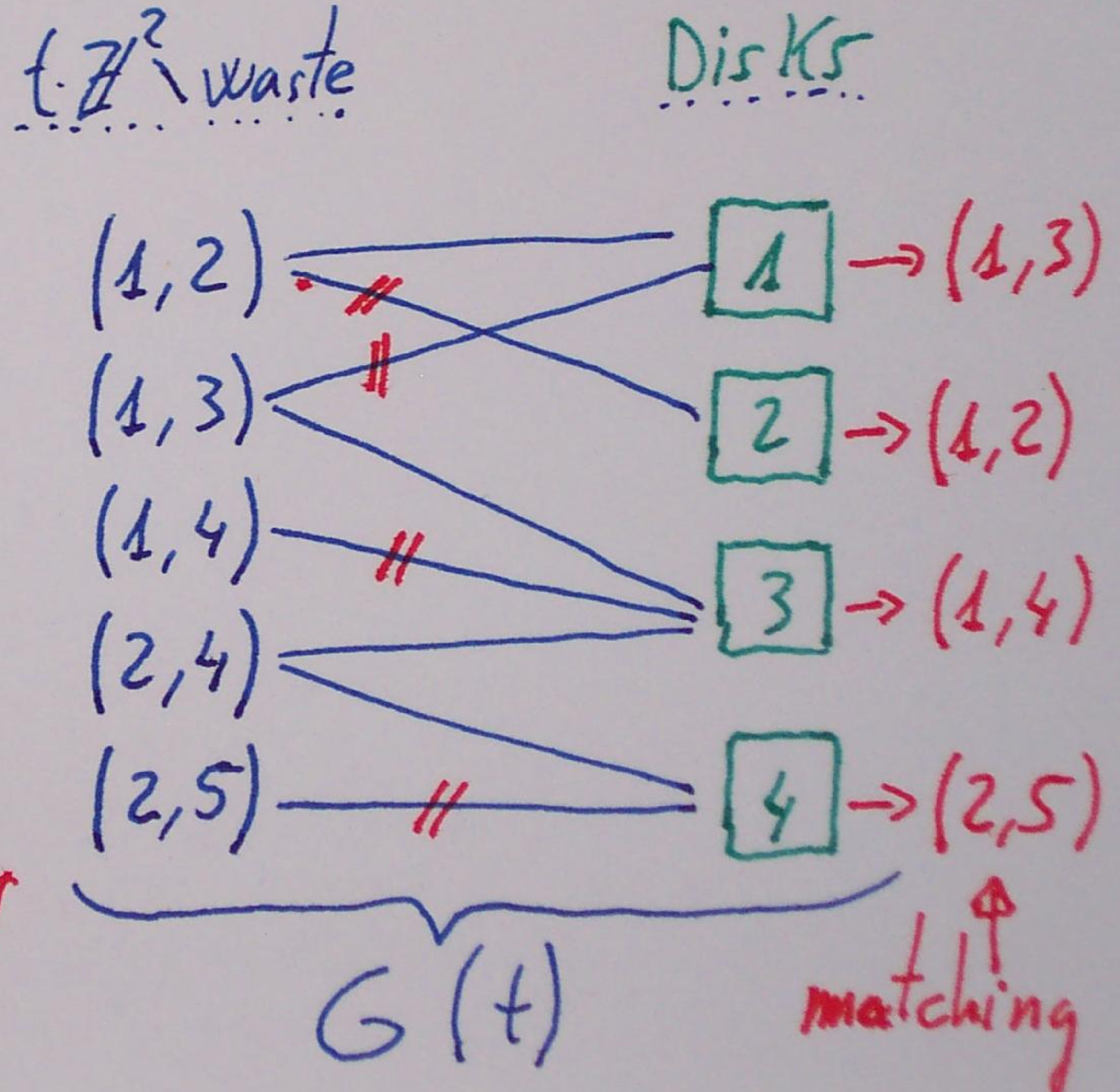
L_∞

2-approximation

PLACEMENT ALGORITHM (t)

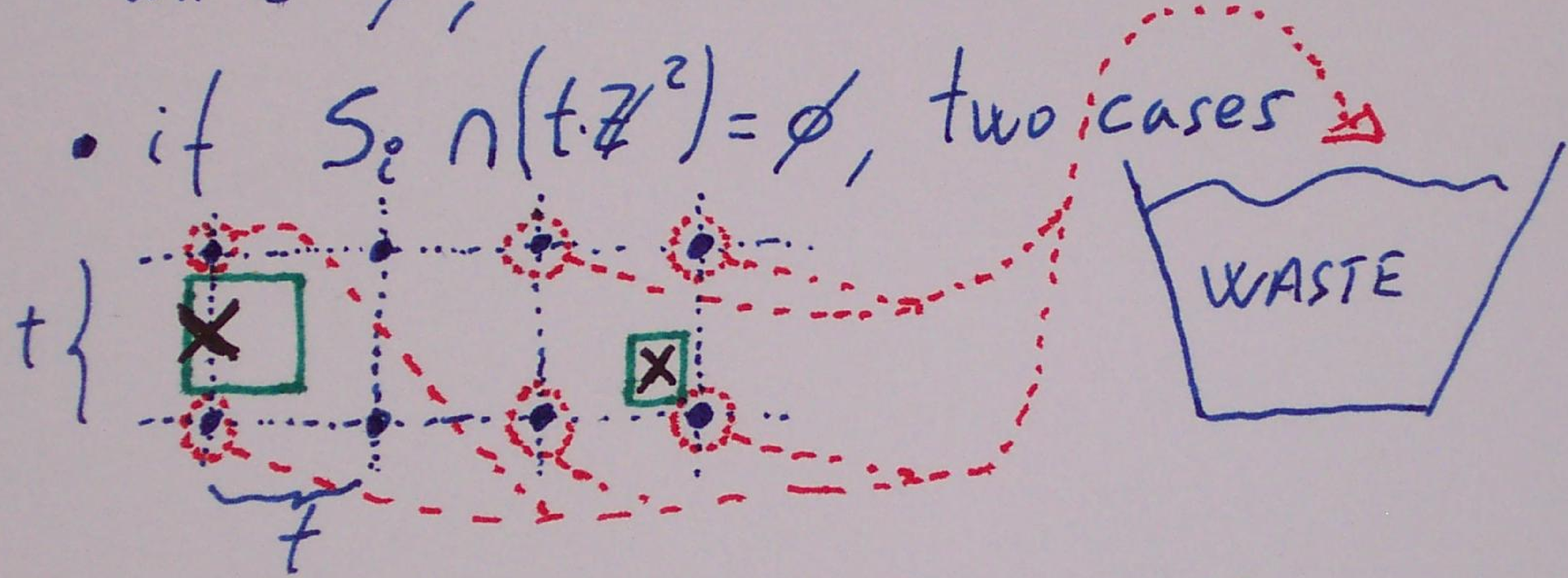
Idea: Try to place points at $t \cdot \mathbb{Z}^2$.





PLACEMENT ALGORITHM (t)

- WASTE := $\emptyset$;
- if $S_i \cap (t \cdot \mathbb{Z}^2) = \emptyset$, two cases



- construct bipartite $G(t) = \left(\{S_1, \dots, S_n\} \,\dot\cup\, (t \cdot \mathbb{Z}^2 \setminus \text{WASTE}),\ \{(p, S_i) \mid p \in S_i\}\right)$,
- matching in $G(t)$ $\hookrightarrow$ Placement.

PLACEMENT ALGORITHM (t)

Let t^* be the optimal solution.

Lemma: If $2t \leq t^*$ then:

- $G(t)$ has a matching $\rightsquigarrow$ we get placement;
- c.p. of placement is t apart.

We may have a huge graph $G(t)$. !

↳ Modify algorithm

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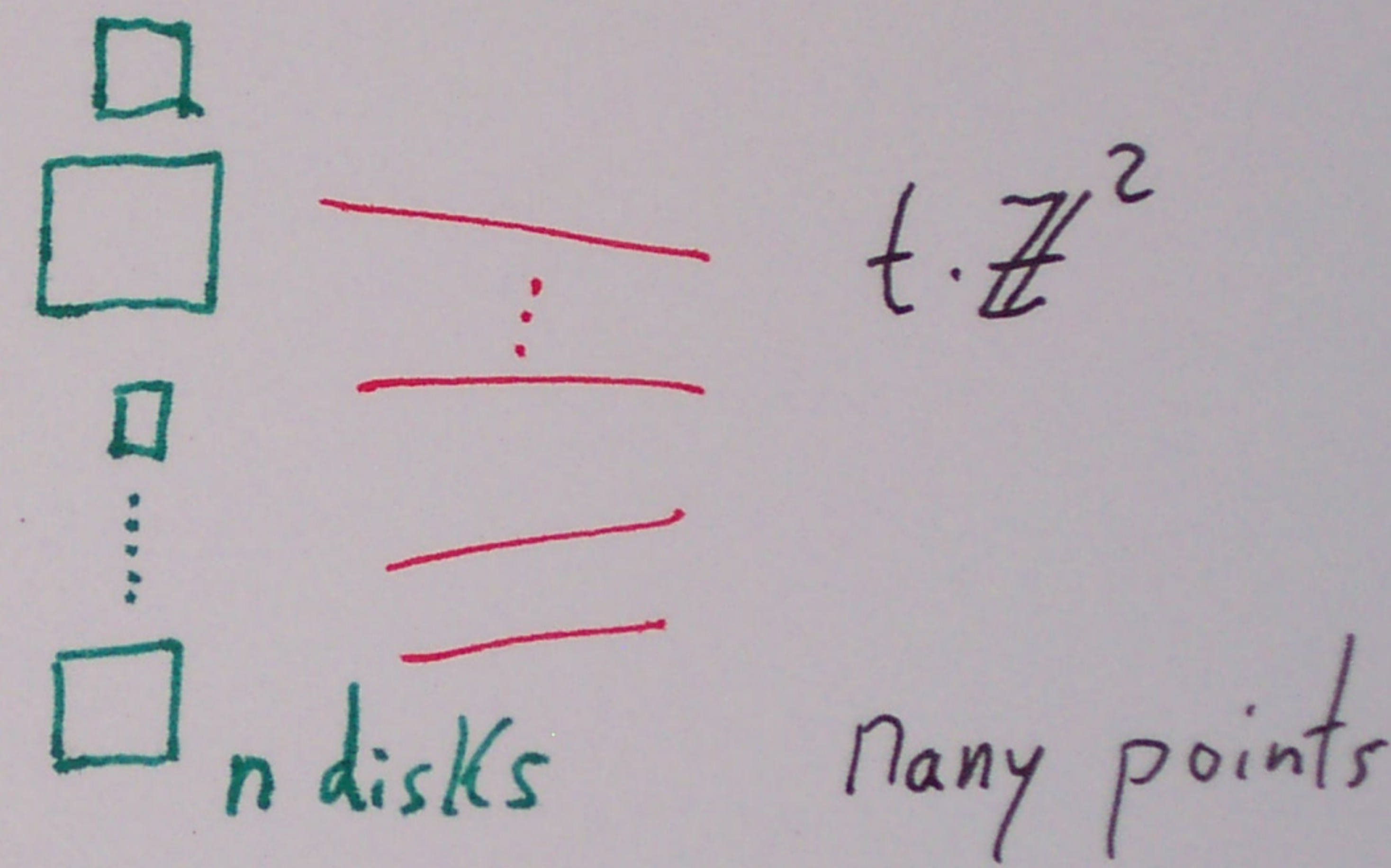
Efficiency Placement (t)

$G(t)$ looks like



⇝ Each disk needs degree $\leq n$

⇝ For each disk, take $5n$ points $(|\text{WASTE}| \leq 4n)$

⇝ $G(t)$ has $\boxed{n} + O(n^2)$ vertices.

Lemma: Placement $\in$ P. $\boxtimes$

EFFICIENCY PLACEMENT (t)

Lemma: PLACEMENT (t) can be done in $O(n\sqrt{n}\log n)$ time.

proof: Two steps:

- considering $O(n\sqrt{n})$ points in total (1)
- geometry helps for matching (2)

  T disks, P points $\rightsquigarrow O(P\log P + \sqrt{T}\cdot T\cdot\log P)$

$\rightsquigarrow O(n\sqrt{n}\log n)$ time $\boxtimes$

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L_∞
2-approximation

DECISION $\rightsquigarrow$ OPTIMIZATION

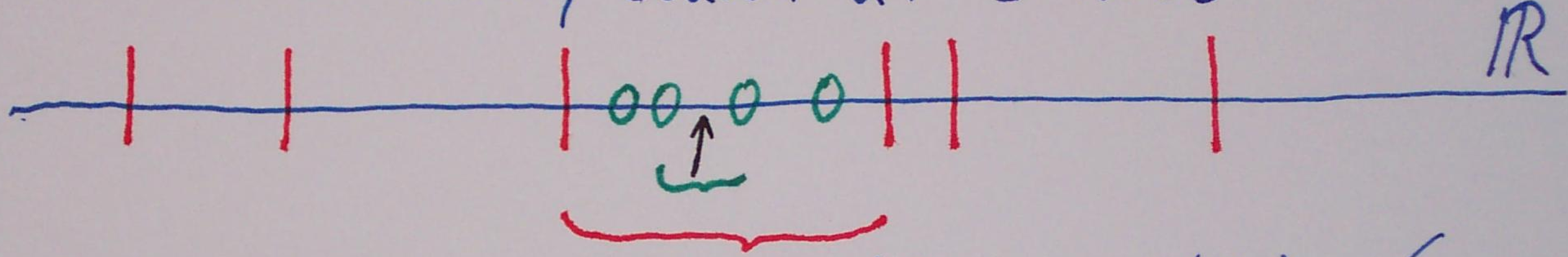
Lemma: If $2t \leq t^*$, PLACEMENT(t) ✓

Corollaries: How to prove PLACEMENT(t) is 2-approxi

- If PLACEMENT(t) ✓ and PLACEMENT(t) ✗ after translation $\rightsquigarrow$ $t > t^*/2$
- If $t > t'$, PLACEMENT(t) ✓, PLACEMENT(t') ✗ $\rightsquigarrow$ $t > t^*/2$
- If PLACEMENT(t) ✓ and PLACEMENT$(t+\varepsilon)$ ✗ for infinitessimal $\varepsilon > 0$ $\rightsquigarrow$ $t \geq t^*/2$

DECISION $\rightsquigarrow$ OPTIMIZATION

Main idea: binary search at 3 levels



Invariant: $t_1 < t_2$ s.t. $\begin{cases} \text{PLACEMENT}(t_1) \ \checkmark \\ \text{PLACEMENT}(t_2) \ \times \end{cases}$

Objective: $t_1 < t_2$ like above and

$$G(t_1 + \varepsilon) \cong G(t_2)$$

$\Rightarrow$ t_1 is a 2-approximation

DECISION $\rightsquigarrow$ OPTIMIZATION

Theorem: In $(\mathbb{R}^2, L_\infty)$, regions are disks, we have a 2-approximation in $O(n\sqrt{n}\log^2 n)$.

proof: $3 \times O(\log n) \times O(n\sqrt{n}\log n)$ time

we find $t_1 < t_2$ s.t.

PLACEMENT (t_1) ✓

PLACEMENT (t_2) ✗

$$G(t_1+\varepsilon) \cong G(t_2)$$

$\boxtimes$

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OPEN PROBLEM

- If S_i's disjoint $\rightsquigarrow$ CENTERS is 2-approximation

Improve it !

Get a T-approximation with $T < 2$
for disjoint disks.