Schematization of Road Networks

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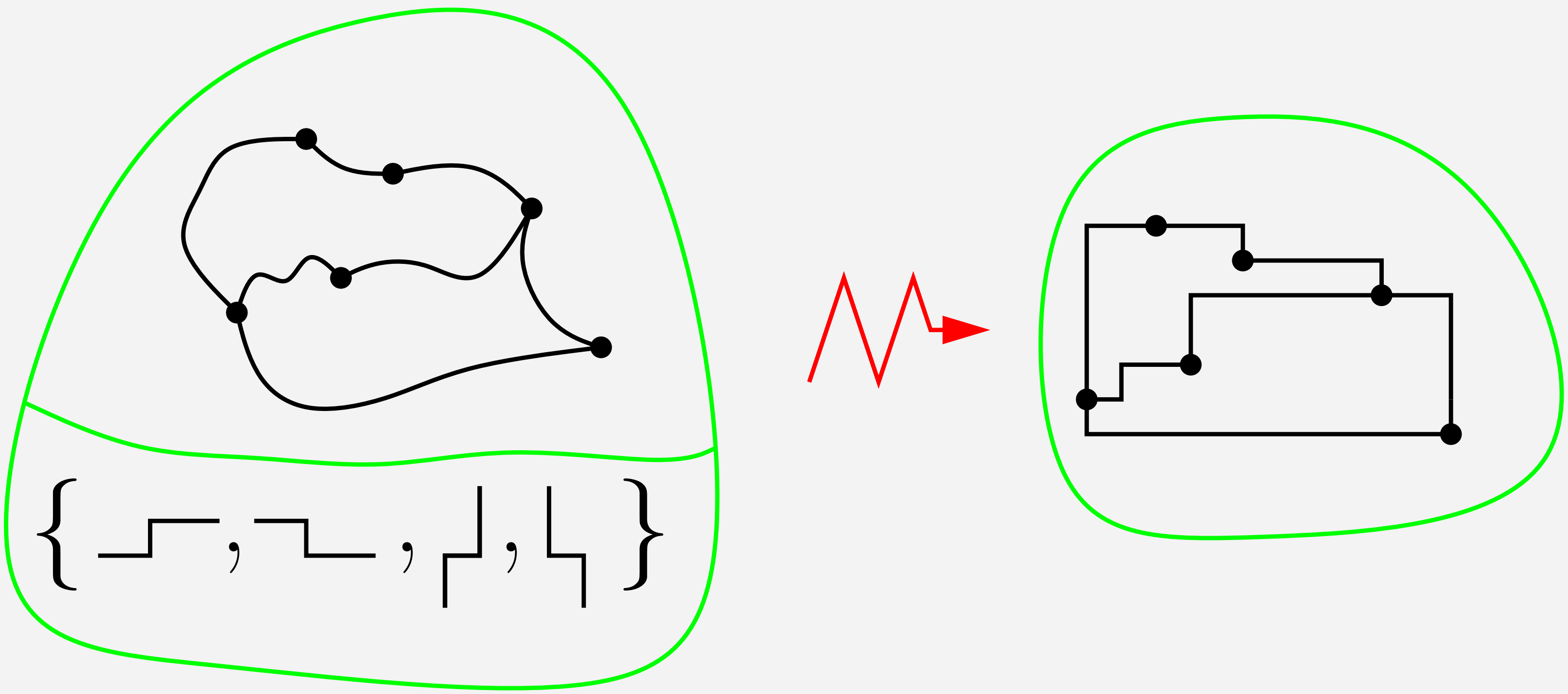
What is a schematic map?

Drastic simplification and conceptualisation of connections.

Framework in our approach



The endpoints of the connections are fixed.

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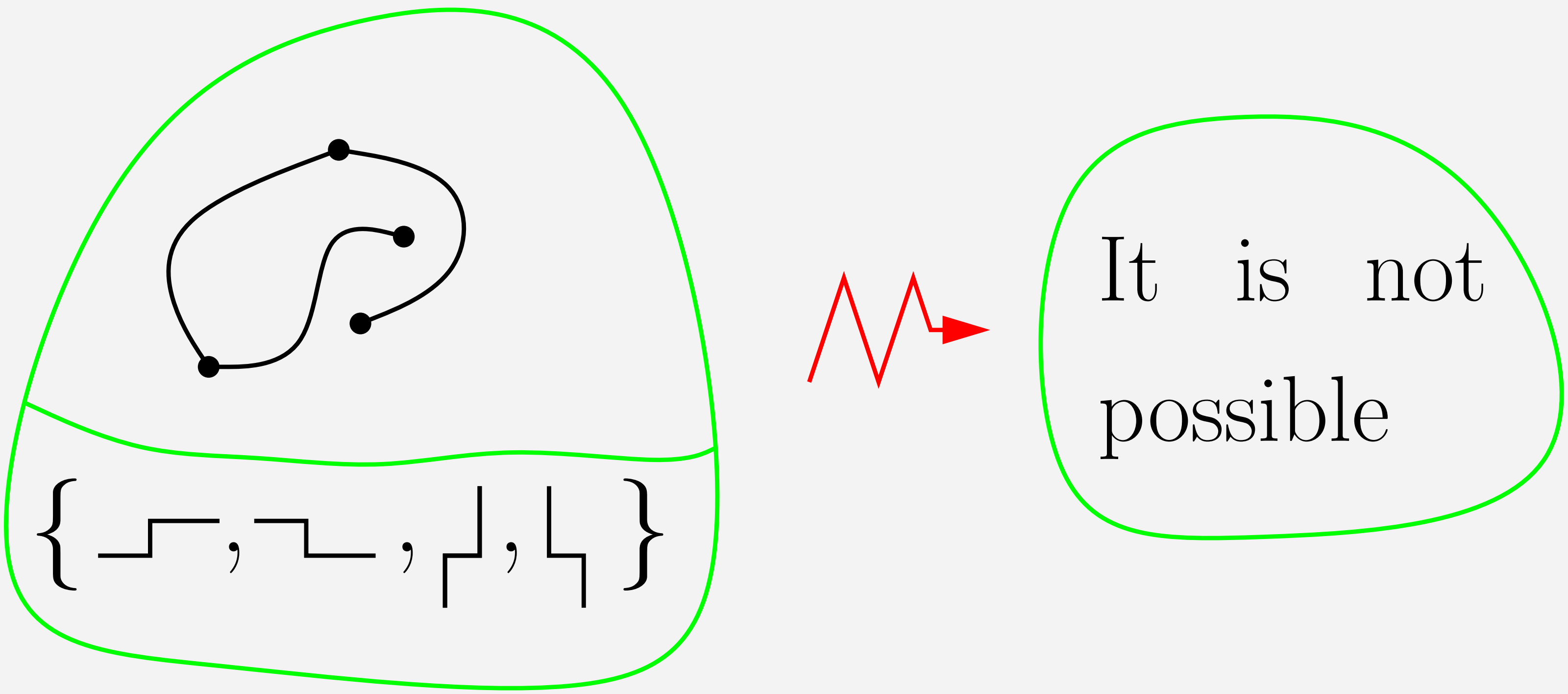

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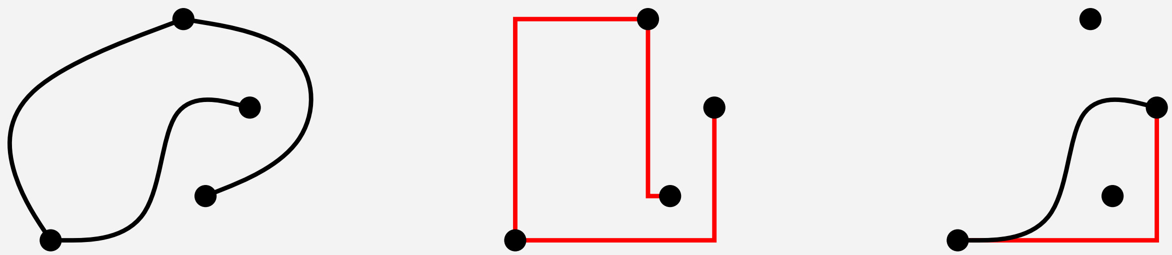
Schematic map: deformation of the original

We want to keep some properties in the schematic map...



$P_M \equiv$ Endpoints of map M.

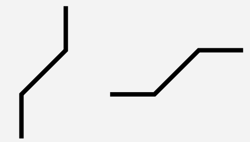
Definition: M and M' are equivalent iff $P_M = P_{M'}$ and each path of M can be continuously deformed in $\mathbb{R}^2 \setminus P_M$ to a path of M'.

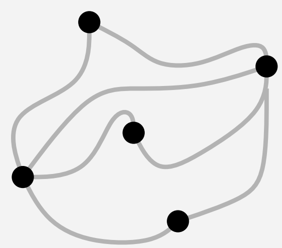
Problem: Given M, find an equivalent M' with 'nice' connections.

Idea behind everything

How to construct the schematic map?

Place the schematic connections in a top to bottom fashion, each connection topmost.

Allowed connections: 



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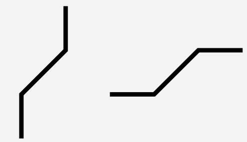
Allowed connections: 

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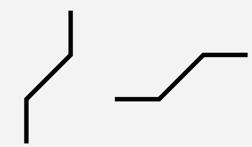

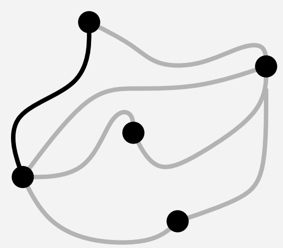

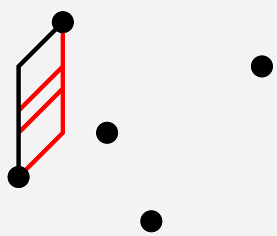

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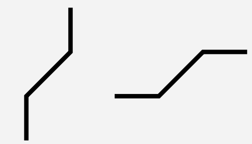

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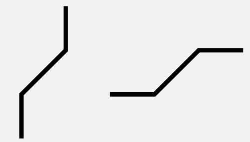
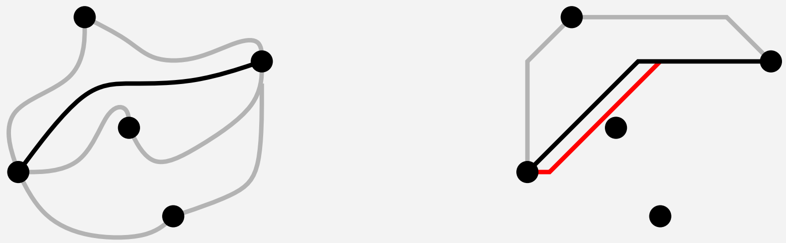
Allowed connections: 



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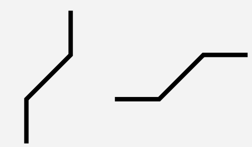
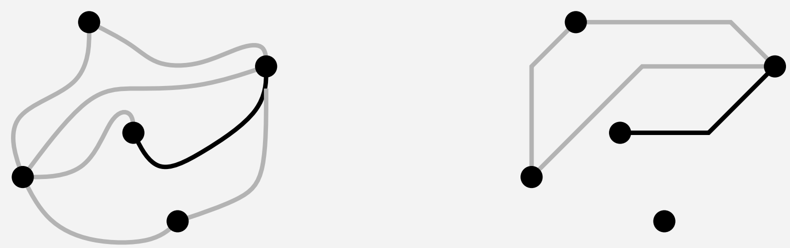
Allowed connections: 



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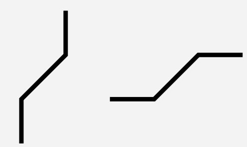
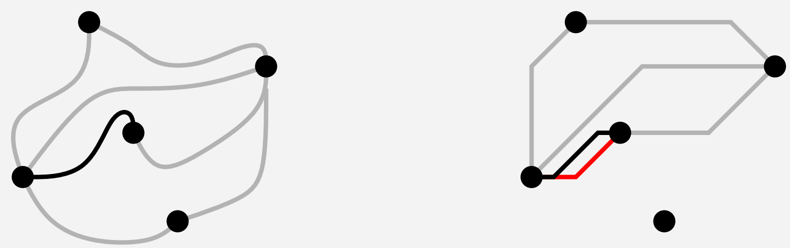
Allowed connections: 



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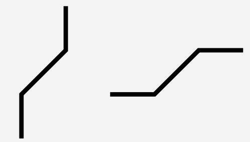
Allowed connections: 

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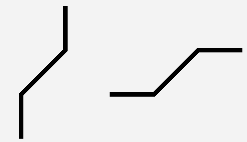
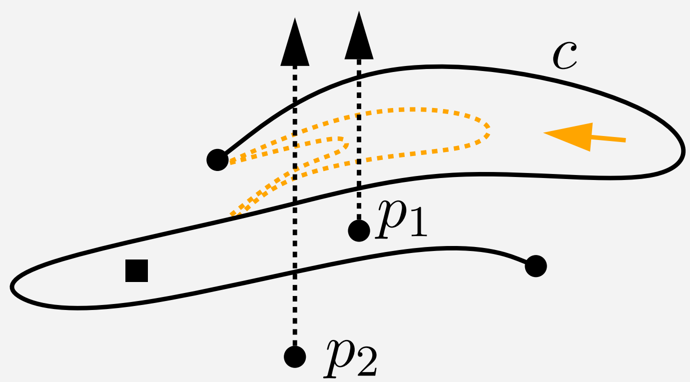
Allowed connections: 

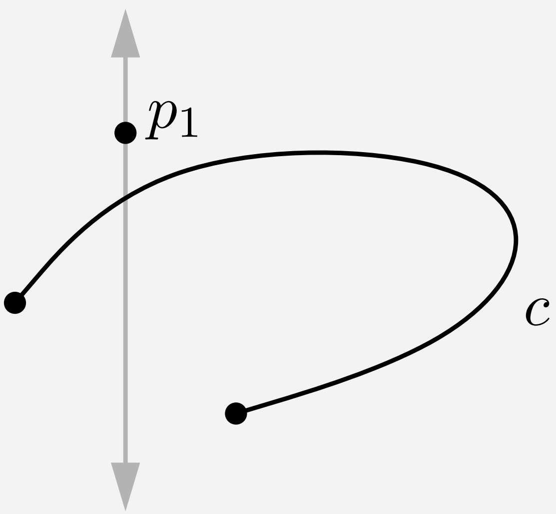
Points above and below a curve

Definition: Point p is below connection c if any continuous deformation in $\mathbb{R}^2 \setminus \{p\}$ of c intersects the vertical upper ray from p. Below is defined similarly.



Remark: For comparing a point with a connection we don't worry of other points.

Example relations point-connection



- p_1 is above c.

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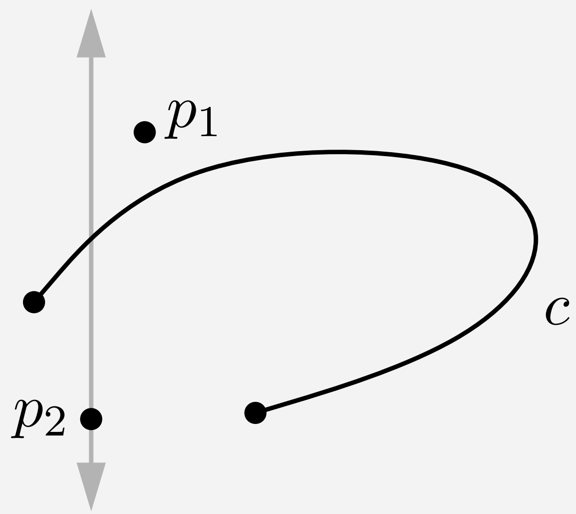

- p_1 is above c.
- p_2 is below c.

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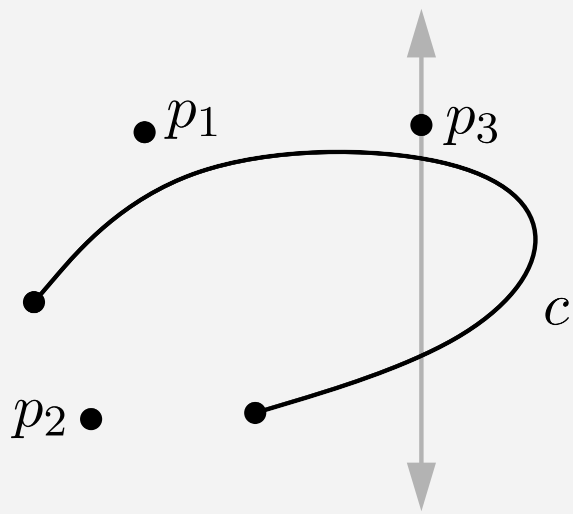

- p_1 is above c.
- p_2 is below c.
- p_3 has no relation with c.

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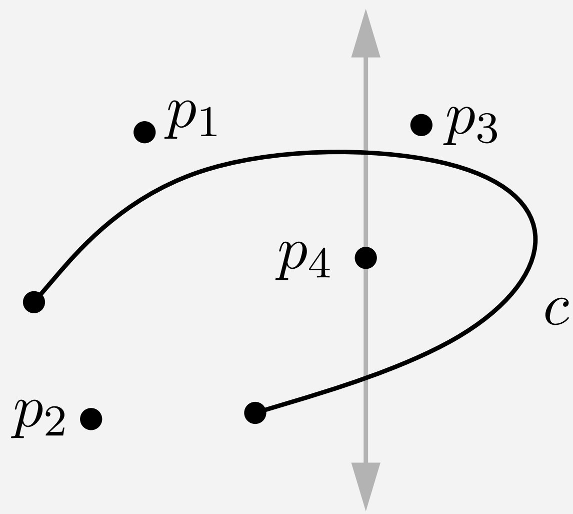
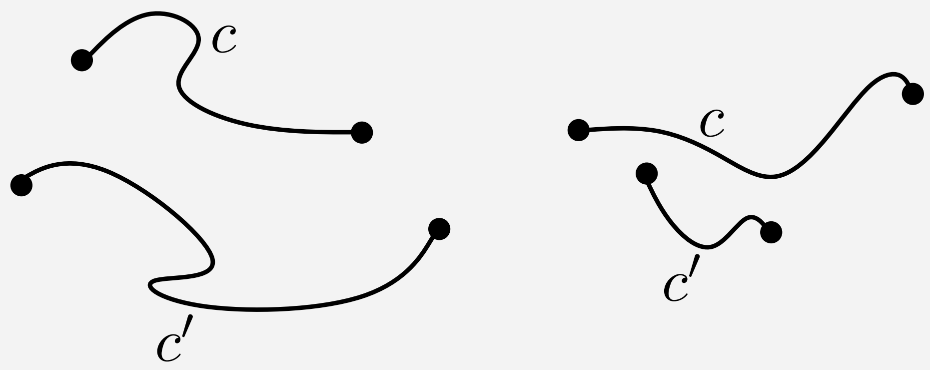


- p_1 is above c.
- p_2 is below c.
- p_3 has no relation with c.
- p_4 is above and below c.

Relation connection-connection

Definition: Connection c is above connection c' if any endpoint of c is above c' or any endpoint of c' is below c



Property: This order is invariant by equivalence of maps.

The order of the input map is preserved in the schematic one.

This is the top-to-bottom order we follow to place connections, leaving always as much freedom as possible.

Our algorithm: (n: complexity of the map)

Input: Original map. Allowed types of connections.

Output: Schematic map, or 'It is not possible'.

Procedure:

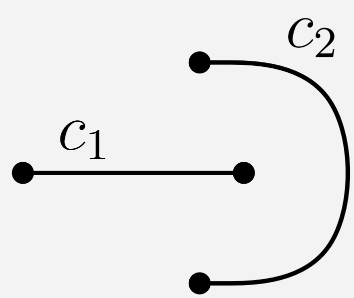
- compute order $\begin{cases} \text{monotone map} & \rightarrow O(n \log n) \\ \text{general map} & \rightarrow O(n \log^3 n) \end{cases}$
- place connections $O(n \log n)$

(Monotone map: each path c is monotone in some direction ϕ_c.)

Remark: 'It is not possible' means that there is no equivalent schematic map with the given types of connections.

When 'It is not possible'?

- We have a cyclic order.



$$c_2 > c_1 > c_2$$

Which one do we place first?

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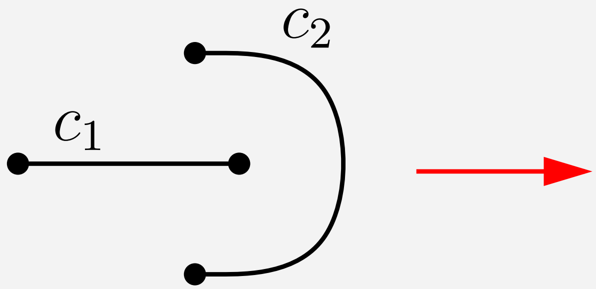
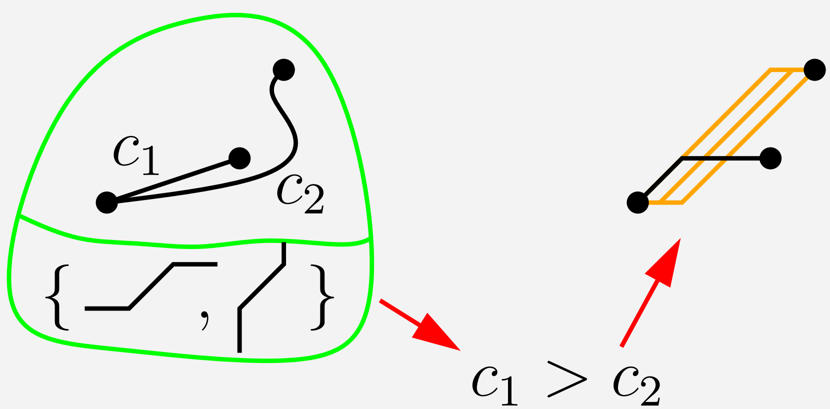


$c_2 > c_1 > c_2$

Which one do we place first?

- We cannot place next connection.



Computing the order

Procedure:

1. compute an intermediate map $\tilde{M}$
   - monotone map $\rightarrow$ x-monotone map in $O(n \log n)$ time
   - general map $\rightarrow$ x-confined map in $O(n \log n)$ time
2. compute the order in $\tilde{M}$
   - x-monotone map $\rightarrow$ order in $O(n \log n)$ time
   - x-confined map $\rightarrow$ order in $O(n \log^3 n)$ time

(x-confined map: each path doesn't cross the vertical lines through its endpoints)

Compute order: reduce points

In a map M of complexity n: $|P_M|$, $|M| \in O(n)$.

Considering all points P_M for a path $c_i \in M$ takes $\Omega(|c_i| + |P_M|)$.

This has $\Omega(n^2)$ worst time for the whole map.

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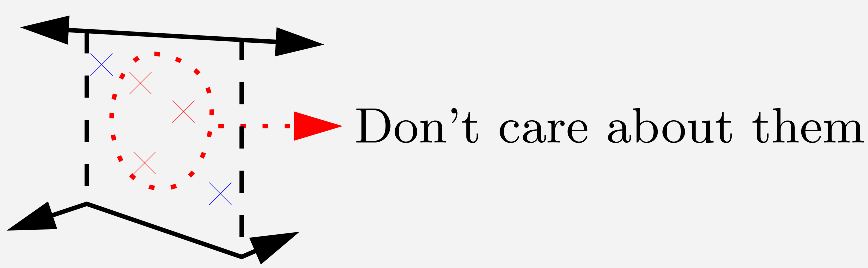
Consider $\mathcal{T}_{c_i}$, trapezoidal decomposition of c_i.

P_{c_i}: set of leftmost and rightmost points in $t \in \mathcal{T}_{c_i}$.

Let $\tilde{c}_i$ path with segments from $\mathcal{T}_{c_i}$. Then $\tilde{c}_i$ and c_i are equivalent in P_M iff equivalent in P_{c_i}.

Now, for a path c_i we only care about $O(c_i)$ points.

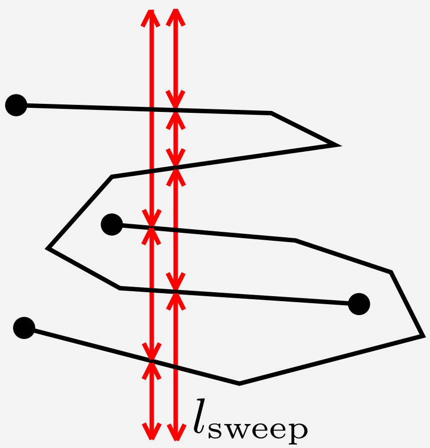


Reducing points

We want to compute P_{c_i}, $\forall c_i \in M$

Leftmost points with l-to-r sweep line simultaneously $\forall c_i \in M$.

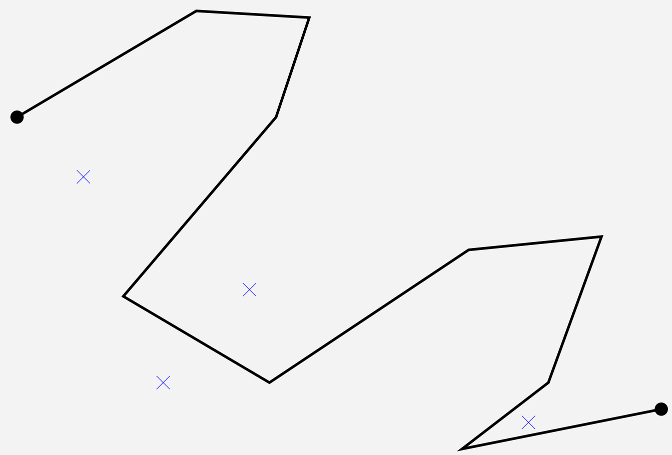
- Maintain the set $l_{\text{sweep}} \cap t$, $t \in \mathcal{T}_{c_i}$ $\forall c_i \in M$ in a tree.



- When find point in P_M report and delete all trapezoids containing it.
- $O(n)$ insertions and deletions $\rightarrow$ $O(n \log n)$

Making an intermediate x-monotone map

Compute for each $c_i \in M$ a monotone path $\tilde{c}_i \in \tilde{M}$:
incremental construction from left to right



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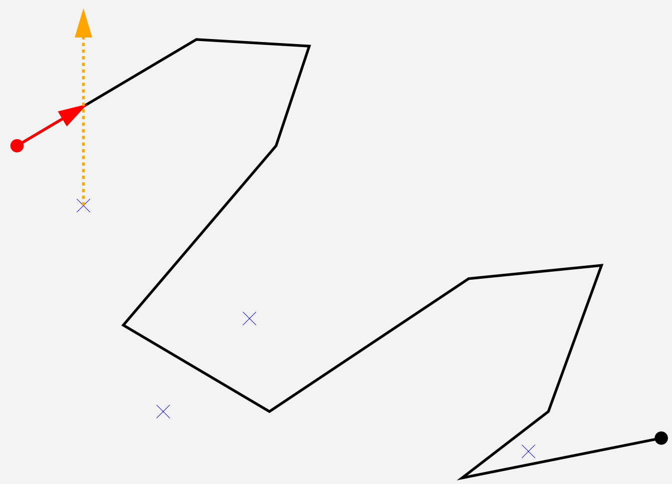

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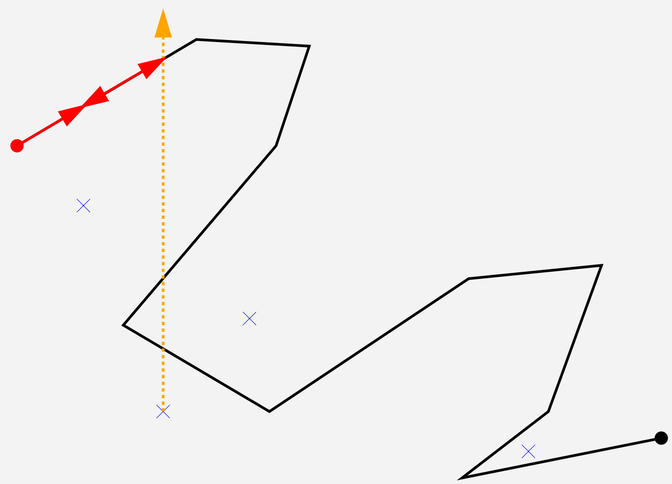

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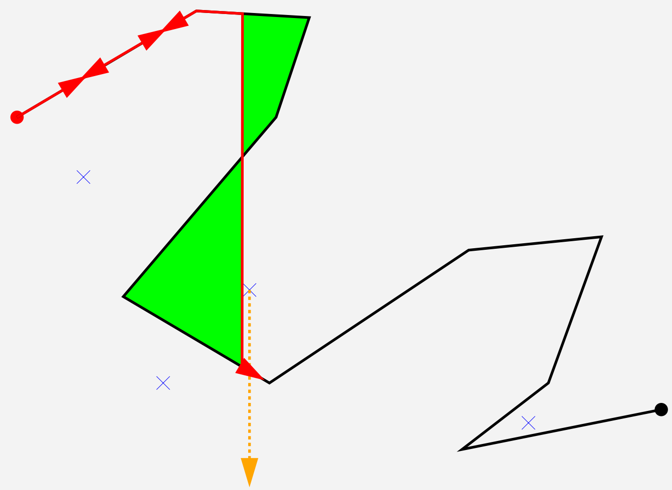

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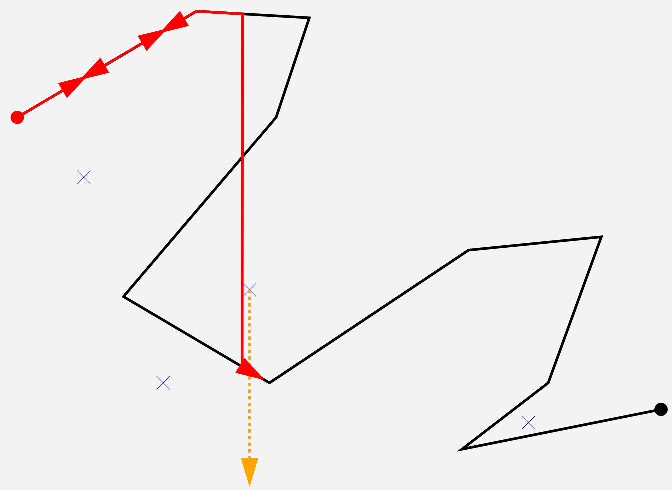

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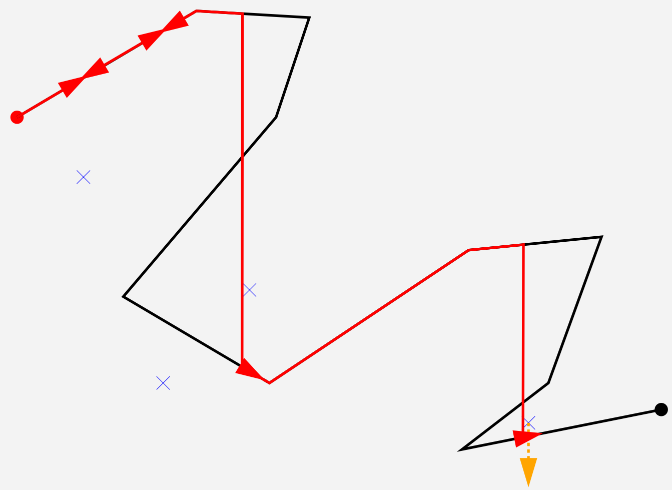

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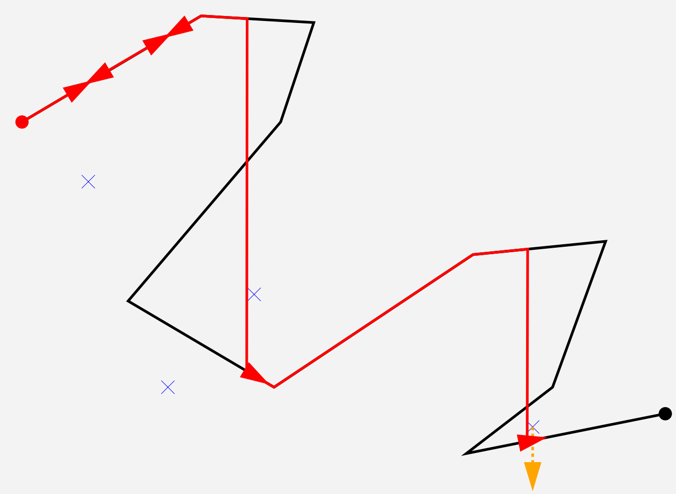
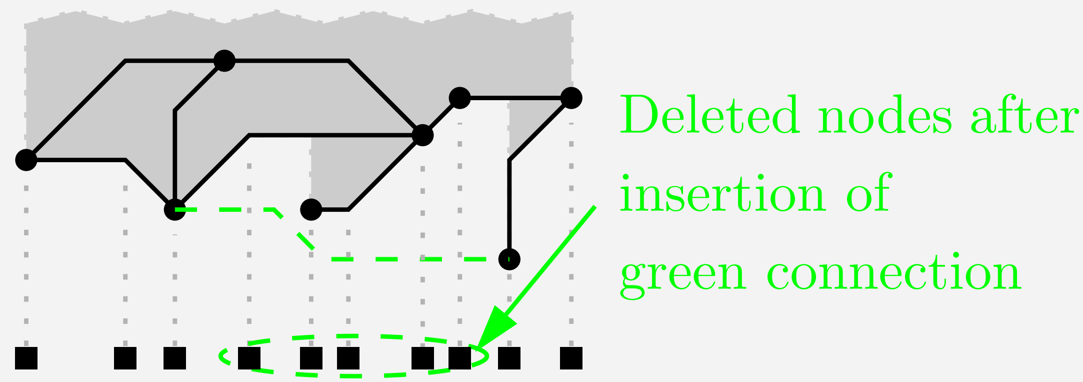


- The output of c_i using P_{c_i} or P_M is the same.
- For c_i, using P_{c_i}, it takes $O(|c_i| \log |c_i|)$ time.
- If c_i and c_j don't intersect, then $\tilde{c}_i$ and $\tilde{c}_j$ don't cross.

Placing connections

Incremental placement following the top to bottom order, maintaining the lower envelope (tree) of placed connections.

Each new placement must be topmost (it has to be defined).



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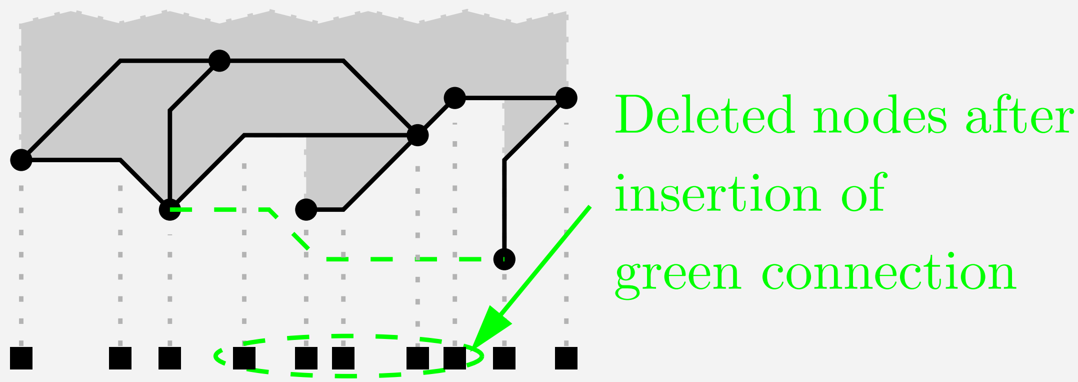

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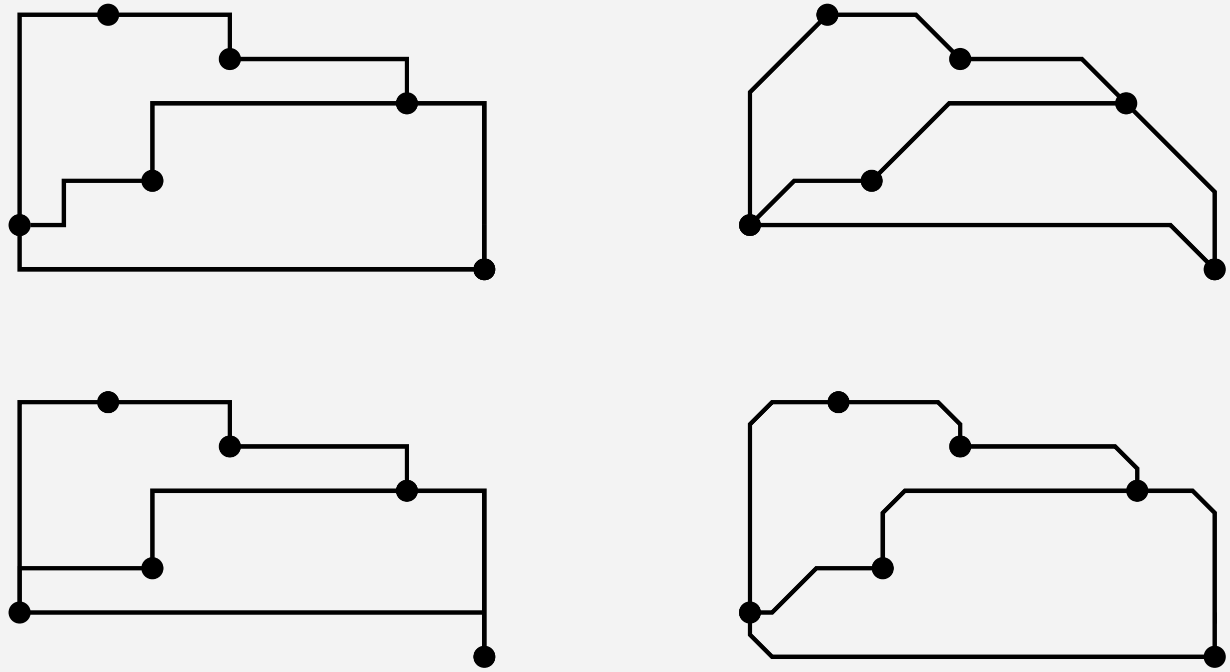
Result: Placement of n connections takes $O(n \log n)$ time.

How nice are the results?



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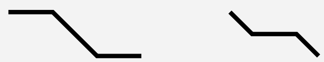
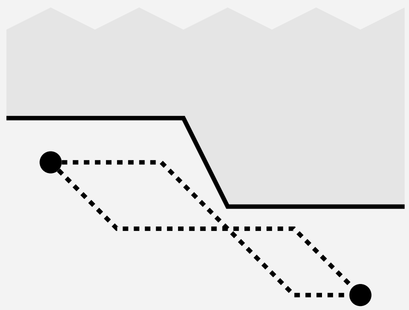
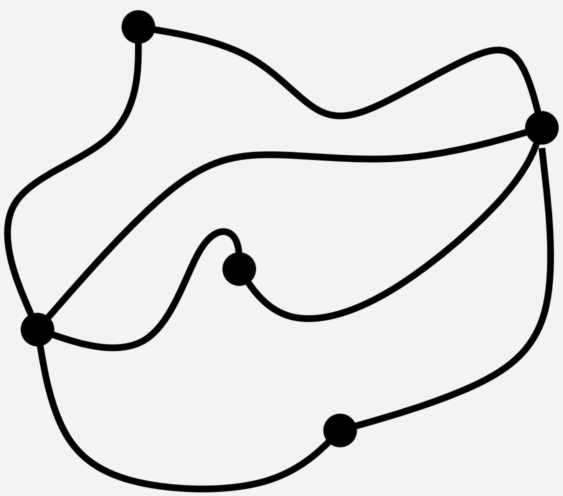


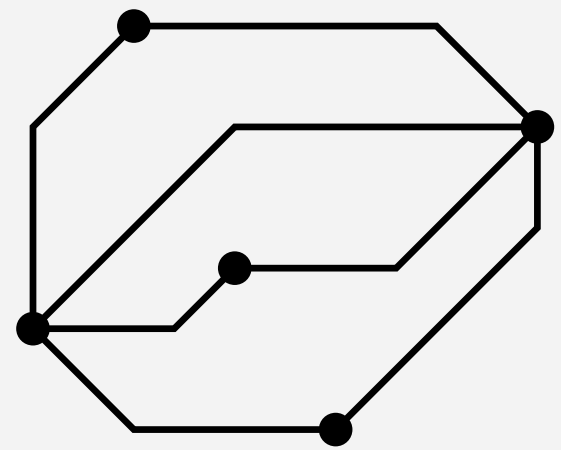


Work to do

- Compute order more quickly in general maps.
- Handle connections with topmost not defined.

- Experimental results.





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