Oka tubes in holomorphic line bundles

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European Research Council
Executive Agency

Established by the European Commission

Conference in memory of Nils Øvrelid
Oslo, 8–10 August 2024

Motivation

This is joint work with **Yuta Kusakabe** from **Kyushu University**.

We are interested in understanding the relationship between the positivity properties of hermitian or Kähler metrics on a compact complex manifold, and on vector bundles over such manifolds, and the Oka property.

Evidence of such a connection comes from the **Frankel Conjecture**, solved affirmatively by **Mori 1979** and **Siu and Yau 1980**, saying that a compact Kähler manifold with positive holomorphic bisectional curvature is biholomorphic to a complex projective space, and hence is an Oka manifold.

Their result was generalised by **Mok 1988** who classified compact complex hermitian manifolds with nonnegative holomorphic bisectional curvature. An inspection of his list shows that every such is an Oka manifold.

Conversely, there are many results, starting with the **Ahlfors lemma**, linking the existence of hermitian metrics with **negative holomorphic sectional curvature** to **Kobayashi hyperbolicity** of the manifold.

Our main result

Theorem (Kusakabe & F., 2024)

Let (E, h) be a semipositive hermitian holomorphic line bundle on a compact complex manifold X.

Assume that for each point $x \in X$ there exists a divisor $D \in |E|$ whose complement $X \setminus D$ is a Stein neighbourhood of x with the density property.

Then, the disc bundle $\Delta_h(E) = \{h < 1\}$ is an Oka manifold while $D_h(E) = \{h > 1\}$ is a pseudoconvex Kobayashi hyperbolic domain.

Varolin 2001 A complex manifold X has the **density property** if every holomorphic vector field on X can be approximated uniformly on compacts by sums of $\mathbb{C}$-complete holomorphic vector fields.

https://arxiv.org/abs/2310.14871

What is an Oka manifold?

A complex manifold Y is called an **Oka manifold** (F. 2009) if maps $X \to Y$ from any Stein manifold (or reduced Stein space) X satisfy the following:

- every continuous map $X \to Y$ is homotopic to a holomorphic map.
- If a continuous map $f_0 : X \to Y$ is holomorphic on a neighbourhood of a compact $\mathscr{O}(X)$-convex set K and on a closed complex subvariety X' of X, then there is a homotopy $f_t : X \to Y$ $(t \in [0,1]$ of maps which are holomorphic near K, close to f_0 on K, they agree with f_0 on X', and such that the map f_1 is holomorphic on X.
- The analogous properties hold for continuous families of maps $X \to Y$.

Oka manifolds lie at the heart of many existence results in complex geometry.

MSC 2020: 32Q56 Oka principle and Oka manifolds

A brief history

- **Oka–Weil, Oka–Cartan** Euclidean spaces $\mathbb{C}^n$ are Oka manifolds.
- **Oka 1939** $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$ is an Oka manifold.
- **Grauert 1958** Every complex Lie group G, and every complex homogeneous manifold G/H is an Oka manifold.
- **Forster & Ramspott 1966** Oka pairs of sheaves.
- **Gromov 1989** Every elliptic complex manifold is an Oka manifold.
- **F. 2005–2009** A complex manifold Y is Oka iff it satisfies the

  Convex approximation property (CAP): Every holomorphic map $K \to Y$ from a compact convex set K in $\mathbb{C}^n$ is a limit of entire maps $\mathbb{C}^n \to Y$.

  By using this axiom, I showed that most of the individual Oka-type conditions are pairwise equivalent.

A localization theorem for Oka manifolds

Kusakabe 2021 A complex manifold Y is Oka iff every holomorphic map $f : K \to Y$ from a compact convex set $K \subset \mathbb{C}^n$ is the core of a dominating holomorphic spray $F : K \times \mathbb{C}^N \to Y$ for some $N \geq \dim Y$:

$$F(\cdot, 0) = f \quad \text{and} \quad \frac{\partial}{\partial z}\Big|_{z=0} F(\zeta, z) : \mathbb{C}^N \to T_{f(\zeta)} Y \ \text{ is surjective for every } \zeta \in K.$$

This is a restricted version of **condition** Ell_1 introduced and studied by Gromov in 1986 and 1989. Kusakabe proved that

$$\mathrm{Ell}_1 \Longrightarrow \mathrm{CAP}.$$

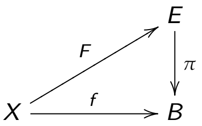
The implications $\mathrm{CAP} \Longleftrightarrow \mathrm{Oka} \Longrightarrow \mathrm{Ell}_1$ were known before (F. 2006).

Kusakabe gave many applications. In particular, he proved

The localization theorem for Oka manifolds: If a complex manifold Y is a union of Zariski-open Oka domains $Y \setminus A_i$, with every A_i a closed complex subvariety of Y, then Y is Oka.

Oka maps

A holomorphic map $\pi : E \to B$ is an **Oka map** if it is a **Serre fibration** and has the **Oka property**: for a Stein X, liftings $X \to E$ of holomorphic maps $X \to B$ satisfy the Oka property with approximation and interpolation.



Such a map $\pi : E \to B$ is a holomorphic submersion and its fibres are Oka manifolds. The Oka property of $E \to B$ is local on the base B.

F, 2006–2010 A holomorphic fibre bundle with Oka fibre is an Oka map. If $E \to B$ is an Oka map, then E is an Oka manifold iff B is an Oka manifold.

Kusakabe, 2021 If a complex manifold Y admits holomorphic maps $\pi_i : Y \to B_i$ with the Oka property such that the kernels of the differentials $d\pi_i$ span TY at every point, then Y is an Oka manifold.

Oka complements of holomorphically convex sets

Kusakabe 2020 (Annals of Math 2024)

- If K is compact polynomially convex set in $\mathbb{C}^n$ $(n > 1)$ then $\mathbb{C}^n \setminus K$ is Oka. The same holds for complements of compact holomorphically convex sets in any Stein manifold with Varolin's density property.

- If S is a closed polynomially convex subset of $\mathbb{C}^n$ such that

$$S \subset \left\{(z, w) \in \mathbb{C}^{n-2} \times \mathbb{C}^2 : |w| \leq c(1 + |z|)\right\}$$

  for some $c > 0$, then $\mathbb{C}^n \setminus S$ is Oka.

- Let $\pi : Y \to Z$ be a holomorphic fibre bundle whose fibre is a Stein manifold with the density property, and let $S \subset Y$ be a closed holomorphically convex set such that $\pi : S \to Z$ is proper. Then, $\pi : Y \setminus S \to Z$ has the Oka property.

Wold & F., 2023 For most closed convex sets $K \subset \mathbb{C}^n$ $(n > 1)$, $\mathbb{C}^n \setminus K$ is Oka.

If K is a compact polynomially convex set in $\mathbb{C}^n$ $(n > 1)$ and H is a complex hyperplane in $\mathbb{C}^n$, then $\mathbb{C}^n \setminus (H \cup K)$ is Oka.

The main problem

Let X be a compact complex manifold, and let $\pi : E \to X$ be a holomorphic vector bundle with a hermitian metric h. Denote by $|e|_h$ the norm of $e \in E$.

If E is a **line bundle** then $h : E \to [0, \infty)$ is a function, and we also write $|e|_h^2 = h(e)$.

Problem

When is the disc bundle

$$\Delta_h(E) = \{e \in E : |e|_h < 1\}$$

an Oka manifold?

In particular, when does the zero section $E(0) = \{e \in E : |e|_h = 0\}$ *admit a basis of open Oka neighbourhoods?*

Some observations

- The total space E is Oka if and only if the base X is Oka.
- For any $c > 0$, $\Delta_{h,c}(E) = \{|e|_h < c\}$ is biholomorphic to $\Delta_h(E)$ by a dilation in the fibres. Hence, if $\Delta_h(E)$ is Oka then $E(0)$ admit a basis of Oka neighbourhoods in E.
- Since $\Delta_h(E)$ admits a holomorphic deformation retraction onto the zero section $E(0) \cong X$, we infer that if $\Delta_h(E)$ is an Oka manifold then so is X.
- The answer is negative for any hermitian metric h on a trivial bundle $E = X \times \mathbb{C}^r$. Indeed, $\Delta_h(E) \subset X \times c\mathbb{B}^r$ for some $c > 0$, where $\mathbb{B}^r$ is the ball in $\mathbb{C}^r$. Since $X \times c\mathbb{B}^r$ admits a bounded plurisubharmonic function which is nonconstant on every nonempty open subset, it does not contain any Oka domains.
- The answer is negative for any negative line bundle $E \to X$. By Grauert, the zero section $E(0) \cong X$ admits a basis of pseudoconvex neighborhoods, and the Grauert reduction of such a neighbourhood (squeezing $E(0)$ to a point) gives a hyperbolic complex space.

Curvature and pseudoconvexity in line bundles

Let $\phi_{i,j} \in \mathscr{O}^*(U_{i,j})$ be a transition 1-cocycle of a line bundle $E \to X$. A hermitian metric h on E is given on any line bundle chart $(x,t) \in U_i \times \mathbb{C}$ by

$$h(x,t) = h_i(x)|t|^2,$$

where the functions $h_i : U_i \to (0,+\infty)$ satisfy the compatibility conditions

$$h_i(x)|\phi_{i,j}(x)|^2 = h_j(x) \quad \text{for } x \in U_{i,j} = U_i \cap U_j.$$

In $(E^{\otimes k}, h^{\otimes k})$, the functions $\phi_{i,j}$ and h_i are raised to the power k.

The bundle (E,h) is curved positively (resp. negatively) if the $(1,1)$-form

$$i\Theta_h = -i\partial\bar\partial \log h_i = -i\,\partial\bar\partial \log h = -\frac{1}{2} dd^c \log h$$

is positive (resp. negative). Hence, the following conditions are equivalent.

1. The metric h is semipositive: $i\Theta_h \geq 0$.
2. The function $-\log h$ is plurisubharmonic on $E \setminus E(0)$.
3. The functions $-\log h_i$ are plurisubharmonic on $U_i \subset X$.
4. The disc bundle $\Delta_h(E) = \{h < 1\}$ is pseudoconcave along $\{h = 1\}$.

Oka tubes in line bundles on projective spaces

Theorem

Given a positive holomorphic line bundle $E = \mathscr{O}_{\mathbb{CP}^n}(k)$ on $\mathbb{CP}^n$ $(n \geq 1,\ k \geq 1)$ and a semipositive hermitian metric h on E, the following assertions hold.

(a) *The punctured disc bundle $\Delta_h^*(E) = \{e \in E : 0 < h(e) < 1\}$ is an Oka manifold, and $\Delta_h(E) = \{e \in E : h(e) < 1\}$ is an Oka-1 manifold.*

(b) *If $n \geq 2$ or $E = \mathscr{O}_{\mathbb{CP}^n}(1)$ then $\Delta_h(E)$ is an Oka manifold.*

(c) *The domain $D_h(E) = E \setminus \overline{\Delta_h(E)} = \{h > 1\}$ is Kobayashi hyperbolic.*

Given a negative holomorphic line bundle $E = \mathscr{O}_{\mathbb{CP}^n}(k)$ $(k \leq -1)$ and a seminegative hermitian metric h on E ($\mathrm{i}\Theta_h \leq 0$), the following hold.

(a′) *The punctured disc bundle $\Delta_h^*(E)$ is Kobayashi hyperbolic.*

(b′) *The domain $D_h(E)$ is Oka.*

These results hold if the metric h is continuous and semipositive (resp. seminegative) in the weak sense. They also hold for the restrictions of these bundles to any affine Euclidean chart in $\mathbb{CP}^n$.

Proof for the hyperplane section bundle

Let $\pi : E = \mathscr{O}_{\mathbb{CP}^n}(1) \to \mathbb{CP}^n$ be the hyperplane section bundle.

Consider $H = \mathbb{CP}^n$ as the hyperplane at infinity in $\mathbb{CP}^{n+1} = \mathbb{C}^{n+1} \cup H$. Let $0 \in \mathbb{C}^{n+1}$ denote the origin. Then:

- the total space of E is $\mathbb{CP}^{n+1} \setminus \{0\}$,
- the zero section is $E(0) = \mathbb{CP}^n = H$, and
- the fibres of π are lines through 0, punctured at 0.

If h is a semipositive hermitian metric on E, then $\Delta_h(E) = \{h < 1\}$ is a pseudoconcave domain in $\mathbb{CP}^{n+1}$ containing $H = \{h = 0\}$, and

$$K := \{h \geq 1\} = \{1/h \leq 1\} \subset \mathbb{C}^{n+1}$$

is a compact set with disc fibres containing 0 in the interior.

Since $1/h = \mathrm{e}^{-\log h}$ is plurisubharmonic on $\mathbb{C}^{n+1}$, K is polynomially convex, so

$$\Delta_h(E) = \mathbb{CP}^{n+1} \setminus K \quad \text{and} \quad \Delta_h^*(E) = \mathbb{C}^{n+1} \setminus K \quad \text{are Oka domains.}$$

We do not know a comparably simple proof for bundles $\mathscr{O}_{\mathbb{CP}^n}(k)$ with $k > 1$.

Special hermitian line bundles on $\mathbb{CP}^n$

Denote by $z = (z_0, z_1, \ldots, z_n)$ the Euclidean coordinates on $\mathbb{C}^{n+1}$ and by $[z] = [z_0 : z_1 : \cdots : z_n]$ the homogeneous coordinates on $\mathbb{CP}^n$.

On the affine chart $U_i = \{[z] \in \mathbb{CP}^n : z_i \neq 0\} \cong \mathbb{C}^n$ $(i = 0, 1, \ldots, n)$ we have the affine coordinates $z^i = (z_0/z_i, \ldots, z_n/z_i)$, where $z_i/z_i = 1$ omitted.

On $E = \mathscr{O}_{\mathbb{CP}^n}(k)$, we have $\phi_{i,j}(z) = (z_j/z_i)^k$. Hence,

$$h([z], t) := \frac{|t|^2}{(1 + |z^i|^2)^k} = \frac{|z_i|^{2k}}{|z|^{2k}}|t|^2 \quad \text{for } [z] \in U_i \text{ and } t \in \mathbb{C} \tag{1}$$

is a hermitian metric on E. We have

$$\mathrm{i}\Theta_h = k\,\mathrm{i}\,\partial\bar{\partial}\log\big(|z|^2\big),$$

which is k-times the Fubini–Study form. The disc tube

$$\Delta_h(E)|_{U_i} = \{(z, t) \in \mathbb{C}^n \times \mathbb{C} : |t| < (1 + |z|^2)^{k/2}\}$$

is a **Hartogs domain** with radius of order $|z|^k$ as $|z| \to \infty$.

Since $\mathbb{CP}^n$ is compact, it follows that every hermitian metric on $\mathscr{O}_{\mathbb{CP}^n}(k)$ grows/decays at this rate at infinity in affine charts.

Oka Hartogs domains in $\mathbb{C}^n \times \mathbb{C}$

This shows that for every **semipositive hermitian metric** h on $E = \mathscr{O}_{\mathbb{CP}^n}(k)$ $(k \geq 1)$ and any affine chart $\mathbb{C}^n \cong U \subset \mathbb{CP}^n$, the restricted disc bundle $\Delta_h(E)|_U$ is a **pseudoconcave Hartogs domain**

$$\Omega = \{(z, t) \in \mathbb{C}^n \times \mathbb{C} : |t| < \phi(z)\},$$

where $\phi > 0$ is a positive function on $\mathbb{C}^n$ such that $\log \phi$ is plurisubharmonic and there is a constant $c > 0$ such that

$$\phi(z) \geq c\,(1 + |z|) \text{ holds for all } z \in \mathbb{C}^n.$$

Lemma

If $n \geq 2$ then every Hartogs domain Ω as above is Oka.

Since $\Delta_h(E)$ is covered by Zariski open Oka domains $\Delta_h(E)|_U$ for affine charts $U \subset \mathbb{CP}^n$, it follows that $\Delta_h(E)$ is Oka by Kusakabe's localization theorem.

Proof of the lemma, 1

Let $T : \mathbb{C}^{n+1} = \mathbb{C}^n \times \mathbb{C} \to \mathbb{C}$ denote the projection $T(z,t) = t$. Set

$$\begin{aligned} S &= \mathbb{C}^{n+1} \setminus \Omega = \{(z,t) \in \mathbb{C}^n \times \mathbb{C} : |t| \ge \phi(z)\} \\ &= \{(z,t) \in \mathbb{C}^n \times \mathbb{C}^* : \log\phi(z) - \log|t| \le 0\}. \end{aligned}$$

Since $\log|t|$ is harmonic on $t \in \mathbb{C}^*$, the function

$$\psi(z,t) = \log\phi(z) - \log|t| \quad \text{is plurisubharmonic on } \mathbb{C}^n \times \mathbb{C}^*.$$

Since ϕ grows at least linearly, the restricted projection $T|_S : S \to \mathbb{C}$ is proper. It follows that for every $r > 0$ the set

$$S_r = \{(z,t) \in S : |t| \le r\} = \{(z,t) \in \mathbb{C}^n \times \mathbb{C}^* : \psi(z,t) \le 0, \ \ \log|t| \le \log r\}$$

is compact and $\mathscr{O}(\mathbb{C}^n \times \mathbb{C}^*)$-convex. By a theorem of Kusakabe,

$$T : (\mathbb{C}^n \times \mathbb{C}^*) \setminus S \to \mathbb{C}^* \text{ has the Oka property.}$$

Since $S \cap \{t = 0\} = \varnothing$, the projection $T : \mathbb{C}^{n+1} \setminus S \to \mathbb{C}$ has the Oka property as well. (For a holomorphic submersion, the Oka property is local on the base.)

Proof of the lemma, 2

Since ϕ grows at least linearly, we have $\Lambda \cap S = \varnothing$ for every complex hyperplane $\Lambda \subset \mathbb{C}^{n+1}$ sufficiently close to $\Lambda_0 = \{t = 0\}$, and there is a path Λ_s $(s \in [0,1])$ of such hyperplanes connecting Λ_0 to Λ.

For any such Λ the set S_r is also $\mathscr{O}(\mathbb{C}^{n+1} \setminus \Lambda)$-convex. (Apply Oka's criterion for holomorphic convexity.)

As $r \to \infty$, the sets S_r exhaust S, so S is $\mathscr{O}(\mathbb{C}^{n+1} \setminus \Lambda)$-convex.

Let $T_\Lambda : \mathbb{C}^{n+1} \to \mathbb{C}$ be a linear projection with $(T_\Lambda)^{-1}(0) = \Lambda$. If Λ is sufficiently close to Λ_0 then the restricted projection $T_\Lambda|_S : S \to \mathbb{C}$ is still proper. As before, we infer that $T_\Lambda : \mathbb{C}^{n+1} \setminus S \to \mathbb{C}$ has the Oka property.

Applying this conclusion with two linearly independent projections T_Λ, we see that $\mathbb{C}^{n+1} \setminus S = \Omega$ is an Oka domain.

Conclusion of proof of the theorem

It remains to show that, for a positive bundle $E = \mathscr{O}_{\mathbb{CP}^n}(k)$ with $k \geq 1$, the exterior tube $D_h(E) = \{h > 1\}$ is hyperbolic.

Note that the dual bundle $E^* = \mathscr{O}_{\mathbb{CP}^n}(-k)$ of a positive bundle is a negative line bundle. When passing to the dual bundle, the (deleted) interior disc tube and the exterior disc tube get reversed.

In particular, the tube $D_h(E) = \{h > 1\}$ is biholomorphic to deleted disc bundle $\Delta^*_{h^*}(E^*) = \{0 < h^* < 1\}$ in the dual bundle, which is hyperbolic by Grauert's theorem when h is semipositive (so h^* is seminegative).

When $k = 1$, $E^* = \mathscr{O}_{\mathbb{CP}^n}(-1)$ is the universal bundle on $\mathbb{CP}^n$ whose total space is $\mathbb{C}^{n+1}$ blown up at the origin, and the tube $\{h^* < 1\}$ for a seminegative metric h^* is a bounded pseudoconvex domain blown up at the origin.

The main result

Theorem

Let (E, h) be a semipositive hermitian holomorphic line bundle on a compact complex manifold X.

Assume that for each point $x \in X$ there exists a divisor $D \in |E|$ whose complement $X \setminus D$ is a Stein neighbourhood of x with the density property.

Then, the disc bundle $\Delta_h(E)$ is an Oka manifold while $D_h(E) = E \setminus \overline{\Delta_h(E)}$ is a pseudoconvex Kobayashi hyperbolic domain.

Proof: It suffices to show that $\Delta_h(E)|_{X \setminus D}$ is Oka for every divisor D as in the theorem. Note that $E|_{X \setminus D} \cong (X \setminus D) \times \mathbb{C}$. Since $|E|$ is base-point-free, there are a holomorphic map $\Phi : X \to \mathbb{CP}^N$ and a hyperplane $H \subset \mathbb{CP}^N$ such that

$$E = \Phi^*(\mathscr{O}_{\mathbb{CP}^N}(1)) \text{ and } \Phi^{-1}(\mathbb{CP}^N \setminus H) = X \setminus D.$$

Since $X \setminus D$ is Stein with the density property, we can apply the proof for $X = \mathbb{CP}^n$ with projections close to $T : (X \setminus D) \times \mathbb{C} \to \mathbb{C}$ to show that $\Delta_h(E)|_{X \setminus D}$ is an Oka domain.

Polarised manifolds

A **polarised manifold** (X, E) is a pair of a compact complex manifold X and an ample line bundle E on X. Such X is necessarily projective, and every projective manifold admits an ample line bundle.

Definition

(a) A polarised manifold (X, E) has the **polarised density property** (PDP) if for each point $x \in X$ there exists a divisor $D \in |E|$ whose complement $X \setminus D$ is a Stein neighbourhood of x with the density property.

(b) A projective manifold X has PDP if (X, E) has PDP for every ample E.

If (X, E) has PDP then X is an Oka manifold by the localization theorem. Every line bundle satisfying the condition of our main theorem is ample. Hence, the theorem can be equivalently stated as follows.

Theorem

If (X, E) has PDP, then for any semipositive hermitian metric h on E the disc bundle $\Delta_h(E)$ is an Oka manifold while $D_h(E) = E \setminus \overline{\Delta_h(E)}$ is hyperbolic.

Some properties of PDP manifolds

- If (X, E) has PDP then so does $(X, E^{\otimes k})$ for every $k > 0$.
- Every ample line bundle on a complex Grassmann manifold of dimension > 1 has PDP.
- If (X_1, E_1) and (X_2, E_2) have PDP then so does their exterior tensor product $(X_1 \times X_2, E_1 \boxtimes E_2)$.
- If (X, E) has PDP then $(X \times \mathbb{CP}^n, E \boxtimes \mathscr{O}_{\mathbb{CP}^n}(k))$ $(n > 0,\ k > 0)$ also has PDP.
- A **rational manifold** is a projective manifold birationally isomorphic to a projective space. If $X_1, \ldots, X_m$ $(m \geq 2)$ are rational manifolds such that every X_i with $\dim X_i > 1$ has PDP, then $X = X_1 \times X_2 \times \cdots \times X_m$ also has PDP.

Griffiths seminegative vector bundles of higher rank

Theorem

If (E, h) is a Griffiths seminegative hermitian holomorphic vector bundle of rank > 1 on an Oka manifold X, then $\Omega_h(E) = \{e \in E : |e|_h > 1\}$ is a pseudoconcave Oka domain.

Proof. Let $\pi : E \to X$ be the bundle projection and set

$$S = \{e \in E : |e|_h \leq 1\}.$$

For each holomorphic chart $\psi : U \to \mathbb{B}^n$ from an open set $U \subset X$ onto the unit ball $\mathbb{B}^n \subset \mathbb{C}^n$ $(n = \dim X)$ and each $0 < r < 1$,

$$\{e \in S|_U : |\psi \circ \pi(e)| \leq r\} \text{ is } \mathscr{O}(E|_U)\text{-convex.}$$

By a theorem of Kusakabe, the restricted projection $\pi : \Omega_h(E) = E \setminus S \to X$ has the Oka property.

This projection is also a topological fibre bundle, and hence an Oka map.

Since X is an Oka manifold, it follows that $\Omega_h(E)$ is an Oka manifold as well.

A problem

Problem

Let (E, h) be a Griffiths semipositive hermitian holomorphic vector bundle of rank > 1 over an Oka manifold X. Let

$$\phi(e) = |e|_h^2, \quad e \in E.$$

Is the tube $\{\phi < 1\}$ an Oka manifold?

The boundary $\{\phi = 1\}$ of this domain is pseudoconcave in the horizontal directions and strongly pseudoconvex in the fibre directions.

There is no example in the literature of a non-pseudoconcave Oka domain.

The problem on the (non) existence of such a domain is known as the

inverse Levi problem for Oka manifolds

THANK YOU

FOR YOUR ATTENTION