Product Graphs: Structure and Recognition

(Misprints and notes)

February 4, 2011

Misprints

- Page 20, line 14 should read: $d_G(u,w) \leq d_G(u,v) + d_G(v,w)$.
- Page 31, line 9: replace $1 = 1$ by $i = 1$.
- Page 49, line 17: replace $=$ by $\leq$.
- Page 65, line -5: replace U_{ba} by W_{ba}.
- Page 72, line -3: replace “has” by “have”.
- Page 85, line 13: replace G_i^* by G/Π_i.
- Page 116, line 3: replace “Theorem2.42” by “Theorem 2.42”.
- Page 127: in Corollary 4.14 replace “transitive Abelian group” by “transitive elementary Abelian two-group”.
- Page 149, line -11: replace “$\ell \leq k =$” by “$\ell - 1 \leq k - 1 =$”.
- Page 176, line -12: replace “Denominator($a_2(y)$” by “Denominator($a_2(y)$)”.
- Page 215, line -12: replace “is 3” by “is at most 3”.
- Page 226, line -10: replace $\sqrt{n}$ by $m/\sqrt{n}$.
- Page 226, line -3: replace “connetion” by “connection”.
- Page 306, line 8: replace “is a more” by “is more”.
- Page 310, line -14: replace $A \cong B$ by $B \cong C$.
- Page 322, line 10: replace “$G \diamond G$” by “the *weak modular product* $G \nabla G$, as defined in Exercise C.5.1”. In the following lines, and in Exercise 6 on the next page, replace $\diamond$ by ∇ (four times altogether).
- Page 323: The text of Exercise 1 should read: The following table defines the *weak modular product* $G \nabla H$. Is it associative?

Notes

- The first author (manuscript, April 2000) showed that the recognition complexity of connected Cartesian product graphs is linear. The new algorithm extends the one of the book.

- Brešar and Klavžar (manuscript, April 2000) showed that certain subdivisions of K_4 disprove the conjecture 2.45 from page 80. They also proved that every counterexample to the conjecture contains a subdivision of K_4.

- The paper containing the answers to Exercises 9, 10, and 11 on page 82 has been published: S. Klavžar and R. Škrekovski, On median graphs and median grid graphs, Discrete Math. 219 (2000) 287–293.

- On page 13 we say: "We ... show that K_5 and $K_{3,3}$ are nonplanar. To see this, we first note that all faces of any planar drawing of K_5 must be triangles and that all faces in a plane drawing of $K_{3,3}$ must be 4-cycles. Now the observation that $|E(K_5)| = 10 \neq 9 = 3\,(|V(K_5)| - 2)/(3 - 2)$ and $|E(K_{3,3})| = 9 \neq 8 = 4\,(|V(K_{3,3})| - 2)/(4 - 2)$ shows that neither of these graphs can be planar."

  Yaokun Wu from Jiao Tong University, Shanghai, suggested the following elegant elementary argument for this fact: "By Euler's formula $f = 7$ for K_5. Hence K_5 has a face with at most two (the integer part of $2m/f$) edges. A contradiction. For $K_{3,3}$ we have $n = 6$, $m = 9$, and $f = 5$. Thus $K_{3,3}$ has a face with at most three edges, which is impossible since $K_{3,3}$ is bipartite."

- Yang Chao from the University of Science and Technology of China noticed that the second proof of Theorem 2.42 (Page 78) is incomplete. After removing all the peripheral subgraphs it is possible that we are left with the empty graph. Consequently, we get nothing from induction. Instead, the last sentence of the proof should read as follows. We only have to note that every automorphism of G preserves the collection of peripheral subgraphs $\langle W_{uv} \rangle$, where $\langle W_{vu} \rangle$ is not peripheral, and the (nonempty) subgraph of G obtained by removing all such peripheral $\langle W_{uv} \rangle$ from G.

  (Note that there is at least one such peripheral subgraph by the assumption of the second case of the proof. That the remaining subgraph X of G is nonempty can be seen as follows. Let Q be a largest hypercube in G and v a vertex of Q with a neighbor not in Q. Then v belongs to X.)

- Don Knuth pointed out that Exercise 5.11 on Page 183 is wrong.

- Algorithm 2.3 (Median graphs 1) also has to check connectedness of all $\langle U_{ab} \rangle$s. Their computation and the check for connectedness does not alter the complexity of the algorithm.