DISCRETE MATHEMATICS

Dominating direct products of graphs

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Abstract

An upper bound for the domination number of the direct product of graphs is proved. It in particular implies that for any graphs G and H, $\gamma(G \times H) \leqslant 3\gamma(G)\gamma(H)$. Graphs with arbitrarily large domination numbers are constructed for which this bound is attained. Concerning the upper domination number we prove that $\Gamma(G \times H) \geqslant \Gamma(G)\Gamma(H)$, thus confirming a conjecture from [R. Nowakowski, D.F. Rall, Associative graph products and their independence, domination and coloring numbers, Discuss. Math. Graph Theory 16 (1996) 53–79]. Finally, for paired-domination of direct products we prove that $\gamma_{\mathrm{pr}}(G \times H) \leqslant \gamma_{\mathrm{pr}}(G)\gamma_{\mathrm{pr}}(H)$ for arbitrary graphs G and H, and also present some infinite families of graphs that attain this bound.
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1. Introduction

Many authors have investigated the behavior of domination and independence parameters in graph products, cf. the survey by Nowakowski and Rall [15]. In this paper we focus on domination in direct products of graphs that was initiated in [6,15].

There is no consistent relation between the domination number of the direct product of two graphs and the product of their domination numbers [15]. In the same paper an example was given that disproved a Vizing-type conjecture from [6]. An infinite series of such examples was presented in [11]. Chérifi et al. [3] determined the domination number of the direct product of two paths with the only exception when one factor is a path on 10, 11, or 13 vertices. Independently, some of these results were obtained by Klobučar in [12,13]. In [15] it was proved that $\gamma(G \times H) \geqslant \rho(G)\gamma_{\mathrm{t}}(H)$, see also [17] for a shorter proof, while in [17] it was observed that

$$\gamma(G \times H) \leqslant 4\gamma(G)\gamma(H).$$

Other domination-type problems in direct products were studied in [4,5,9,14,19]. In this note we further investigate the domination number, and present, to our knowledge, the first results on the paired-domination and upper domination numbers of direct products of graphs.

In the next section we define the concepts needed and recall two known results to be used later. In Section 3 we prove an upper bound on $\gamma(G \times H)$ that in particular implies that

$$\gamma(G \times H) \leqslant 3\gamma(G)\gamma(H).$$

We also construct graphs G and H with arbitrarily large domination numbers for which this bound is attained and show that in many cases it can be further improved. In the fourth section we prove that for any graphs G and H,

$$\Gamma(G \times H) \geqslant \Gamma(G)\Gamma(H),$$

where $\Gamma(G)$, as usual, denotes the upper domination number of a graph G. This result was conjectured in [15]. In the last section we consider paired-domination in direct products. We show that for any graphs G and H

$$\gamma_{\mathrm{pr}}(G \times H) \leqslant \gamma_{\mathrm{pr}}(G)\gamma_{\mathrm{pr}}(H),$$

present some infinite families of graphs for which the equality is attained and pose a problem concerning a possible lower bound for $\gamma_{\mathrm{pr}}(G \times H)$.

2. Preliminaries

All the graphs considered will be simple, undirected graphs with no isolated vertices. Let $G = (V, E)$ be a graph. A set $S \subseteq V$ is a *dominating set* if each vertex in $V \setminus S$ is adjacent to at least one vertex of S. If, in addition, each vertex of S has a neighbor in S, then S is called a *total dominating set*. Furthermore, as in [8], a dominating set S is called a *paired-dominating set* if it induces a subgraph with a perfect matching. Equivalently, S is the set of endvertices of a matching of G that is also a dominating set of G. If A and B are subsets of V we say that A *dominates* B if every vertex of B has a neighbor in A or is a vertex of A. We also say that B is *dominated* by A. We will make use of the following well-known result.

Theorem 1 (*Ore [16]*). *A dominating set S of a graph G is minimal if and only if for every vertex $u \in S$ one of the following two conditions holds*:

 (i) *u is not adjacent to any vertex of S,*
(ii) *there exists a vertex $v \in V(G) \setminus S$ such that u is the only neighbor of v from S.*

The *domination* (resp. *total domination, paired-domination*) *number* $\gamma(G)$ (resp. $\gamma_{\mathrm{t}}(G)$, $\gamma_{\mathrm{pr}}(G)$) of a graph G is the minimum cardinality of a dominating (resp. total dominating, paired-dominating) set. A dominating set of size $\gamma(G)$ is called a γ-*set*. Analogously we define a γ_{t}-*set* and a γ_{pr}-*set*. Note that for any graph G we have $\gamma(G) \leqslant \gamma_{\mathrm{t}}(G) \leqslant \gamma_{\mathrm{pr}}(G)$. The *upper domination number* $\Gamma(G)$ of G is the size of a largest minimal dominating set. The 2-*packing number* $\rho(G)$ of G is the maximum cardinality of a set $S \subset V(G)$ such that any two vertices in S are at distance at least three. Equivalently, the vertices of S have pairwise disjoint closed neighborhoods. The *open packing number* $\rho^o(G)$ is the maximum cardinality of a set of vertices whose open neighborhoods are pairwise disjoint. For more information on domination in graphs we refer to [7].

For graphs G and H, the *direct product* $G \times H$ (also known as the tensor product, cross product, cardinal product, categorical product, . . .) is the graph with vertex set $V(G) \times V(H)$ where two vertices (x, y) and (v, w) are adjacent if and only if $xv \in E(G)$ and $yw \in E(H)$. The *Cartesian product* $G \square H$ is defined on the same vertex set where two vertices (x, y) and (v, w) are adjacent if and only if either $x = v$ and $yw \in E(H)$, or $y = w$ and $xv \in E(G)$.

We will also need the following result from [1].

Lemma 2. *Let $X = G \times H$ and let (x, y), (v, w) be vertices of X. Then $d_X((x, y), (v, w))$ is the smallest d such that there is an x, v-walk of length d in G and a y, w-walk of length d in H.*

An explicit formula for the distance function in the direct product was first obtained by Kim [10] in a somehow more involved form. Note that Lemma 2 in particular implies that if walks of the same parity do not exist, then the corresponding vertices are in different connected components of the direct product.

3. Domination in direct products of graphs

Let G be a graph. For a total dominating set D of G let $\gamma_D(G)$ denote the size of a smallest subset of D that dominates G. Note that $\gamma_D(G) \leqslant |D|$ and so $\gamma_D(G)$ is well-defined.

Theorem 3. *For any graphs G and H,*

$$\gamma(G \times H) \leqslant \min\{\gamma_D(G)|D'| + \gamma_{D'}(H)|D| - \gamma_D(G)\gamma_{D'}(H)\},$$

where the minimum is taken over all total dominating sets D of G and D' of H.

Proof. Let D be a total dominating set of G and D' a total dominating set of H. Let $A \subseteq D$ be a dominating set for G of size $\gamma_D(G)$ and $B \subseteq D'$ a dominating set for H of size $\gamma_{D'}(H)$. We claim that $X = (A \times D') \cup (D \times B)$ is a dominating set of $G \times H$. Let $(a, b) \in V(G \times H) \backslash X$. Suppose $a \in D \backslash A$ and $b \in D' \backslash B$. Since A is a dominating set for G there exists a vertex c in A such that $ac \in E(G)$. Similarly, there exists $d \in B$ such that $bd \in E(H)$. Hence, $(c, d) \in X$ and is adjacent to (a, b) in $G \times H$. Suppose that $a \in V(G) \backslash D$ and b is any vertex of H. As A dominates G and D' is a total dominating set for H, a has a neighbor $c \in A$ and b has a neighbor $d \in D'$. It follows that $(c, d) \in X$ and dominates (a, b). The argument for $(a, b) \in V(G) \times (V(H) \backslash D')$ is similar, thus we have shown that X is a dominating set for $G \times H$.

Now,

$$\begin{aligned} \gamma(G \times H) \leqslant |X| &= |A \times D'| + |D \times B| - |(A \times D') \cap (D \times B)| \\ &= |A||D'| + |D||B| - |A||B| \\ &= \gamma_D(G)|D'| + \gamma_{D'}(H)|D| - \gamma_D(G)\gamma_{D'}(H). \qquad \square \end{aligned}$$

It was previously known [17] that $\gamma(G \times H) \leqslant 4\gamma(G)\gamma(H)$ for every pair of graphs G and H. If S is any dominating set for a graph G, then S is a subset of a total dominating set for G. Consider the vertices of S sequentially and enlarge the set to include a neighbor of any $x \in S$ that is isolated in the subgraph induced by S. The resulting set is a total dominating set D for G that may not have cardinality $\gamma_t(G)$, but $|D| \leqslant 2|S|$. By starting with a minimum dominating set A for G and a minimum dominating set B for H we can thus enlarge A to a total dominating set D for G and B to a total dominating set D' for H such that $|D| \leqslant 2\gamma(G)$ and $|D'| \leqslant 2\gamma(H)$. This establishes the following corollary.

Corollary 4. *For any graphs G and H, $\gamma(G \times H) \leqslant 3\gamma(G)\gamma(H)$.*

Let G and H be graphs with at least two edges and let $\gamma(G) = \gamma(H) = 1$. Then $\gamma(G \times H) = 3$. Indeed, it is easily seen that $\gamma(G \times H) \geqslant 3$, so Corollary 4 yields the equality. On the other hand, it follows immediately from the construction preceding the above corollary that $\gamma(G) > \rho(G)$ implies $\gamma(G \times H) < 3\gamma(G)\gamma(H)$ for an arbitrary graph H. This raises the question, whether there are any pairs of graphs with domination number greater than 1 such that equality is attained in Corollary 4. The answer is positive, and moreover, there are such graphs with arbitrarily large domination number.

Let G be a graph with $\gamma(G) = \rho(G)$ such that there exists a maximum 2-packing of G which is at the same time a γ-set of G. By G^+ we denote the graph obtained from G by attaching to each vertex of this minimum dominating set two pendant vertices. Clearly, $\gamma(G) = \gamma(G^+)$.

Theorem 5. *Let G and H be connected graphs each having a maximum 2-packing which is at the same time a minimum dominating set. Then $\gamma(G^+ \times H^+) = 3\gamma(G^+)\gamma(H^+)$.*

Proof. Let D and D' be minimum dominating sets of G and H. Note that D and D' are unique minimum dominating sets of G^+ and H^+. For a vertex $x \in D \subset V(G^+)$ denote by x_p and x_r the corresponding pendant vertices attached to x. Similarly, let y_p and y_r be the pendant vertices of $y \in D' \subset V(H^+)$. Set $X = G^+ \times H^+$.

Consider a vertex $(x, y) \in D \times D'$. Then the vertices (x_p, y_p), (x_p, y_r), (x_r, y_p), and (x_r, y_r) are pendant vertices in $G^+ \times H^+$ that are adjacent to (x, y). It follows that $D \times D'$ must be a subset of any minimum dominating set of X.

Let (x, y) and (v, w) be different vertices of $D \times D'$. Then the vertices (x, y_p), (x, y_r), (x_p, y), and (x_r, y) induce a subgraph $C(x, y)$ of X while the vertices (v, w_p), (v, w_r), (v_p, w), and (v_r, w) induce a subgraph $C(v, w)$, where both $C(x, y)$ and $C(v, w)$ are isomorphic to C_4. By considering the cases where the vertices (x, y) and (v, w) differ in both or in only one coordinate, we see that $d_X((x, y), (v, w)) \geqslant 3$. Lemma 2 then implies that the distance between an arbitrary vertex of $C(x, y)$ and an arbitrary vertex of $C(v, w)$ is at least four. If follows that no vertex of X can dominate a vertex of $C(x, y)$ and a vertex of $C(v, w)$. Observe next that at least two vertices are required to dominate $C(x, y)$. Indeed, to dominate (x, y_p) and (x, y_r) we need to use a vertex from $V(G^+) \times \{y\}$, but such a vertex will not dominate (x_p, y) and (x_r, y). Moreover, these two vertices cannot be in $D \times D'$. Indeed, there is no edge between (a, b), with $b \in D'$, and any vertex of $D \times D'$, since D' is a 2-packing. (And similarly, if $a \in D$.)

In conclusion, let S be a minimum dominating set of X. Then by the above, $D \times D' \subseteq S$. Moreover, to any vertex (x, y) of $D \times D'$ there are at least two additional vertices in S that dominate $C(x, y)$. As $|D \times D'| = \gamma(G^+)\gamma(H^+)$ we infer that $\gamma(G^+ \times H^+) \geqslant 3\gamma(G^+)\gamma(H^+)$. Corollary 4 completes the proof. $\quad\square$

The family of graphs in Theorem 5 is in fact quite large. Take an arbitrary (connected) graph and subdivide each edge of it by two vertices. Then all the original vertices form a maximum 2-packing which is at the same time a minimum dominating set.

In some special cases Theorem 3 assumes a more explicit form. For instance:

Corollary 6. *Suppose that each of G and H contains a γ-set that can be extended to a γ_t-set. Then*

$$\gamma(G \times H) \leqslant \gamma(G)\gamma_t(H) + \gamma_t(G)\gamma(H) - \gamma(G)\gamma(H).$$

Proof. Let A and A' be γ-sets of G and H as stated above and let D and D' be γ_t-sets of G and H with $A \subseteq D$ and $A' \subseteq D'$. Then apply Theorem 3. $\quad\square$

Since $\gamma_t(G) \leqslant 2\gamma(G)$ and $\gamma_t(H) \leqslant 2\gamma(H)$, the bound of Corollary 6 is at least as good as the one from Corollary 4 as soon as Corollary 6 is applicable.

Let S_n be the graph that is obtained from $K_{1,n}$ by subdividing each of its edges. Then $\gamma(S_n) = n$, $\gamma_t(S_n) = n + 1$, and Corollary 6 can be used. Hence $\gamma(S_n \times S_n) \leqslant n^2 + 2n$ which is close to the real value, since $\gamma(S_n \times S_n) \geqslant \rho(S_n)\gamma_t(S_n) = n^2 + n$. On the other hand, Corollary 4 only yields $\gamma(S_n \times S_n) \leqslant 3n^2$.

To conclude this section note that if $\gamma(G) = \gamma_t(G)$ and $\gamma(H) = \gamma_t(H)$ then Corollary 6 implies $\gamma(G \times H) \leqslant \gamma(G)\gamma(H)$. If, in addition, $\rho(G) = \gamma(G)$ or $\rho(H) = \gamma(H)$ then $\gamma(G \times H) = \gamma(G)\gamma(H)$. This last observation appeared already in [17].

4. Upper domination in direct products

In this section we prove the before mentioned conjecture on the upper domination number of direct products. The proof follows similar lines as the proof of a related conjecture for the Cartesian product given in [2]. For this purpose, we first present a partition of the vertex set of a graph depending on a given minimal dominating set. This partition is in part based on Theorem 1.

Let D_G be a minimal dominating set of a graph G. If condition (ii) of Theorem 1 holds for a vertex $u \in D_G$, then we say that v is a *private neighbor* of u (that is, v is adjacent only to u among vertices of D_G). Denote by D'_G the set of vertices of D_G that are not isolated in D_G, and note that they must have private neighbors. Let P_G be the set of private neighbors of vertices of D'_G, and let N_G denote the set of vertices of $V(G) \backslash D_G$ which are adjacent to a vertex of D'_G but are not private neighbors of any vertex of D'_G. Set $D''_G = D_G \backslash D'_G$ denoting the vertices of D_G which are isolated in D_G, and finally let the remaining set be R_G, that is, $R_G = V(G) \backslash (D_G \cup P_G \cup N_G)$. We skip the indices if the graph G is understood from the context. Note that given a minimal dominating set D of a graph G, the sets D', D'', P, N and R form a partition of the vertex set $V(G)$. In addition, some pairs of sets must clearly have adjacent vertices (like D' and P), while some other pairs of sets clearly do not have any adjacent vertices. The situation is presented in Fig.1, where a bold arrow from one set to another indicates that every vertex in the first set must be adjacent to a vertex of the second,
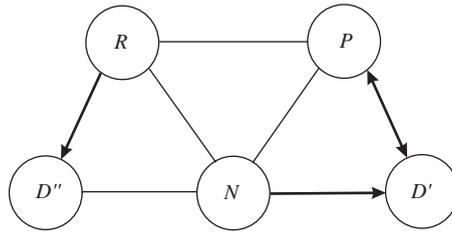
Fig. 1. Partition derived from a minimal dominating set.

a normal line indicates that between the two sets edges are possible (but are not necessary), and no line between two sets means no edges are possible. Note that every vertex of R is adjacent to a vertex of D'', and that every vertex of $N \cup P$ is adjacent to a vertex of D'. Of course, some of the sets could also be empty for a dominating set D.

Theorem 7. *For arbitrary graphs G and H,*

$$\Gamma(G \times H) \geqslant \Gamma(G)\Gamma(H).$$

Proof. We will construct a minimal dominating set D of $G \times H$ that has enough vertices. Let D_G and D_H be minimal dominating sets of G and H, respectively, with maximum cardinality, that is $|D_G| = \Gamma(G)$ and $|D_H| = \Gamma(H)$.

Consider the following set of vertices D in $G \times H$:

$$D = (D_G \times D_H) \cup I,$$

where I is a maximum independent set of the subgraph induced by

$$[(V(G)\backslash D_G) \times D''_H] \cup [D''_G \times (V(H)\backslash D_H)]$$

(if the subgraph is empty then we define I to be empty as well). We claim that D is a minimal dominating set of G, while it is clear that $|D| \geqslant \Gamma(G)\Gamma(H)$.

First let us show that D is a dominating set of $G \times H$. Observe that $D'_G \times D'_H$ dominates $(D'_G \cup P_G \cup N_G) \times (D'_H \cup P_H \cup N_H)$, that $D''_G \times D'_H$ dominates $R_G \times (D'_H \cup P_H \cup N_H)$, analogously $D'_G \times D''_H$ dominates $(D'_G \cup P_G \cup N_G) \times R_H$, and that $D''_G \times D''_H$ dominates $R_G \times R_H$. Finally, if the remainder $[(V(G)\backslash D_G) \times D''_H] \cup [D''_G \times (V(H)\backslash D_H)]$ is nonempty then I dominates it.

To see that D is minimal, first observe that each vertex (u, v) of $D'_G \times D'_H$ has a private neighbor in $P_G \times P_H$. Indeed, if x is a private neighbor of u with respect to D_G and y a private neighbor of v with respect to D_H then (x, y) is adjacent only to (u, v) among vertices of $D'_G \times D'_H$. Other vertices of D are in $D''_G \times V(H)$ or in $V(G) \times D''_H$, and since D'' has no adjacencies with P we infer that (x, y) is a private neighbor of (u, v) with respect to D. It is easy to check that vertices of $D\backslash(D'_G \times D'_H)$ are isolated in D (that is, they enjoy property (i) of Ore's theorem), by considering different cases with respect to the partitions of $V(G)$ and $V(H)$. (See Fig. 2, where the gray squares denote subsets of vertices of $G \times H$ which are all in D, while dotted areas indicate I.) □

Suppose $\Gamma(G \times H) = \Gamma(G)\Gamma(H)$ for some pair of graphs G and H. Recall that all graphs considered have no isolated vertices, and thus $\Gamma(G)$ must be strictly larger than $\alpha(G)$, the vertex independence number of G. This follows since

$$\Gamma(G)\Gamma(H) = \Gamma(G \times H) \geqslant \alpha(G \times H) \geqslant \alpha(G)|H|.$$

In addition, it follows from the proof of Theorem 7 that the set D''_G, and hence also the set R_G, must be empty. Both of these restrictions must similarly hold for H. However, these conditions are not sufficient to force equality in Theorem 7 since both are satisfied for $G = K_2 \Box K_3$, and yet $\Gamma(G \times G) \geqslant \alpha(G \times G) \geqslant 12 > \Gamma(G)\Gamma(G)$. On the other hand we suspect that the bound of Theorem 7 is sharp. Possible candidates for graphs that attain the bound are the Cartesian products $G_n = K_2 \Box K_n$, $n \geqslant 4$. Clearly, $\Gamma(G_n) = n$, but we were not able to prove that $\Gamma(G_n \times G_n) = n^2$.
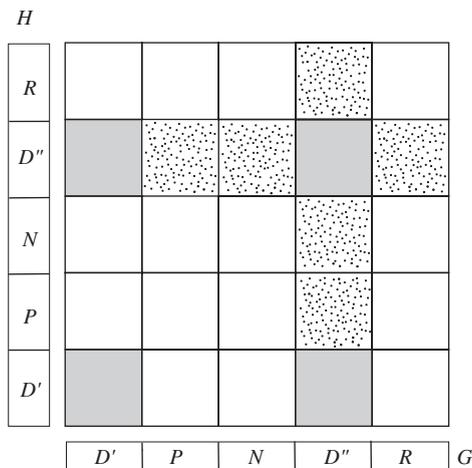
Fig. 2. Upper domination of the direct product.

5. Paired-domination in direct products

For total domination of the direct product of arbitrary graphs G and H the following inequality was shown: $\gamma_t(G \times H) \leqslant \gamma_t(G)\gamma_t(H)$ [15]. Similarly, if D_G is a γ_{pr}-set of G and D_H be a γ_{pr}-set of H, then it is straightforward to see that $D_G \times D_H$ is a dominating set of $G \times H$ which at the same time induces a subgraph with a perfect matching. Hence, for any graphs G and H,

$$\gamma_{pr}(G \times H) \leqslant \gamma_{pr}(G)\gamma_{pr}(H). \tag{1}$$

It is easy to see that for any graphs G and H with $\gamma_{pr}(G) = 2 = \gamma_{pr}(H)$, we obtain $\gamma_{pr}(G \times H) = 4$. (In fact, note that for any nontrivial graphs G and H, $\gamma_{pr}(G \times H) \geqslant 4$.) To present a nontrivial infinite family of graphs for which the above bound is achieved we will make use of the following inequality from [17]:

$$\gamma_t(G \times H) \geqslant \max\{\rho^o(G)\gamma_t(H), \rho^o(H)\gamma_t(G)\}. \tag{2}$$

Proposition 8. *Let G be a graph with $\rho^o(G) = \gamma_{pr}(G)$, and H a graph with $\gamma_t(H) = \gamma_{pr}(H)$. Then $\gamma_{pr}(G \times H) = \gamma_{pr}(G)\gamma_{pr}(H)$.*

Proof. By combining (1) with (2) we obtain

$$\gamma_{pr}(G)\gamma_{pr}(H) \geqslant \gamma_{pr}(G \times H) \geqslant \gamma_t(G \times H) \geqslant \rho^o(G)\gamma_t(H) = \gamma_{pr}(G)\gamma_{pr}(H),$$

and hence

$$\gamma_{pr}(G \times H) = \gamma_{pr}(G)\gamma_{pr}(H). \quad \square$$

Since $\rho^o(G) \leqslant \gamma_t(G)$ holds for an arbitrary graph G (see e.g. [17]), the assumption concerning G in the above proposition implies $\rho^o(G) = \gamma_t(G) = \gamma_{pr}(G)$. This family of graphs includes paths P_n where $n \not\equiv 1 \pmod 4$, double-stars, cycles C_n where $n \equiv 0 \pmod 4$, etc. The complete characterization of trees with equal total and paired-domination numbers is given in [18]. Since $\rho^o(T) = \gamma_t(T)$ for any tree T [17], this characterization yields all trees that satisfy the condition imposed on the graph G in the above proposition.

On the other hand the paired-domination number of the direct product of graphs can be arbitrarily smaller than the upper bound in (1). For example, consider again the subdivided star S_n. Denote its vertex of degree n by x, vertices adjacent to x by $y_1, \ldots, y_n$, and for each i let z_i be a leaf adjacent to y_i. Note that $D = \{x, y_1, \ldots, y_n\} \times \{x, y_1, \ldots, y_n\}$ dominates $S = S_n \times S_n$. Each (y_i, y_j) is adjacent to (z_i, z_j) that is a leaf of S. A matching of S that includes the vertices of

D can be obtained from $\bigcup_i \{(x, y_i)(y_i, x)\}$ by adding $\bigcup_{i,j} \{(y_i, y_j)(z_i, z_j)\}$ and then replacing the edge $(y_1, y_1)(z_1, z_1)$ by $(x, x)(y_1, y_1)$. Clearly, the endvertices of the matching induce a paired-dominating set of size $2n^2 + 2n$, while $(\gamma_{\text{pr}}(S_n))^2 = 4n^2$. We are not aware of any graphs G and H for which $\gamma_{\text{pr}}(G \times H) \leqslant \frac{1}{2}\gamma_{\text{pr}}(G)\gamma_{\text{pr}}(H)$ so we pose:

Problem 9. *Does*

$$\gamma_{\text{pr}}(G \times H) > \tfrac{1}{2}\gamma_{\text{pr}}(G)\gamma_{\text{pr}}(H)$$

hold for all graphs G and H?

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