

GRUPE REDA p^4

Imamo 5 neizomorfih komutativnih grup – za vsako particijo števila 4 eno.

Poleg tega je za $p \geq 3$ še 10 neizomorfih nekomutativnih grup:

- (1) $\langle x, y | x^{p^3} = 1, y^p = 1, y^{-1}xy = x^{1+p^2} \rangle \cong M(p^4),$
- (2) $\langle x, y | x^{p^2} = 1, y^{p^2} = 1, y^{-1}xy = x^{1+p} \rangle,$
- (3) $\langle x, y, z | x^{p^2} = 1, y^p = 1, z^p = 1, y^{-1}xy = x, z^{-1}yz = yx^p, z^{-1}xz = x \rangle,$
- (4) $\langle x, y, z | x^{p^2} = 1, y^p = 1, z^p = 1, x^{-1}yx = y, z^{-1}yz = y, z^{-1}xz = x^{1+p} \rangle \cong M(P^3) \times \mathbb{Z}_p,$
- (5) $\langle x, y, z | x^{p^2} = 1, y^p = 1, z^p = 1, y^{-1}xy = x, z^{-1}yz = y, z^{-1}xz = xy \rangle,$
- (6) $\langle x, y, z | x^{p^2} = 1, y^p = 1, z^p = 1, y^{-1}xy = x^{1+p}, z^{-1}yz = y, z^{-1}xz = xy \rangle,$
- (7) $\langle x, y, z | x^{p^2} = 1, y^p = 1, z^p = x^p, y^{-1}xy = x^{1+p}, z^{-1}yz = y, z^{-1}xz = xy \rangle,$
- (8) $\langle x, y, z | x^{p^2} = 1, y^p = 1, z^p = x^{2p}, y^{-1}xy = x^{1+p}, z^{-1}yz = y, z^{-1}xz = xy \rangle,$
- (9) $\langle v, x, y, z | v^p = 1, x^p = 1, y^p = 1, z^p = 1, v^{-1}zv = zx, v^{-1}yv = y, v^{-1}xv = x, y^{-1}xy = x, z^{-1}yz = y, z^{-1}xz = x \rangle,$
- (10) $p = 3:$ $\langle x, y, z | x^9 = 1, y^3 = 1, z^3 = 1, y^{-1}xy = x, z^{-1}xz = xy, z^{-1}yz = x^{-3}y \rangle,$
 $p > 3:$ $\langle v, x, y, z | v^p = 1, x^p = 1, y^p = 1, z^p = 1, v^{-1}zv = zy, v^{-1}yv = yx, v^{-1}xv = x, y^{-1}xy = x, z^{-1}yz = y, z^{-1}xz = x \rangle.$

Za $p = 2$ imamo 9 neizomorfih nekomutativnih grup reda 16. Prvih 5 se ujema s prvimi petimi v gornjem seznamu (za $p = 2$). Ostale 4 so:

- (6) $\langle x, y, z | x^4 = 1, y^4 = 1, z^2 = 1, y^{-1}xy = x^{-1}, y^2 = x^2, z^{-1}yz = y, z^{-1}xz = x \rangle,$
- (7) $\langle x, y | x^8 = 1, y^2 = 1, y^{-1}xy = x^{-1} \rangle,$
- (8) $\langle x, y | x^8 = 1, y^2 = 1, y^{-1}xy = x^3 \rangle,$
- (9) $\langle x, y | x^8 = 1, y^4 = 1, y^{-1}xy = x^{-1}, x^2 = y^4 \rangle.$

LITERATURA

W. Burnside. Theory of groups of finite order. Dover Publ. 1955, strani 145-146.