Domača naloga : 6

6. aprila 2016

1. Pokaži, da je
$$II(\underline{\underline{a}}) = \frac{1}{2}\left(\left(I(\underline{\underline{a}})\right)^2 - I(\underline{\underline{a}}^2)\right)$$
in izrazi $III(\underline{\underline{a}})$ kot funkcijo spremenljivk $I(\underline{\underline{a}}^k), k = 1, 2, 3$. Tu so $I(\underline{\underline{a}})$, $II(\underline{\underline{a}})$ in $III(\underline{\underline{a}})$ prva, druga in tretja skalarna invarianta tenzorja $\underline{\underline{a}}$.

2. Definiramo $\underline{\underline{C}} = \underline{\underline{F}}^T\underline{\underline{F}}$ in tenzorje $\underline{\underline{a}}_k$ (*Rivlin Ericksenove tenzorje*) s predpisom
$$\frac{D^n\underline{\underline{C}}}{Dt^n} = \underline{\underline{F}}^T\underline{\underline{a}}_k\,\underline{\underline{F}}, \quad k = 0, 1, \ldots$$
Dokaži:
   (i)
   $$\underline{\underline{a}}_1 = 2\underline{\underline{d}}$$
   (ii)
   $$\underline{\underline{a}}_k = \underline{\underline{a}}_{k-1}^{\diamond},$$
   kjer je $\underline{\underline{a}}_{k-1}^{\diamond}$ konvektivni odvod tenzorja $\underline{\underline{a}}_{k-1}$.
   (iii)
   $$\underline{\underline{a}}_2 = \operatorname{grad}\frac{D\,\vec{v}}{Dt} + \left(\operatorname{grad}\frac{D\,\vec{v}}{Dt}\right)^2 + 2\underline{\underline{l}}^T\,\underline{\underline{l}}$$

3. Za tenzor $\underline{\underline{a}}$ definiramo korotacijski odvod s predpisom
$$\underline{\underline{a}}^{\circ} = \frac{D\,\underline{\underline{a}}}{Dt} - \underline{\underline{w}}\,\underline{\underline{a}} + \underline{\underline{a}}\,\underline{\underline{w}}.$$
Dokaži, da je korotacijski odvod koordinatno neodvisen. Tu je $\underline{\underline{w}}$ poševno simetrični del gradienta hitrosti. Izračunaj $\underline{\underline{w}}^{\circ}$.

4. Za konstitutivne zveze
   (i) $\underline{\underline{t}} = -\gamma(t)\underline{\underline{I}}$;
   (ii) $\underline{\underline{t}} = \alpha\left(\underline{\underline{F}} + \underline{\underline{F}}^T\right)$;
   (iii) $\underline{\underline{t}} = \underline{\underline{f}}(\vec{v})$;
   (iv) $\underline{\underline{t}} = \underline{\underline{a}}_2$;
   (v) $\frac{D\,\underline{\underline{t}}}{Dt} = \underline{\underline{w}}\underline{\underline{t}} - \underline{\underline{t}}\underline{\underline{w}} + \alpha\,\mathrm{sl}\left(\underline{\underline{d}}\right)\underline{\underline{I}} + \beta\underline{\underline{d}}$

   ugotovi katere so koordinatno neodvisne.