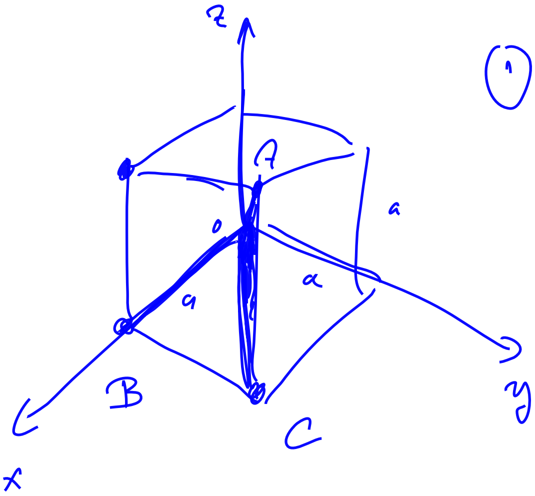
Vaje 18. februar 2021

1. Osnove vektorskega računa: Izračunaj:
   - dolžno glavne diagonale kocke;
   - kot med glavno diagonalo in stranico kocke;
   - kot med glavno diagonalo in diagonalo osnovne ploskve;
   - površino trikotnika med glavno diagonalo in diagonalo osnovne ploskve.



(1) $O(0,0,0)$

$A(a,a,a)$

$\vec d = \vec{OA} = A - O$

$= a\vec i + a\vec j + a\vec k$

$|\vec d| = \sqrt{a^2 + a^2 + a^2} = \sqrt{3a^2} = a\sqrt{3}$

(2) $B(a,0,0)$; $\vec b = \vec{OB} = B - O = a\vec i$

$\vec d \cdot \vec b = |\vec d||\vec b|\cos\varphi$ $\quad |\vec b| = a$

$\cos\varphi = \dfrac{\vec d\cdot\vec b}{|\vec d||\vec b|} = \dfrac{a^2}{\sqrt{3}\,a^2}$

$\vec d\cdot\vec b = a(\vec i + \vec j + \vec k)\cdot a\vec i = a^2(\vec i + \vec j + \vec k)\cdot\vec i = a^2$

$\cos\varphi = \dfrac{1}{\sqrt{3}}$; $\boxed{\varphi = \arccos\dfrac{1}{\sqrt{3}}}$

(3) $C(a,a,0)$ $\quad \vec c = a\vec i + a\vec j$ $\quad |\vec c| = \sqrt{a^2 + a^2} = a\sqrt{2}$

$\vec c\cdot\vec d = |\vec c||\vec d|\cos\psi$

$\cos\psi = \dfrac{\vec c\cdot\vec d}{|\vec c||\vec d|} = \dfrac{a^2\,2}{a^2\sqrt{2}\sqrt{3}} = \dfrac{2}{\sqrt{6}} = \dfrac{\sqrt{2}}{\sqrt{3}} = \sqrt{\dfrac{2}{3}}$

$\boxed{\psi = \arccos\sqrt{\dfrac{2}{3}}}$

(4) $p = \dfrac{1}{2}|\vec c\times\vec d|$ $\quad\Big|\quad p = \dfrac{1}{2}|\vec c|\cdot v = \dfrac{1}{2}a\sqrt{2}\cdot a = \dfrac{a^2}{\sqrt{2}}$

$$\vec{c} \times \vec{d} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & a & 0 \\ a & a & a \end{vmatrix} = a^2 \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

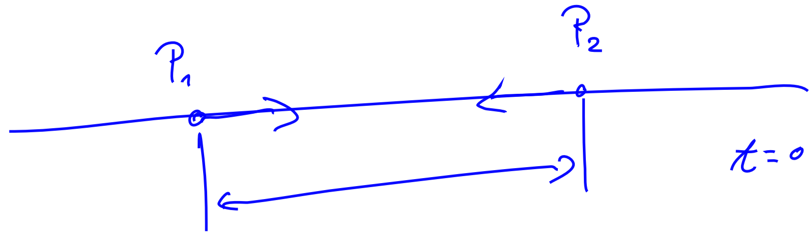
$$= a^2 \left(\vec{i} + \vec{j} + 0\vec{k} \right) = a^2 \left(\vec{i} + \vec{j} \right)$$

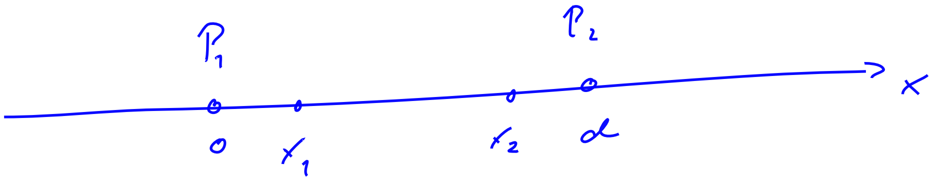
$$\left| \vec{c} \times \vec{d} \right| = a^2 \left| \vec{i} + \vec{j} \right| = a^2 \sqrt{1+1} = a^2 \sqrt{2}$$

$$P = \frac{1}{2} \cdot a^2 \sqrt{2} = \frac{a^2}{\sqrt{2}}$$

2. Točki P_1 in P_2 se gibljeta premočrtno ena proti drugi. Določi čas kdaj in kje se srečata.

- Če se obe gibljeta enakomerno.
- Če se ena giblje enakomerno, druga pa enakomerno pospešeno.
- Določi pospešek iz točke b) tako, da se točki srečata v času iz točke a). Začetna oddaljenost točk je d.





$x_1 = v_1 t$ $\qquad t=0 \qquad x_1(t=0) = 0$

$x_2 = -v_2 t + d$ $\qquad x_2(t=0) = d$

$x = v_0 t + x_0$

$x_1 = x_2$

$v_1 t = -v_2 t + d \Rightarrow (v_1 + v_2) t = d$ $\qquad \boxed{t_1 = \frac{d}{v_1 + v_2}}$

$x_1(t = t_1) = v_1 \cdot \frac{d}{v_1 + v_2}$ $\qquad v_1 = v_2 = v_0$

$x_1 = \frac{d}{2}$ ✓ $\qquad t_1 = \frac{d}{2 v_0}$

$x_1 = v_1 t$

$\ddot{x}_2 = a \Leftrightarrow \dot{x}_2 = a t - v_2$ $\qquad \dot{x}_2(t=0) = -v_2$

$v_2 > 0$

$x_2 = \frac{1}{2} a t^2 - v_2 t + d$ $\qquad x_2(t=0) = d$

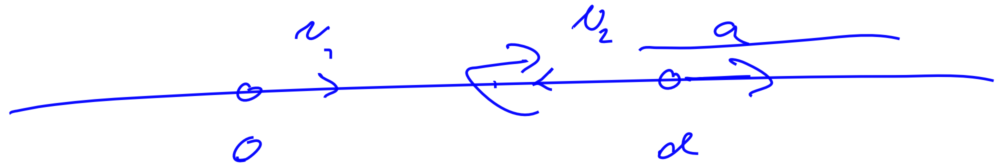
$v_1 t = \frac{1}{2} a t^2 - v_2 t + d$

$0 = \frac{1}{2} a t^2 - (v_1 + v_2) t + d$

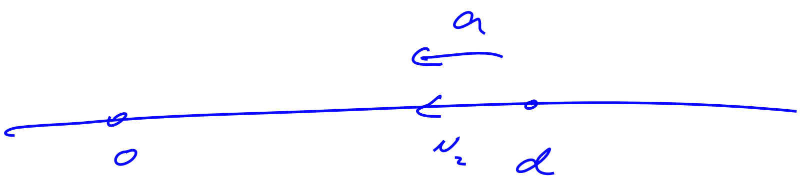
$$t_{1,2} = \frac{(v_1+v_2) \pm \sqrt{(v_1+v_2)^2 - 4\cdot\frac{1}{2}ad}}{2\,\frac{1}{2}a} = \frac{v_1+v_2 \pm \sqrt{(v_1+v_2)^2 - 2ad}}{a}$$

$$(v_1+v_2)^2 - 2ad \geq 0 \Rightarrow (v_1+v_2)^2 \geq 2ad$$

$$\boxed{a \leq \frac{(v_1+v_2)^2}{2d}}$$



$a > 0$



$$\boxed{a < 0}$$

$$(v_1+v_2)^2 - 2ad > (v_1+v_2)^2$$

$$t_1 = \frac{v_1+v_2 + \sqrt{(v_1+v_2)^2 - 2ad}}{a}$$

$$x_2(t = t_1) = v_1 t_1 =$$

(3)

$$\frac{d}{v_1+v_2} = \frac{v_1+v_2 + \sqrt{(v_1+v_2)^2 - 2ad}}{a}$$

$$ad = (v_1+v_2)(v_1+v_2+\sqrt{\quad})$$

$$ad - (v_1+v_2)^2 = (v_1+v_2)\sqrt{(v_1+v_2)^2 - 2ad}$$

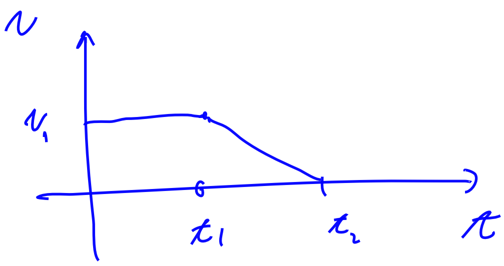
$$\left(ad - (v_1+v_2)^2\right)^2 = (v_1+v_2)^2\left((v_1+v_2)^2 - 2ad\right)$$

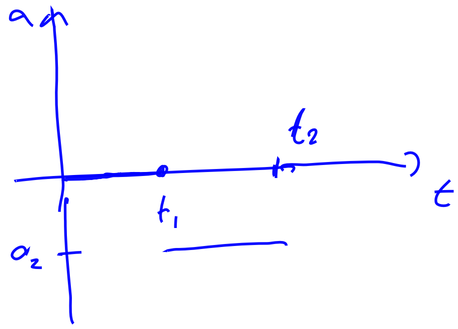
Dobimo kvadratno enačbo za a.

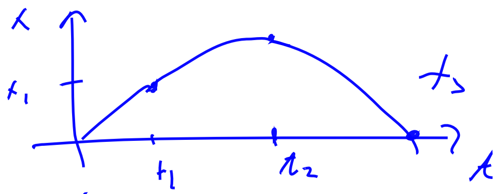
3. Točka se giblje premočrtno po osi x. V času od 0 do t_1 se giblje s konstantno brzino v_1, v času od t_1 do t_2 enakomerno zavira tako, da ima v času t_2 trenuto brzino nič.

- Izračunaj do kod pride v času t_1.
- Izračunaj pospešek zaviranja.
- Do kod pride v času t_2.
- Kdaj se vrne v začetni položaj?
- Izračunaj za konkretne vrednosti $v_1 = 2m/s$, $t_1 = 10s$, $t_2 = 20s$. Nariši tudi diagram hitrosti in položaja v odvisnosti od časa.

x







$x_1 = v_1 t_1 = 20m$

2. $t \in [t_1, t_2]$ velja

$v = v_1 + (t - t_1) a_2$ $\qquad v(t = t_1) = v_1$; $\dot{v} = a_2$

$v(t_2) = 0 \qquad v_1 + (t_2 - t_1) a_2 = 0$

$a_2 = -\frac{v_1}{t_2 - t_1} = -\frac{2m}{s(10)s} = -\frac{1}{5} m s^{-2} = -0.2 m s^{-2}$

$\dot{x} = v = v_1 + (t - t_1) a_2$

$x = \frac{1}{2}(t - t_1)^2 a_2 + v_1 (t - t_1) + x_1$ ✓

$x(t = t_1) = x_1$ ✓ $\qquad \dot{x} = \frac{1}{2} 2 (t - t_1) a_2 + v_1$

$\frac{d}{dt}(t - t_1)^2 = 2(t - t_1)1$

$t \xmapsto{g} t - t_1 \to (t - t_1)^2 \qquad (f(g(t)))^{\cdot} = \dot{f}(g(t))\,\dot{g}(t)$

$$x(t=t_2) = -\frac{1}{2}\cdot 100\,s^2 \cdot \frac{1}{5}\frac{m}{s^2} + 2\frac{m}{s}\,10\,s + 20\,m = -10\,m + 40\,m = \underline{30\,m}$$

$$0 = \frac{1}{2}\underline{(t_3 - t_1)}^2 a_2 + v_1 (t_3 - t_1) + x_1$$

$$(t_3 - t_1)_{1,2} = \frac{-v_1 \pm \sqrt{v_1^2 - 2x_1 a_2}}{2\,\frac{1}{2}\,a_2}$$

$$t_3 - t_1 = \frac{-v_1 + \sqrt{v_1^2 - 2x_1 a_2}}{a_2}$$

$$t_3 = t_1 + \frac{-v_1 + \sqrt{v_1^2 - 2x_1 a_2}}{a_2}$$