Vaje 22. april 2021

1. Podan je napetostni tenzor

$$\underline{\underline{t}} = \begin{bmatrix} -24 & 16 & -8 \\ 16 & 24 & 0 \\ -8 & 0 & 0 \end{bmatrix} \text{MPa}.$$

Izračunaj normalno in strižno napetost na ravnino, ki ima normalo v smeri vektorja $\vec{\imath} - \vec{k}$.

$$\vec{t} = \underline{\underline{t}}\,\vec{n} \quad \checkmark$$

$$t_n = \sigma = \vec{t} \cdot \vec{n}$$

$$t_s^2 = |\vec{t}|^2 - t_n^2$$

$$\vec{n} = \frac{\vec{\imath} - \vec{k}}{|\vec{\imath} - \vec{k}|} = \frac{1}{\sqrt{2}}(\vec{\imath} - \vec{k}) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\vec{t} = \begin{bmatrix} -24 & 16 & -8 \\ 16 & 24 & 0 \\ -8 & 0 & 0 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \text{MPa} =$$

$$= \frac{1}{\sqrt{2}}\, 8 \begin{bmatrix} -3 & 2 & -1 \\ 2 & 3 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \frac{8}{\sqrt{2}} \begin{bmatrix} -3+1 \\ 2 \\ -1 \end{bmatrix} = \frac{8}{\sqrt{2}} \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix} \text{MPa}$$

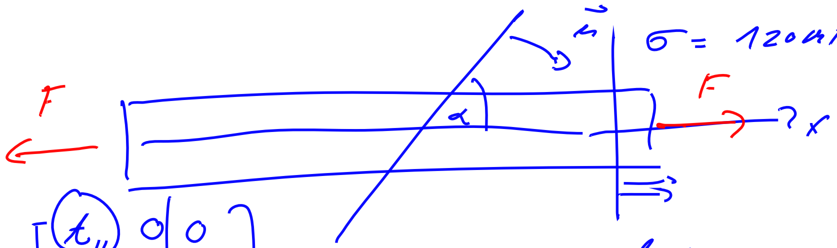
$$t_n = \vec{t} \cdot \vec{n} = \frac{8}{\sqrt{2}} \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \frac{1}{\sqrt{2}} = \frac{8}{2}(-2+1) = -4 \text{ MPa}$$

$$t_s^2 = |\vec{t}|^2 - t_n^2 = \frac{64}{2}(4+4+1) - 16 = 32 \cdot 9 - 16 = 16(18-1) =$$

$$= 16 \cdot 17$$

$$t_s = 4\sqrt{17} \text{ MPa}$$

2. Na preseku osnega elementa pod kotom $\pi/4$ je normalna napetost enaka $\sigma = 120\text{MPa}$. Določi osno silo in strižno napetost, če je napetostno stanje enoosno v smeri osi elementa. Osni element ima površino 4cm^2.

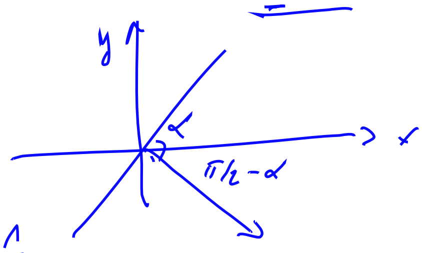


$\sigma = 120\text{MPa}$

$F = A\sigma_0 = A t_{11}$

$$\underline{\underline{t}} = \begin{bmatrix} t_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$t_n = \vec{n} \cdot \underline{\underline{t}}\, \vec{n} = \sigma = 120\text{MPa}$



$$\vec{n} = \cos(\pi/2 - \alpha)\,\vec{i} - \sin(\pi/2 - \alpha)\,\vec{j}$$

$$= \sin\alpha\,\vec{i} - \cos\alpha\,\vec{j} = \frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}$$

$$t_n = \begin{bmatrix} \sin\alpha \\ \cos\alpha \\ 0 \end{bmatrix} \cdot \underbrace{\begin{bmatrix} t_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \sin\alpha \\ \cos\alpha \\ 0 \end{bmatrix}} = \begin{bmatrix} \sin\alpha \\ \cos\alpha \\ 0 \end{bmatrix} \cdot \underbrace{\begin{bmatrix} t_{11}\sin\alpha \\ 0 \\ 0 \end{bmatrix}} = t_{11}\sin^2\alpha$$

$t_n = \frac{1}{2} t_{11}$

$\alpha = \pi/4$

$$\frac{1}{2} t_{11} = \sigma \Rightarrow t_{11} = 2\sigma = 240\text{MPa}$$

$$t_s^2 = |\vec{t}|^2 - t_n^2 = t_{11}^2 \sin^2\alpha - t_{11}^2 \sin^4\alpha = t_{11}^2 \sin^2\alpha\,(1 - \sin^2\alpha) =$$

$$= t_{11}^2 \sin^2\alpha \cos^2\alpha =$$

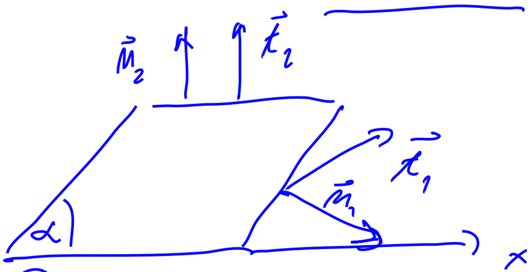
$$= (240)^2 \frac{1}{2} \cdot \frac{1}{2}$$

$t_s = 120\text{MPa}$

$$F = 4\text{cm}^2 \cdot 240\text{MPa} = 960 \cdot 10^{-4}\text{m}^2 \cdot 10^6 \frac{\text{N}}{\text{m}^2} = 96 \cdot 10^3\text{N}$$

$1\text{cm} = 10^{-2}\text{m}$

3. Na rombu z vmesnim kotom α in stranico v smeri osi x je na vodoravni stranici napetost enaka $\vec{t}_2 = \tau \vec{\jmath}$ na poševni stranici pa je napetost enaka $\vec{t}_1 = \frac{\tau}{2}(\vec{\imath} + \vec{\jmath})$. Določi pogoj na kot α, ki dopušča dane napetosti.

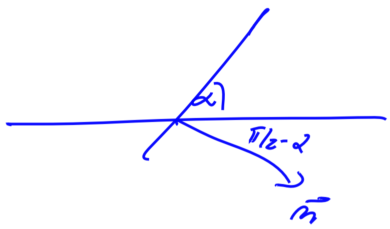


$$\vec t_1 = \underline{t} \cdot \vec n_1$$
$$\vec t_2 = \underline{t} \cdot \vec n_2$$

$$\vec t_1 = \begin{bmatrix} t'_x \\ t'_y \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{12} & t_{22} \end{bmatrix} \begin{bmatrix} n'_x \\ n'_y \end{bmatrix}$$

2 enačbi + 2 enačbi = 4 enačbe

$$\vec t_3 = \underline{t} \cdot \vec n_3$$
$$\min \sum_{i=1}^{N} \left| \vec t_i - \underline{t}\, \vec n_i \right|$$



$$\vec n_1 = \sin\alpha\, \vec\imath - \cos\alpha\, \vec\jmath$$
$$\vec n_2 = \vec\jmath$$

$$\underline{t}\, \vec n_1 = \vec t_1 \Rightarrow \begin{bmatrix} t_{11} & t_{12} \\ t_{12} & t_{22} \end{bmatrix} \begin{bmatrix} \sin\alpha \\ -\cos\alpha \end{bmatrix} = \frac{\tau}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\underline{t}\, \vec n_2 = \vec t_2 = \underbrace{\begin{bmatrix} t_{11} & t_{12} \\ t_{12} & t_{22} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\begin{bmatrix} t_{12} \\ t_{22} \end{bmatrix}} = \tau \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow \begin{matrix} t_{12} = 0 \\ t_{22} = \tau \end{matrix}$$

$$\begin{bmatrix} t_{11} & 0 \\ 0 & \tau \end{bmatrix} \begin{bmatrix} \sin\alpha \\ -\cos\alpha \end{bmatrix} = \frac{\tau}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$t_{11} \sin\alpha = \frac{\tau}{2}, \quad -\tau\cos\alpha = \frac{\tau}{2} \Rightarrow \cos\alpha = -\frac{1}{2}$$
$$\alpha = \frac{2\pi}{3}$$

$$t_{11} \frac{\sqrt{3}}{2} = \frac{\tau}{2} \Rightarrow t_{11} = \frac{\tau}{\sqrt{3}}$$

$$\underline{t} = \tau \begin{bmatrix} \frac{1}{\sqrt{3}} & 0 \\ 0 & 1 \end{bmatrix}$$

4. Z meritvami smo dobili napetosti $\vec{t}(\vec{\imath}) = (10\vec{\imath} + 3\vec{\jmath})$ MPa, $|\vec{t}(\vec{\jmath})| = \sqrt{34}$ MPa in $\vec{t}(\vec{k}) = \vec{0}$ MPa.

- Določi napetostni tenzor.
- Skiciraj napetosti na kvadratu s stranicami v smereh koordinatnih osi x in y.
- Določi ekstremalni normalni napetosti in njuni smeri.
- Skiciraj Mohrovo krožnico.
- Skiciraj napetosti na kvadratu s stranicami v smereh diagonal prvega in drugega kvadranta.
- Določi normalno in strižno napetost v ravnini, ki ima normalo v smeri vektorja $\vec{\imath} + \vec{\jmath} + \vec{k}$.

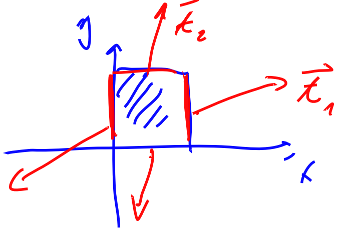
$$\underline{\underline{t}}\,\vec{\imath} = (10\vec{\imath} + 3\vec{\jmath})\,\text{MPa} \Rightarrow \begin{aligned} t_{11} &= 10\,\text{MPa} \\ t_{12} &= 3\,\text{MPa} \\ t_{13} &= 0 \end{aligned}$$

$$|\underline{\underline{t}}\cdot\vec{\jmath}| = |t_{12}\vec{\imath} + t_{22}\vec{\jmath} + t_{23}\vec{k}| = \sqrt{34}\,\text{MPa}$$

$$\underline{\underline{t}}\,\vec{k} = t_{13}\vec{\imath} + t_{23}\vec{\jmath} + t_{33}\vec{k} = \vec{0} \Rightarrow t_{13} = t_{23} = t_{33} = 0$$

$$t_{12}^2 + t_{22}^2 = 34\ ; \quad t_{22}^2 = 34 - 9 = 25 \Rightarrow t_{22} = \pm 5\,\text{MPa}$$

$$\underline{\underline{t}} = \begin{bmatrix} 10 & 3 & 0 \\ 3 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{MPa}$$



$$\sigma_{ext} = \frac{1}{2}(t_{11} + t_{22}) \pm \sqrt{\left(\frac{1}{2}(t_{11} - t_{22})\right)^2 + t_{12}^2} =$$

$$= \left(\frac{1}{2}\,15 \pm \sqrt{\frac{25}{4} + 9}\right)\text{MPa} =$$

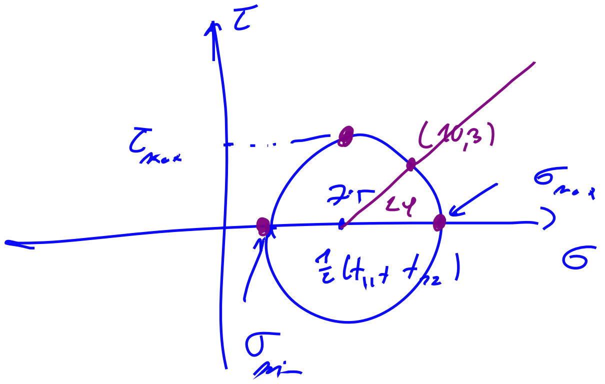
$$= \frac{1}{2}\left(15 \pm \sqrt{25 + 36}\right) =$$

$$\frac{1}{2}\left(15 \pm \sqrt{61}\right)\text{MPa}$$

$$\varphi_{ext_1} = \frac{1}{2}\,\text{arctg}\,\frac{2t_{12}}{t_{11} - t_{22}} \qquad \varphi_{ext_2} = \varphi_{ext_1} + \pi/2$$

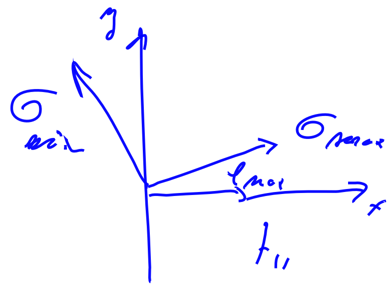
$$\varphi_{ext_1} = \frac{1}{2}\,\text{arctg}\,\frac{2\cdot 3}{5} = \frac{1}{2}\,\text{arctg}\,\frac{6}{5} \qquad t_{11} > t_{22}$$

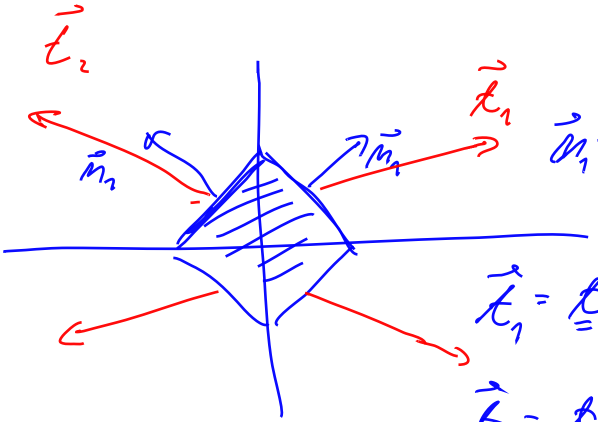
$$\varphi_{max} = \frac{1}{2}\,\text{arctg}\,\frac{6}{5} \qquad \varphi_{min} = \frac{1}{2}\,\text{arctg}\,\frac{6}{5} + \frac{\pi}{2}$$



$g = \frac{1}{2}\sqrt{61} \doteq 3{,}9$

$p(t_{11}, t_{12})$





$\vec n_1 = \frac{1}{\sqrt{2}}(\vec i + \vec j) \qquad \vec n_2 = \frac{1}{\sqrt{2}}(-\vec i + \vec j)$

$$\vec t_1 = \underline{t}\,\vec n_1 = \frac{1}{\sqrt{2}}\begin{bmatrix}10 & 3\\ 3 & 5\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}13\\ 8\end{bmatrix} \text{ MPa}$$

$$\vec t_2 = \underline{t}\,\vec n_2 = \frac{1}{\sqrt{2}}\begin{bmatrix}10 & 3\\ 3 & 5\end{bmatrix}\begin{bmatrix}-1\\ 1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}-7\\ 2\end{bmatrix} \text{ MPa}$$

$$\vec n = \frac{\vec i + \vec j + \vec k}{|\vec i + \vec j + \vec k|} = \frac{1}{\sqrt{3}}(\vec i + \vec j + \vec k)$$

$$\vec t = \underline{t}\,\vec n = \frac{1}{\sqrt{3}}\begin{bmatrix}10 & 3 & 0\\ 3 & 5 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}1\\ 1\\ 1\end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix}13\\ 8\\ 0\end{bmatrix}$$

$$t_n = \vec t \cdot \vec n = \frac{1}{\sqrt{3}}\begin{bmatrix}13\\ 8\\ 0\end{bmatrix} \cdot \frac{1}{\sqrt{3}}\begin{bmatrix}1\\ 1\\ 1\end{bmatrix} = \frac{1}{3}(13 + 8) = 7 \text{ MPa}$$

$$t_s^2 = |\vec t|^2 - t_n^2 = \frac{1}{3}(169 + 64) - 49 = \frac{1}{3}\,233 - 49 = 28{,}6$$

$$\underline{\underline{t_s \doteq 5{,}35 \text{ MPa}}}$$

$$t_{11} + t_{22} = 0$$

5. Naj za ravninsko napetostno stanje velja, da je sl $\underline{\underline{t}} = 0$. Pokaži, da obstaja KS v katerem sta diagonalni komponenti napetostnega tenzorja enaki nič.

$$t_{11}' = \frac{1}{2}(t_{11} + t_{22}) + \frac{1}{2}(t_{11} - t_{22})\cos 2\varphi + t_{12}\sin 2\varphi$$

$$t_{22}' = \frac{1}{2}(t_{11} + t_{22}) - \frac{1}{2}(t_{11} - t_{22})\cos 2\varphi - t_{12}\sin 2\varphi$$

$$0 = t_{11}' + t_{22}' = t_{11} + t_{22}$$

$$t_{11}' = \frac{1}{2}(t_{11} - t_{22})\cos 2\varphi + t_{12}\sin 2\varphi$$

$$t_{11}' = 0 \quad \checkmark \quad \Rightarrow \quad \operatorname{tg} 2\varphi = -\frac{\frac{1}{2}(t_{11} - t_{22})}{t_{12}} \Rightarrow \varphi = \ldots$$

$$t_{22}' = -t_{11}' = 0$$

Napetostno stanje je strižno $\Leftrightarrow$ $\text{sl}\, \underline{\underline{t}} = 0$

$$\underline{\underline{t}} = \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad \underline{\underline{t}} = \begin{bmatrix} 0 & \tau_{12} & \tau_{13} \\ & 0 & \tau_{23} \\ & & 0 \end{bmatrix}$$