Predavanje ~~21~~ 28. april 2021

Mohrove krožnice osnovnih napetostnih stanj

- Enoosno napetostno stanje

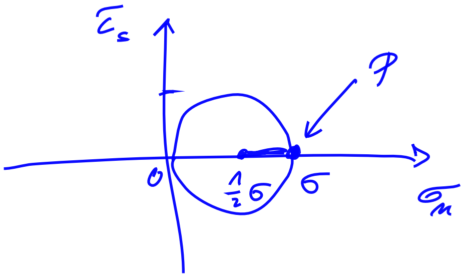
$\sigma > 0$

$$\underline{\underline{t}} = \left[\begin{array}{cc|c} \sigma & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}\right]$$

$$s = \frac{1}{2}(t_{11} + t_{22}) = \frac{1}{2}\sigma$$

$$\rho = \sqrt{\left(\frac{1}{2}(t_{11} - t_{22})\right)^2 + t_{12}^2} = \frac{1}{2}|\sigma|$$

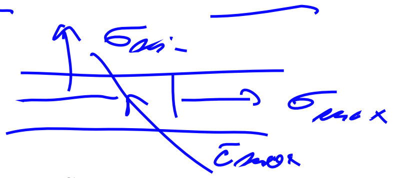
$$\sqrt{\sigma^2} = |\sigma|$$



$P(t_{11}, t_{12})$

$\sigma_{min} = 0$; $\sigma_{max} = \sigma$; $\tau_{max} = \frac{1}{2}\sigma$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$$



- Strižno napetostno stanje

$Sl\ \underline{\underline{t}} = 0$

$$\underline{\underline{t}} = \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

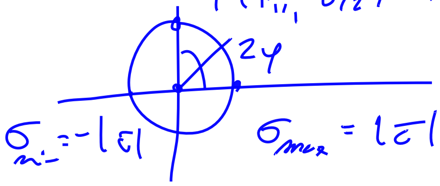
$$\begin{bmatrix} 0 & t_{12} & t_{13} \\ t_{12} & 0 & t_{23} \\ t_{13} & t_{23} & 0 \end{bmatrix}$$

$$s = \frac{1}{2}(t_{11} + t_{22}) = 0$$

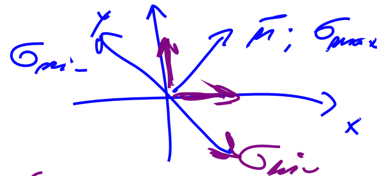
$$\rho = \sqrt{\left(\frac{1}{2}(t_{11} - t_{22})\right)^2 + t_{12}^2} = |\tau|$$

$$P(t_{11}, t_{12}) = (0, \tau)$$

$\tau > 0$



$$\tau_{max} = |\tau|$$



$$\vec{n} \cdot (\underline{\underline{t}}\,\vec{n}) = (-\vec{n}) \cdot \underline{\underline{t}}(-\vec{n})$$

- Hidrostatično napetostno stanje

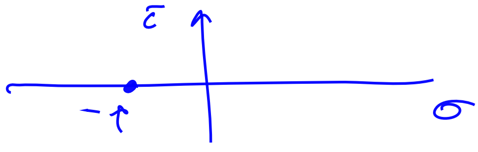
$$\underline{\underline{t}} = \begin{bmatrix} -p & 0 & 0 \\ 0 & -p & 0 \\ 0 & 0 & -p \end{bmatrix}$$

$$\underline{\underline{t}} = \begin{bmatrix} -p & 0 \\ 0 & -p \end{bmatrix} = -p\,I$$

$$s = \frac{1}{2}(t_{11} + t_{22}) = -p$$

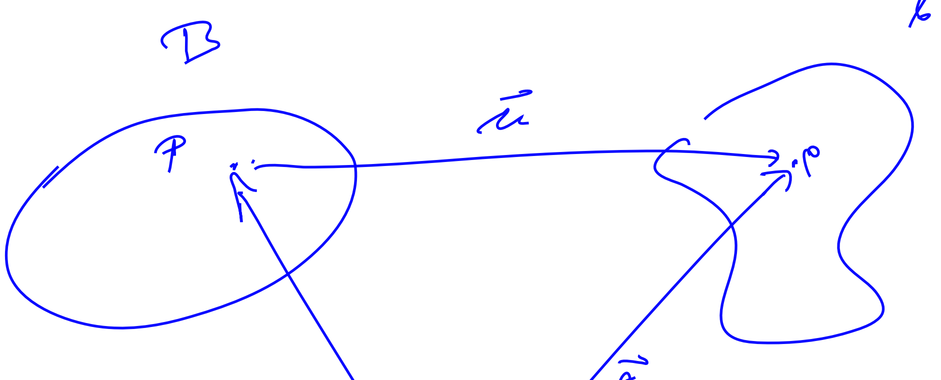
$$\rho = \sqrt{\left(\frac{1}{2}(\underbrace{t_{11} - t_{22}}_{0})\right)^2 + \underbrace{t_{12}^2}_{0}} = 0$$

$p > 0$



$t_n = -p$

$\tau = 0$

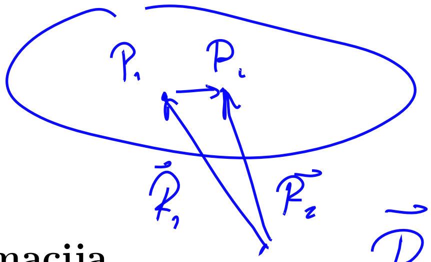


$$\vec{r} = \vec{R} + \underline{\vec{U}(\vec{R})}$$

funkcija pomika

$$\Delta\vec{R} = \vec{R}_2 - \vec{R}_1$$

$$\Delta\vec{R} = d\vec{R}$$



Prostorska deformacija

$$\vec{R}_1 \to \vec{r}_1 = \vec{R}_1 + \vec{U}(\vec{R}_1)$$

$$\vec{R}_2 \to \vec{r}_2 = \vec{R}_2 + \vec{U}(\vec{R}_2)$$

Opis deformacije z vektorjem pomika $\vec{r} = \vec{R} + \vec{u}$.

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\varepsilon_1 = \frac{|\Delta\vec{r}| - |\Delta\vec{R}|}{|\Delta\vec{R}|}$$

$$\Delta\vec{r} = \vec{R}_2 + \vec{U}(\vec{R}_2) - \vec{R}_1 - \vec{U}(\vec{R}_1) = $$

$$= \Delta\vec{R} + \boxed{\vec{U}(\vec{R}_2) - \vec{U}(\vec{R}_1)}$$

Lokalizacija mere deformacije, sprememba dolžin za infinitezimalno bližnje točke.

$$\vec{U}(\vec{R}) = \begin{bmatrix} U_1(\vec{R}) \\ U_2(\vec{R}) \\ U_3(\vec{R}) \end{bmatrix} \qquad U_k(\vec{R}_2) - U_k(\vec{R}_1) \qquad k = 1, 2, 3$$

$$\vec{R}_1 = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \qquad \vec{R}_2 = \vec{R}_1 + \Delta\vec{R} = \begin{bmatrix} X + \Delta X \\ Y + \Delta Y \\ Z + \Delta Z \end{bmatrix}$$

$$U_1(\vec{R}_2) - U_1(\vec{R}_1) = U_1(X+\Delta X, Y+\Delta Y, Z+\Delta Z) - U_1(X, Y, Z) =$$

$$\boxed{f(x+\Delta x) - f(x) \doteq f'(x)\Delta x} \; ; \qquad f(x+\Delta x) - f(x) = f'(x_1)\Delta x$$

$$x < x_1 < x + \Delta x$$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} \approx f'(x) \qquad \frac{\partial U_1}{\partial X}\Delta X$$

$$= U_1(X+\Delta X, Y+\Delta Y, Z+\Delta Z) - U_1(X, Y+\Delta Y, Z+\Delta Z) + U_1(X, Y+\Delta Y, Z+\Delta Z)$$

$$- U_1(X, Y, Z+\Delta Z) + U_1(X, Y, Z+\Delta Z) - U_1(X, Y, Z)$$

$$\text{''} \; \frac{\partial U_1}{\partial Y}\Delta Y \qquad \text{''} \; \frac{\partial U_1}{\partial Z}\Delta Z$$

$$U_k(\vec{R}_2) - U_k(\vec{R}_1) \doteq \frac{\partial U_k}{\partial X}\Delta X + \frac{\partial U_k}{\partial Y}\Delta Y + \frac{\partial U_k}{\partial Z}\Delta Z$$

$$\vec{U}(\vec{R}_2) - \vec{U}(\vec{R}_1) = \begin{bmatrix} \frac{\partial U_1}{\partial X}\Delta X + \frac{\partial U_1}{\partial Y}\Delta Y + \frac{\partial U_1}{\partial Z}\Delta Z \\ \frac{\partial U_2}{\partial X}\Delta X + \frac{\partial U_2}{\partial Y}\Delta Y + \frac{\partial U_2}{\partial Z}\Delta Z \end{bmatrix} =$$

$$\frac{\partial U_3}{\partial X}\Delta X + \frac{\partial U_3}{\partial Y}\Delta Y + \frac{\partial U_3}{\partial Z}\Delta Z$$

Gradinet pomika

$$\operatorname{Grad}\vec{u} = \begin{bmatrix} \frac{\partial u_1}{\partial X} & \frac{\partial u_1}{\partial Y} & \frac{\partial u_1}{\partial Z} \\ \frac{\partial u_2}{\partial X} & \frac{\partial u_2}{\partial Y} & \frac{\partial u_2}{\partial Z} \\ \frac{\partial u_3}{\partial X} & \frac{\partial u_3}{\partial Y} & \frac{\partial u_3}{\partial Z} \end{bmatrix}.$$

$$\begin{bmatrix} \frac{\partial U_1}{\partial X} & \frac{\partial U_1}{\partial Y} & \frac{\partial U_1}{\partial Z} \\ \frac{\partial U_2}{\partial X} & \frac{\partial U_2}{\partial Y} & \frac{\partial U_2}{\partial Z} \\ \frac{\partial U_3}{\partial X} & \frac{\partial U_3}{\partial Y} & \frac{\partial U_3}{\partial Z} \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}$$

$$\vec{U}(\vec{R}_2) - \vec{U}(\vec{R}_1) = \operatorname{Grad}\vec{U}\,\Delta\vec{R}$$

$$\Delta\vec{r} = \Delta\vec{R} + \operatorname{Grad}\vec{U}\,\Delta\vec{R} = (I + \operatorname{Grad}\vec{U})\,\Delta\vec{R}$$

$\underline{\underline{F}}$ gradient deformacije

Gradinet deformacije

$$\Delta\vec{r} = \underline{\underline{F}}\Delta\vec{R}, \quad \underline{\underline{F}} = (\underline{\underline{I}} + \operatorname{Grad}\vec{u}).$$

$$\boxed{\Delta\vec{r} = \underline{\underline{F}}\,\Delta\vec{R}} \qquad d\vec{r} = \underline{\underline{F}}\,d\vec{R}$$

$$\varepsilon_2 = \frac{|d\vec{r}|^2 - |d\vec{R}|^2}{|d\vec{R}|^2} = \frac{|d\vec{r}|^2}{|d\vec{R}|^2} - 1$$

$$|d\vec{r}|^2 = d\vec{r}\cdot d\vec{r} = \underline{\underline{F}}d\vec{R}\cdot\underline{\underline{F}}d\vec{R}$$

$A;\ A_{ij} \to A_{ji}$

$$\boxed{a\cdot Ab = A^T a\cdot b}$$

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \cdot \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\|$$

$$\begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Transponiranje; osnovni lastnosti

- $\vec{a}\cdot\underline{\underline{A}}\,\vec{b} = \underline{\underline{A}}^T\,\vec{a}\cdot\vec{b}$;
- $(\underline{\underline{A}}\,\underline{\underline{B}})^T = \underline{\underline{B}}^T\underline{\underline{A}}^T$.

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \cdot \begin{bmatrix} A_{11}b_1 + A_{12}b_2 \\ A_{21}b_1 + A_{22}b_2 \end{bmatrix} =$$

$$A_{11}a_1b_1 + A_{12}a_1b_2 + A_{21}a_2b_1 + A_{22}a_2b_2$$

$$\begin{bmatrix} A_{11}a_1 + A_{21}a_2 \\ \end{bmatrix} \begin{bmatrix} b_1 \\ \end{bmatrix} = A_{11}a_1b_1 + A_{21}a_2b_1 + A_{12}a_1b_2 + A_{22}a_2b_2$$

$$\left[A_{12}a_1 + A_{22}a_2\right]\left[l_2\right]$$

$$d\vec{r}\cdot d\vec{r} = F d\vec{R}\cdot F d\vec{R} = F^T F\, d\vec{R}\cdot d\vec{R}$$

Deformacijski tenzor

$$\underline{\underline{E}} \quad \underline{\underline{\epsilon}} = \frac{1}{2}\left(\underline{\underline{F}}^T\underline{\underline{F}} - \underline{\underline{I}}\right).$$

$$\varepsilon_2 = \frac{|d\vec{r}|^2 - |d\vec{R}|^2}{|d\vec{R}|^2} = \frac{1}{|d\vec{R}|^2}\left(F^T F\, d\vec{R}\cdot d\vec{R} - d\vec{R}\cdot d\vec{R}\right) = \frac{1}{|d\vec{R}|^2}\left(F^T F - I\right) d\vec{R}\cdot d\vec{R}$$

$$\varepsilon_2 = \left(F^T F - I\right)\frac{d\vec{R}}{|d\vec{R}|}\cdot\frac{d\vec{R}}{|d\vec{R}|}$$

$$\vec{e} = d\vec{R}/|d\vec{R}|$$

$$\varepsilon_2 \doteq 2\varepsilon_1$$

$$\underline{\underline{E}} = \frac{1}{2}\left(F^T F - I\right)$$ deformacijski tenzor

Deformacijski tenzor je simetričen. ✓

$$\underline{\underline{E}}^T = \underline{\underline{E}}\,; \quad \underline{\underline{E}}^T = \frac{1}{2}\left(\left(F^T F\right)^T - I^T\right) = \frac{1}{2}\left(F^T \left(F^T\right)^T - I\right) = \underline{\underline{E}}.$$

$$\left(F^T\right)^T = F$$

$$\vec{U} = 2\vec{U}_0$$

$$\boxed{\varepsilon_2 = 2\underline{\underline{E}}\,\vec{e}\cdot\vec{e}} \Rightarrow \varepsilon_1 = \underline{\underline{E}}\,\vec{e}\cdot\vec{e}$$

Zapis deformacijskega tenzorja s pomikom $\underline{\underline{E}} = \frac{1}{2}(\mathrm{Grad}\,\vec{u} + (\mathrm{Grad}\,\vec{u})^T + (\mathrm{Grad}\,\vec{u})^T(\mathrm{Grad}\,\vec{u}))$.

$$\underline{\underline{F}} = I + \mathrm{Grad}\,\vec{U}\,; \quad \underline{\underline{E}} = \frac{1}{2}\left(\left(I + (\mathrm{Grad}\,\vec{U})^T\right)\left(I + \mathrm{Grad}\,\vec{U}\right) - I\right) \Rightarrow$$

$$\underline{\underline{E}} = \frac{1}{2}\left((\mathrm{Grad}\,\vec{U})^T + \mathrm{Grad}\,\vec{U} + (\mathrm{Grad}\,\vec{U})^T(\mathrm{Grad}\,\vec{U})\right)$$

Infinitezimalni deformacijski tenzor

$$\underline{\underline{\epsilon}} = \frac{1}{2}\left(\mathrm{Grad}\,\vec{u} + (\mathrm{Grad}\,\vec{u})^T\right).$$

Če so elementi gradienta pomika majhni, je

$$\underline{\underline{E}} \doteq \frac{1}{2}\left(\mathrm{Grad}\,\vec{U} + (\mathrm{Grad}\,\vec{U})^T\right)$$

$$\underline{\underline{E}} \doteq \underline{\underline{\epsilon}}$$

$$\frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x}$$

Komponentni zapis infinitezimalnega deformacijskega tenzorja

$$\underline{\underline{\epsilon}} = \begin{bmatrix} \epsilon_x & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{xy}}{2} & \epsilon_y & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{xz}}{2} & \frac{\gamma_{yz}}{2} & \epsilon_z \end{bmatrix} = \begin{bmatrix} \frac{\partial u_1}{\partial X} & \frac{1}{2}(\frac{\partial u_1}{\partial Y} + \frac{\partial u_2}{\partial X}) & \frac{1}{2}(\frac{\partial u_1}{\partial Z} + \frac{\partial u_3}{\partial X}) \\ \frac{1}{2}(\frac{\partial u_2}{\partial X} + \frac{\partial u_1}{\partial Y}) & \frac{\partial u_2}{\partial Y} & \frac{1}{2}(\frac{\partial u_2}{\partial Z} + \frac{\partial u_3}{\partial Y}) \\ \frac{1}{2}(\frac{\partial u_3}{\partial X} + \frac{\partial u_1}{\partial Z}) & \frac{1}{2}(\frac{\partial u_3}{\partial Y} + \frac{\partial u_2}{\partial Z}) & \frac{\partial u_3}{\partial Z} \end{bmatrix}$$

$$\varepsilon_{12} = \frac{1}{2}\gamma_{xy}$$

Zapis infinitezimalne deformacije v smeri enotskega vektorja $\vec{n}$

$$\epsilon_1 = \epsilon_1(\vec{n}) = \vec{n} \cdot \underline{\underline{\epsilon}}\,\vec{n}.$$

$$\vec{n} = \vec{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \vec{e}_1 \cdot (\underline{\underline{\varepsilon}} \cdot \vec{e}_1) = \varepsilon_{11} = \frac{\partial u_1}{\partial X}$$

Pomen komponent deformacijskega tenzorja

- diagonalni elementi so enaki relativnim spremembam v smereh koordinatnih osi; ✓
- izven diagonalni elementi so enaki polovični spremembi kota med koordinatnimi osmi.

$$d\vec{R}_1 \rightarrow d\vec{r}_1 \qquad \cos\varphi = \frac{d\vec{r}_1 \cdot d\vec{r}_2}{|d\vec{r}_1| \cdot |d\vec{r}_2|} = \frac{F d\vec{R}_1 \cdot F d\vec{R}_2}{\sqrt{F d\vec{R}_1 \cdot F d\vec{R}_1}\sqrt{F d\vec{R}_2 \cdot F d\vec{R}_2}} =$$

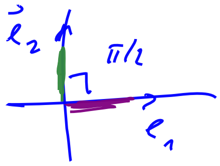
$$\underline{\underline{E}} = \frac{1}{2}(F^T F - \underline{\underline{I}})$$

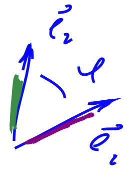
(Deformacija je homogena, če je deformacijski tenzor konstanten.)

$$= \frac{F^T F\, d\vec{R}_1 \cdot d\vec{R}_2}{\sqrt{F^T F\, d\vec{R}_1 \cdot d\vec{R}_1}\sqrt{F^T F\, d\vec{R}_2 \cdot d\vec{R}_2}} = \frac{(2\underline{\underline{E}} + \underline{\underline{I}})\, d\vec{R}_1 \cdot d\vec{R}_2}{\sqrt{(2\underline{\underline{E}} + \underline{\underline{I}})\, d\vec{R}_1 \cdot d\vec{R}_1}\sqrt{(2\underline{\underline{E}} + \underline{\underline{I}})\, d\vec{R}_2 \cdot d\vec{R}_2}}$$

$$\frac{d\vec{R}_i}{|d\vec{R}_i|} = \vec{e}_i \qquad = \frac{2E_{12}}{\sqrt{2E_{11} + 1}\sqrt{2E_{22} + 1}} = \cos\varphi$$

$$\underline{\underline{E}}\,\vec{e}_1 \cdot \vec{e}_2 = E_{12}$$





Pri majhni deformaciji je $|\Delta\varphi| \ll 1$

$\cos\varphi \doteq 2\varepsilon_{12}$

$\sin\varphi \doteq \Delta\varphi$

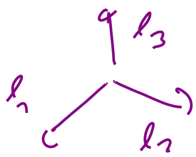
$\Delta\varphi = \varphi - \pi/2$ $\quad -\sin\Delta\varphi = \cos\left(\frac{\pi}{2}+\Delta\varphi\right) \doteq 2\varepsilon_{12}$

$-\Delta\varphi \doteq 2\varepsilon_{12}\,; \quad -\varepsilon_{12} \doteq \frac{1}{2}\Delta\varphi$

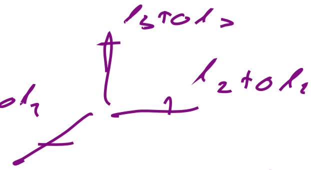
Sprememba volumna.

$$\frac{\Delta v - \Delta V}{\Delta V} = \frac{\Delta v}{\Delta V} - 1 = \epsilon_x + \epsilon_y + \epsilon_z = \mathrm{sl}\,\underline{\underline{\epsilon}}.$$

Relativna sprememba volumna je enaka vsoti diagonalnih elementov deformacijskega tenzorja oziroma sledi deformacijskega tenzorja.



$V = l_1 l_2 l_2$



$v = (l_1+\Delta l_1)(l_2+\Delta l_2)(l_3+\Delta l_3)$

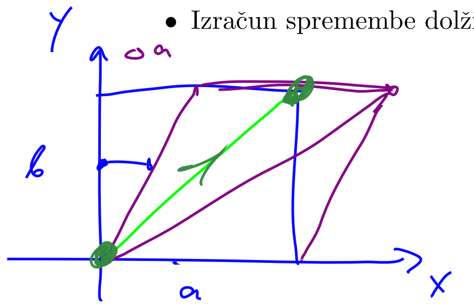
$$v = l_1 l_2 l_3 + (l_2 l_3 \Delta l_1 + l_1 l_3 \Delta l_2 + l_1 l_2 \Delta l_3) + l_1 \Delta l_2 \Delta l_3 + l_2 \Delta l_1 \Delta l_3 + l_3 \Delta l_1 \Delta l_2 + \Delta l_1 \Delta l_2 \Delta l_3$$

$$\frac{v - V}{V} = \frac{1}{l_1 l_2 l_3}\left(l_2 l_3 \Delta l_1 + l_1 l_3 \Delta l_2 + l_1 l_2 \Delta l_3 + \dots\right) = \frac{\Delta l_1}{l_1} + \frac{\Delta l_2}{l_2} + \frac{\Delta l_3}{l_3}$$

$$= \varepsilon_x + \varepsilon_y + \varepsilon_z = \mathrm{sl}\,\underline{\underline{\varepsilon}}$$

Primer: strižna deformacija pravokotnika.

- Določitev pomika iz slike.
- Izračun spremembe dolžine diagonale s pomočjo slike.
- Izračun deformacijskega tenzorja.
- Izračun spremembe dolžine diagonale s pomočjo deformacijskega tenzorja.



$$x = X + \frac{\Delta a}{b} Y \qquad u_1 = \frac{\Delta a}{b} Y$$
$$y = Y \qquad u_2 = 0$$
$$z = Z \qquad u_3 = 0$$

$$\mathrm{Grad}\,\vec{u} = \begin{bmatrix} 0 & \frac{\Delta a}{b} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(\mathrm{Grad}\,\vec{u})^T = \begin{bmatrix} 0 & 0 & 0 \\ \frac{\Delta a}{b} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(\mathrm{Grad}\,\vec{u})^T (\mathrm{Grad}\,\vec{u}) = \begin{bmatrix} 0 & 0 & 0 \\ \frac{\Delta a}{b} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & \frac{\Delta a}{b} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \left(\frac{\Delta a}{b}\right)^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\underline{\underline{E}} = \frac{1}{2} \begin{bmatrix} 0 & \frac{\Delta a}{b} & 0 \\ \frac{\Delta a}{b} & \left(\frac{\Delta a}{b}\right)^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \underline{\underline{\varepsilon}} = \frac{1}{2} \begin{bmatrix} 0 & \frac{\Delta a}{b} & 0 \\ \frac{\Delta a}{b} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \left|\frac{\Delta a}{b}\right| \ll 1$$

$$D = \sqrt{a^2 + b^2} \qquad d = \sqrt{(a+\Delta a)^2 + b^2}$$

$$\varepsilon_2 = \frac{d^2 - D^2}{D^2} = \frac{(a+\Delta a)^2 + b^2 - a^2 - b^2}{a^2+b^2} = \frac{2a\Delta a + (\Delta a)^2}{a^2+b^2} \qquad \Big| \quad \varepsilon_2 = 2\varepsilon_1$$

$$\varepsilon_2 = 2\,\underline{\underline{E}}\,\vec{e}\cdot\vec{e} \quad ; \quad \vec{e} = \frac{a\vec{i} + b\vec{j}}{\sqrt{a^2+b^2}}$$

$$= 2\frac{1}{\sqrt{a^2+b^2}} \begin{bmatrix} a \\ b \end{bmatrix} \cdot \frac{1}{2} \begin{bmatrix} 0 & \frac{\Delta a}{b} & 0 \\ \frac{\Delta a}{b} & \left(\frac{\Delta a}{b}\right)^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{1}{\sqrt{a^2+b^2}} \begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{a^2+b^2} \begin{bmatrix} a \\ b \end{bmatrix} \cdot \begin{bmatrix} \Delta a \\ \frac{\Delta a\, a}{b} + \frac{(\Delta a)^2}{b} \end{bmatrix} =$$

$$= \frac{1}{a^2+b^2} \left(a\Delta a + a\Delta a + (\Delta a)^2\right) =$$

$$= \frac{1}{a^2+b^2} \left(2a\Delta a + (\Delta a)^2\right) \checkmark$$

. Osnovni načini deformacij:

- enoosna;
- enakomerni razteg ali skrčitev, $\underline{\underline{\epsilon}} = \epsilon \underline{\underline{I}}$;
- strižna deformacija, sl $\underline{\underline{\epsilon}} = 0$;
- ravninska deformacija; $u_1 = u_1(X, Y)$, $u_2 = u_2(X, Y)$, $u_3 = 0$.