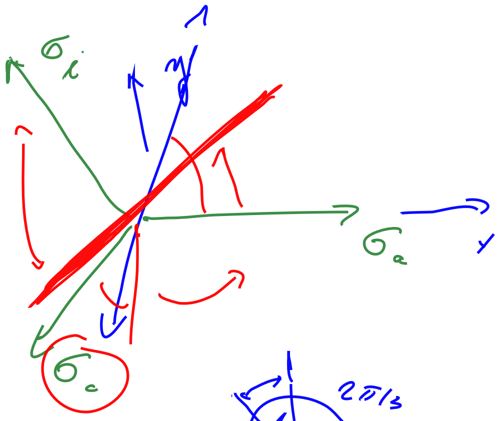
Vaje 29. april 2021

1. V treh smereh, ki z osjo x oklepajo kote $\phi = 0, 2\pi/3$ in $4\pi/3$ smo izmerili normalne napetosti σ_a, σ_b in σ_c.
   - Določi napetostni tenzor.
   - Nariš Mohrovo krožnico za primer $\sigma_\mathrm{a} = \sigma_0$, $\sigma_\mathrm{b} = 2\sigma_0$, $\sigma_\mathrm{c} = 3\sigma_0$.

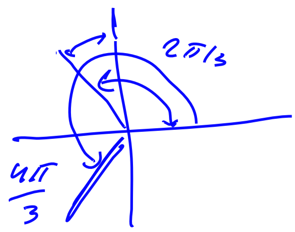
$$\vec n \cdot \underline{t}\,\vec n = (-\vec n)\,\underline{t}\,(-\vec n)$$

$$t'_{n}(\varphi) = \frac12(t_{11}+t_{22}) + \frac12(t_{11}-t_{22})\cos 2\varphi + t_{12}\sin 2\varphi$$



$$\varphi = 0 \qquad t_{11} = t'_{11}(\varphi = 0) = \sigma_a$$

$$\left[\begin{array}{l|l} \cos\frac{4\pi}{3} = -\frac12 & -\frac12 = \cos\frac{8\pi}{3} = \cos\frac{2\pi}{3} \\ \sin\frac{4\pi}{3} = -\frac{\sqrt3}{2} & \frac{\sqrt3}{2} = \sin\frac{2\pi}{3} \end{array}\right.$$



$$\sigma_b = t'_{11}\left(\frac{2\pi}{3}\right) = \frac12(t_{11}+t_{22}) + \frac12(t_{11}-t_{22})\left(-\frac12\right) - t_{12}\frac{\sqrt3}{2}$$

$$\sigma_c = t'_{11}\left(\frac{4\pi}{3}\right) = \underline{\frac12(t_{11}+t_{22}) + \frac12(t_{11}-t_{22})\left(-\frac12\right)} + t_{12}\frac{\sqrt3}{2}$$

$$\sigma_c - \sigma_b = t_{12}\frac{\sqrt3}{2} + t_{12}\frac{\sqrt3}{2} = \sqrt3\, t_{12} \Rightarrow t_{12} = \frac{\sigma_c - \sigma_b}{\sqrt3}$$

$$\sigma_b + \sigma_c = t_{11} + t_{22} - \frac12(t_{11}-t_{22}) = \frac12 t_{11} + \frac32 t_{22}$$

$$t_{22} = \frac23\left(\sigma_b + \sigma_c - \frac12\sigma_a\right)$$

$$\underline{t} = \begin{bmatrix} \sigma_a & \frac{1}{\sqrt3}(\sigma_c - \sigma_b) \\ & \frac23\left(\sigma_b + \sigma_c - \frac12\sigma_a\right) \end{bmatrix} = \sigma_0 \begin{bmatrix} 1 & \frac{1}{\sqrt3} \\ \frac{1}{\sqrt3} & \frac23\underbrace{\left(5 - \frac12\right)}_{\frac92} = 3 \end{bmatrix}$$

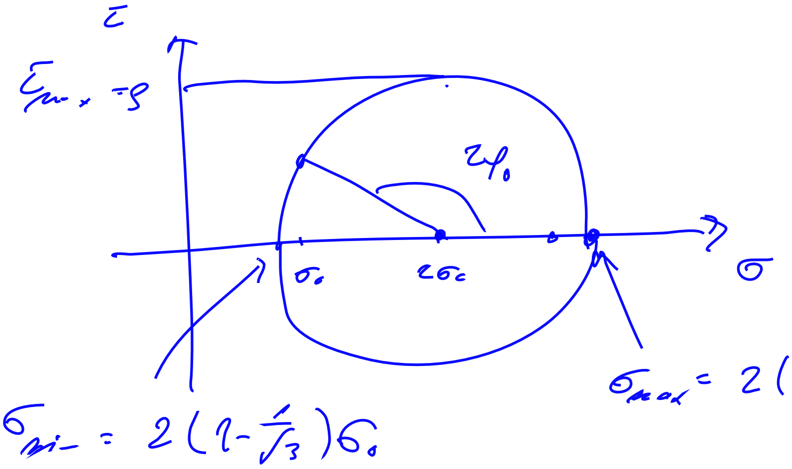
$$s = \frac12(t_{11}+t_{22}) = \sigma_0\,\frac12(1+3) = 2\sigma_0$$

$1 \cdot 1.154\sigma_0$

$$\rho = \sqrt{\left(\frac12(t_{11}-t_{22})\right)^2 + t_{12}^2} = \sigma_0\sqrt{1 + \frac13} = \sigma_0\sqrt{\frac43} = \frac{2}{\sqrt3}\sigma_0$$

$\sigma_0 = 1$

$P(t_{11}, t_{12})$



$$\varphi_0 = \frac{1}{2} \operatorname{arctg} \frac{2t_{12}}{t_{11}-t_{22}} \qquad t_{22} > t_{11}$$

$$\varphi_{max} = \varphi_0 + \frac{\pi}{2} \qquad \varphi_0 = \frac{1}{2} \operatorname{arctg} \frac{6\sigma_0}{-\sigma_0} = -\frac{1}{2} \operatorname{arctg} 6$$

$$\varphi_0 \doteq -40^\circ$$

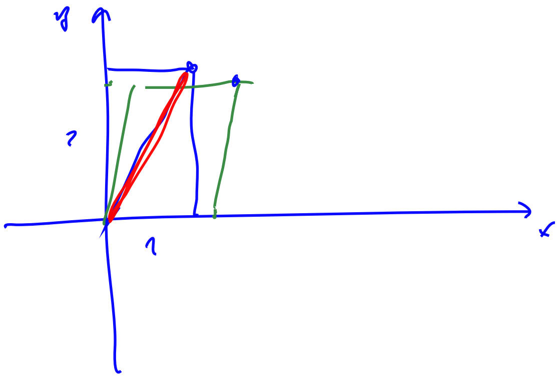
$$\varphi_{max} \doteq 50^\circ$$

2. Ravninska deformacija deformira pravokotnik s stranicama $a = 1$cm in $b = 2$cm v romboid s stranicama $1.02cm$ in $1.97cm$ in vmesnim kotom med stranicama $89.7°$.

   - Zapiši deformacijski tenzor.
   - Določi osno deformacijo diagonale pravokotnika.

$\Delta a = 0.02\,cm$

$\Delta b = -0.03\,cm$



$$\underline{E} = \begin{bmatrix} E_{11} & E_{12} \\ E_{12} & E_{22} \end{bmatrix}$$

$$E_{11} = \frac{\Delta a}{a} = \frac{0.02\,cm}{1\,cm} = 0.02$$

$$E_{22} = \frac{\Delta b}{b} = \frac{-0.03\,cm}{2\,cm} = -0.015$$

$$-E_{12} = \frac{1}{2}(89.7° - 90°) = -\frac{1}{2}\,0.3° \Rightarrow E_{12} = 0.15° = \frac{0.15\cdot\pi}{180}$$

$$180° = \pi \Rightarrow 1° = \frac{\pi}{180}$$

$$E_{12} = \frac{0.01\cdot\pi}{120}$$

$$\underline{\varepsilon} \doteq \underline{E} \doteq 10^{-2}\begin{bmatrix} 2 & \frac{\pi}{12} \\ \frac{\pi}{12} & -1.5 \end{bmatrix} \doteq 10^{-2}\begin{bmatrix} 2 & 1/4 \\ 1/4 & -3/2 \end{bmatrix}$$

$$\vec{n} = \frac{\vec{i} + 2\vec{j}}{\sqrt{5}}$$

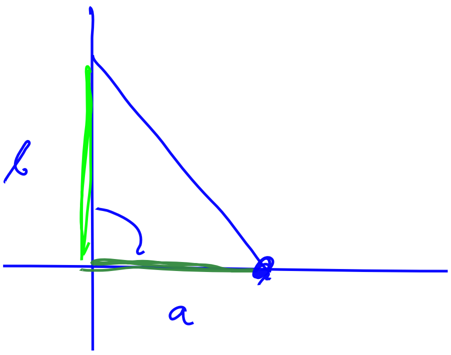
$$\varepsilon \doteq \vec{n}\cdot\underline{E}\,\vec{n} = \vec{n}\cdot\underline{\varepsilon}\,\vec{n}$$

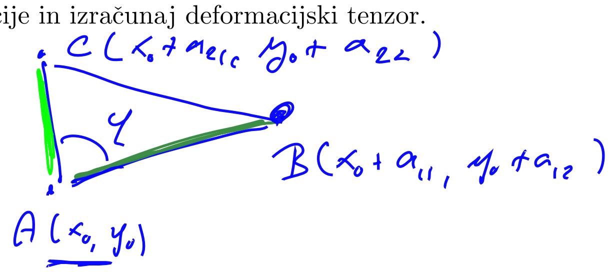
$$\varepsilon = \frac{1}{\sqrt{5}}\begin{bmatrix} 1 \\ 2 \end{bmatrix}\cdot 10^{-2}\begin{bmatrix} 2 & 1/4 \\ 1/4 & -3/2 \end{bmatrix}\frac{1}{\sqrt{5}}\begin{bmatrix} 1 \\ 2 \end{bmatrix} =$$

$$= \frac{1}{5}\cdot 10^{-2}\begin{bmatrix} 1 \\ 2 \end{bmatrix}\begin{bmatrix} 2 + \frac{1}{2} = \frac{5}{2} \\ \frac{1}{4} - 3 = -\frac{11}{4} \end{bmatrix} = \frac{1}{5}\cdot 10^{-2}\Big(\underbrace{\frac{5}{2} - \frac{11}{2}}_{-\frac{6}{2}}\Big) = -6\cdot 10^{-3}$$

3. Ravninska deformacija deformira pravokotni trikotnik z dolžinama stranic a in b v smeri koordinatnih osi v trikotnik z oglišči v točkah $A = (x_0, y_0)$, $B = (x_0 + a_{11}, y_0 + a_{12})$ in $C = (x_0 + a_{21}, y_0 + a_{22})$.

   - Določi deformacijski tenzor na geometrijski način.
   - Zapiši gradient deformacije in izračunaj deformacijski tenzor.





$$|\vec{AB}| = \sqrt{a_{11}^2 + a_{12}^2} \qquad |\vec{AC}| = \sqrt{a_{21}^2 + a_{22}^2}$$

$$\varepsilon_{11} = \frac{|\vec{AB}| - a}{a} = \frac{1}{a}\sqrt{a_{11}^2 + a_{12}^2} - 1$$

$$\varepsilon_{22} = \frac{1}{b}\sqrt{a_{21}^2 + a_{22}^2} - 1$$

$$\cos\varphi = \frac{\vec{AB}\cdot\vec{AC}}{|\vec{AB}|\,|\vec{AC}|} = \frac{a_{11}a_{21} + a_{12}a_{22}}{\sqrt{a_{12}^2 + a_{11}^2}\,\sqrt{a_{21}^2 + a_{22}^2}}$$

$$\varepsilon_{12} = -\frac{1}{2}\left(\varphi - \frac{\pi}{2}\right) \qquad \Delta\varphi = \varphi - \frac{\pi}{2} \Rightarrow \varphi = \Delta\varphi + \frac{\pi}{2}$$

$$\cos\varphi = \cos\left(\Delta\varphi + \frac{\pi}{2}\right) = -\sin\Delta\varphi \doteq -\Delta\varphi$$

$$\varepsilon_{12} = -\frac{1}{2}\Delta\varphi = \frac{1}{2}\cos\varphi = \frac{1}{2}\,\frac{a_{11}a_{21} + a_{12}a_{22}}{\sqrt{a_{12}^2 + a_{11}^2}\,\sqrt{a_{21}^2 + a_{22}^2}}$$

To velja za majhne deformacije

$$\cos\varphi = \frac{a_{21}/a_{22} + a_{12}/a_{11}}{\sqrt{1 + (a_{12}/a_{11})^2}\,\sqrt{1 + (a_{21}/a_{22})^2}}$$

$|\cos\varphi| \ll 1$, če sta $\left|\frac{a_{21}}{a_{22}}\right| \ll 1$ in $\left|\frac{a_{12}}{a_{11}}\right| \ll 1$

$$\varepsilon_{11} = \frac{1}{a}\sqrt{a_{11}^2 + a_{12}^2} - 1 = \frac{|a_{11}|}{a}\sqrt{1 + \left(\frac{a_{12}}{a_{11}}\right)^2} - 1 \doteq \frac{|a_{11}|}{a} - 1$$

$$\frac{|a_{11}|}{a} \doteq 1 \qquad \frac{|a_{22}|}{b} \doteq 1$$

$$\vec{R} \to \vec{r}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} \to \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} X \\ Y \end{bmatrix} - \begin{bmatrix} X \\ Y \end{bmatrix}$$

$$\begin{bmatrix} x_0 + a_{11} \\ y_0 + a_{12} \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} a \\ 0 \end{bmatrix} = \begin{bmatrix} x_0 + a A_{11} \\ y_0 + a A_{21} \end{bmatrix}$$

$$a_{11} = a A_{11} \Rightarrow A_{11} = \frac{a_{11}}{a} \qquad A_{\cancel{12}\,2} = \frac{a_{\cancel{22}\,21}}{b}$$

$$a_{12} = a A_{21} \qquad A_{21} = \frac{a_{12}}{a} \qquad A_{22} = \frac{a_{22}}{b}$$

$$\vec{r} = \vec{R} + \vec{U} = \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} A_{11}-1 & A_{12} \\ A_{21} & A_{22}-1 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

$$\vec{r} = \vec{f}(\vec{R}) = \vec{R} + \vec{f}(\vec{R}) - \vec{R}$$

$$x = x_0 + \left(\frac{a_{11}}{a} - 1\right) X + \frac{a_{12}}{a} Y$$

$$y = y_0 + \frac{a_{12}}{b} X + \left(\frac{a_{22}}{b} - 1\right) Y$$

$$\operatorname{Grad} \vec{U} = \begin{bmatrix} \frac{a_{11}}{a} - 1 & \frac{a_{\cancel{22}\,21}}{a} \\ \frac{a_{12}}{b} & \frac{a_{22}}{b} - 1 \end{bmatrix}$$

$$\underline{\varepsilon} = \frac{1}{2}\left(\operatorname{Grad}\vec{U} + (\operatorname{Grad}\vec{U})^T \right) = \begin{bmatrix} \frac{a_{11}}{a} - 1 & \frac{1}{2}\left(\frac{a_{\cancel{24}\,21}}{a} + \frac{a_{12}}{b}\right) \\ & \frac{a_{22}}{b} - 1 \end{bmatrix}$$

$$\underline{E} = \underline{\varepsilon} + \frac{1}{2} (\operatorname{Grad}\vec{U})^T \operatorname{Grad}\vec{U}$$

$$\varepsilon_{11} = \frac{a_{11}}{a} - 1 \quad ✓$$

$$\vec{U} = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \qquad \underline{\text{grad}\,\vec{U}} = \begin{bmatrix} \frac{\partial U_1}{\partial x} & \frac{\partial U_1}{\partial y} \\ \frac{\partial U_2}{\partial x} & \frac{\partial U_2}{\partial y} \end{bmatrix}$$

4. Z ekstenziometrom smo izmerili osne deformacije $3 \cdot 10^{-3}$, $\sqrt{3} \cdot 10^{-3}$ in $-\sqrt{3} \cdot 10^{-3}$ v smereh, ki oklepajo kot $2\pi/3$.

   - Določi deformacijski tenzor.
   - Določi osno deformacijo v smeri diagonale prvega koordinatnega kvadranta.

5. Podana je funkcija pomika $\vec{U} = \alpha(XY^2\vec{\imath} + X^2Y\vec{\jmath})$.

   (a) Skiciraj, kam se preslika kvadrat $[0, a] \times [0, a]$.
   (b) Izračunaj infinitezimalni deformacijski tenzor.
   (c) Izračunaj deformacijski tenzor in ugotovi kdaj je deformacija majhna.