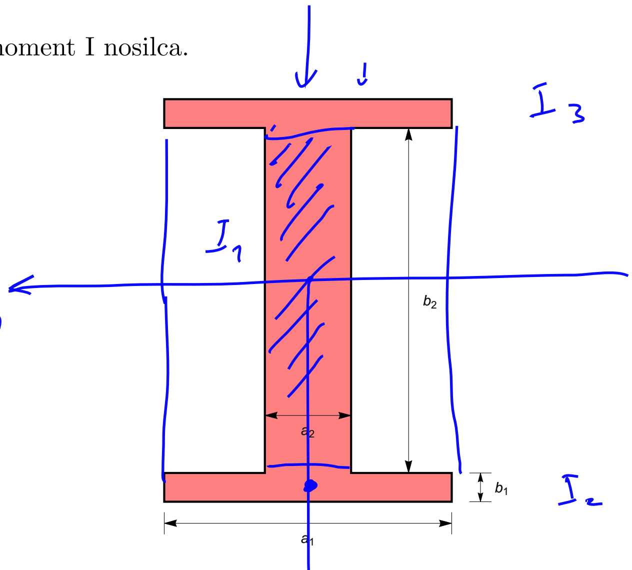
$\sigma = \frac{M}{I} z$ $\qquad I = \int_A z^2 \, dA$

Predavanje 19. maj 2021

$I = \frac{1}{12} a b^3$

Primer: ploskovni moment I nosilca.



Slika 1: I nosilec.

$z_1^* = -z_2^*$

$I = I_1 + I_2 + I_3$

$I_1 = \frac{1}{12} a_2 b_2^3$

$I_2 = (z_2^*)^2 A_2 + \frac{1}{12} a_1 b_1^3 = \frac{1}{12} a_1 b_1^3 + \left(\frac{1}{2}(b_2 + b_1)\right)^2 a_1 b_1$

$I_3 = (z_1^*)^2 A_1 + \frac{1}{12} a_1 b_1^3 = I_2$

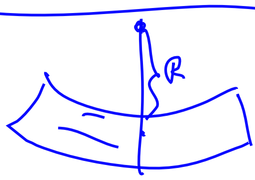
$I = \frac{1}{12} a_2 b_2^3 + \frac{1}{6} a_1 b_1^3 + \frac{1}{2}(b_1 + b_2)^2 a_1 b_1$

$b_1 \ll b_2$

Aproksimacija za tankostensk. nosilec

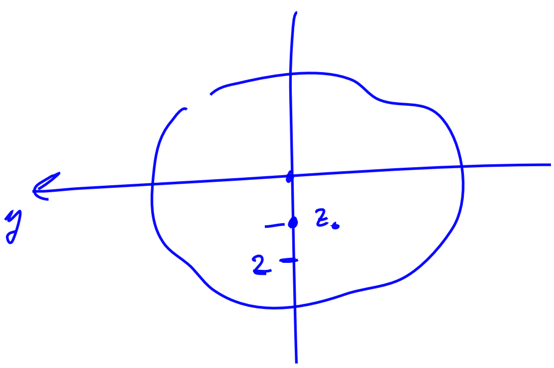
$I \doteq \frac{1}{12} a_2 b_2^3 + \frac{1}{2} a_1 b_1 b_2^2$

$\left| \frac{M}{I} z \right| < \sigma_0$



$\varepsilon = \frac{d}{R} = \frac{z - z_0}{R}$

$\sigma = E\varepsilon = \frac{E}{R}(z - z_0)$



$$0=\int_A \sigma\, dA = \int_A \frac{E}{R}(z-z_0)\,dA = \frac{1}{R}\int_A E(z-z_0)\,dA$$

Določitev z_0 koordinate nevtralne osi v referenčnem položaju. Pogoj

$$0=\int_A E(z-z_0)\,dA.$$

$z_* |A|$

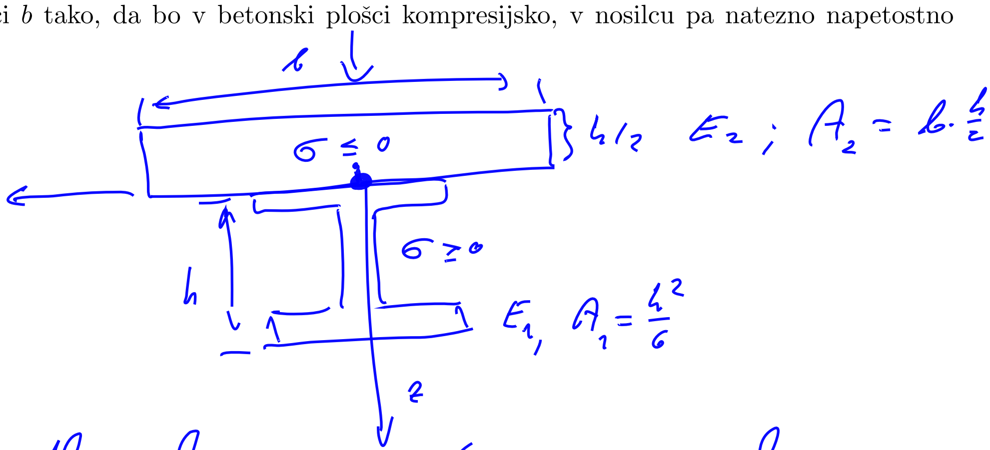
E konstanta $\Rightarrow \int_A (z-z_0)\,dA = 0$; $\int_A z\,dA = z_0\int_A dA = z_0|A|$

$z_* = z_0$

Za nekonstantni E (kompozitni nosilec) moramo vzeti

$$\int_A E(z-z_0)\,dA = 0$$

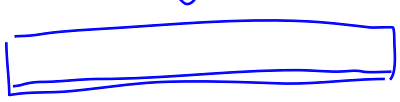
Primer: na jekleni I nosilec višine h in površine $h^2/6$ je postavljena betonska plošča dimenzije $b\times h/2$. Določi b tako, da bo v betonski plošči kompresijsko, v nosilcu pa natezno napetostno stanje.

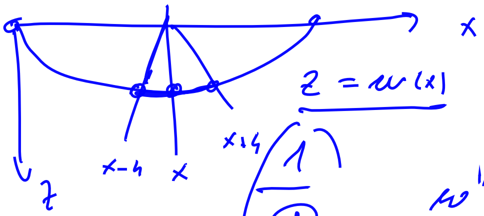


$$0=\int_A E z\,dA = \int_{A_1} E_1 z\,dA + \int_{A_2} E_2 z\,dA = E_1\underbrace{\int_{A_1} z\,dA}_{z_1^*|A_1|} + E_2\underbrace{\int_{A_2} z\,dA}_{z_2^*|A_2|}$$

$$0 = E_1\cdot\frac{h}{2}\cdot\frac{h^2}{6} + E_2\left(-\frac{h}{4}\right)b\cdot\frac{h}{2}$$

$$E_2\cdot\frac{1}{8}h^2 b = E_1\,\frac{1}{12}\,h^3 \Rightarrow b = \frac{E_1}{E_2}\,h\cdot\frac{8}{12} = \frac{2E_1 h}{3E_2}$$





$z = w(x)$ $\quad R$

$$\frac{1}{R} = -\frac{w''(x)}{\left(1 + w'^2(x)\right)^{3/2}}$$

$|w'| \ll 1$

Upogib nevtralne osi

Upogib nevtralne osi $w(x)$, aproksimacija $\frac{1}{R} = -\frac{\partial^2 w}{\partial x^2}$.

$$\frac{1}{R} \doteq -\frac{d^2 w}{dx^2}$$

$$\varepsilon = \frac{z}{R}; \quad \sigma = E\frac{z}{R}; \quad M = \frac{EI}{R} = -EI\frac{d^2 w}{dx^2}$$

$$\frac{dM}{dx} = Q; \quad \frac{dQ}{dx} = -q(x)$$

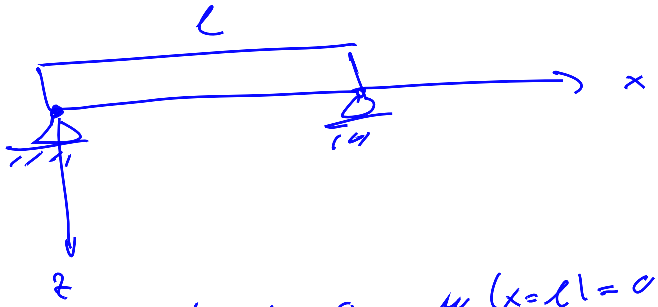
$$\frac{d^2 M}{dx^2} = \frac{dQ}{dx} = -q(x)$$

$$\frac{d^2 M}{dx^2} = -q(x) = -\frac{d^2}{dx^2}\left(EI\frac{d^2 w}{dx^2}\right)$$

$$\frac{d^2}{dx^2}\left(EI\frac{d^2 w}{dx^2}\right) = q(x)$$

$$EI\frac{d^4 w}{dx^4} = q(x)$$

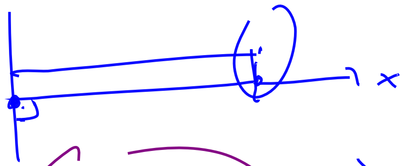
Enačba upogiba $\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 w}{\partial x^2}\right) = q$.



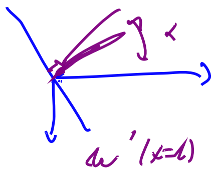
Robni pogoji enačbe upogiba nosilca:

$$w(x=0)=0\,;\quad w(x=l)=0$$
$$M(x=0)=0 \iff w''(x=0)$$

- členkasta podpora;
- konzolno vpetje;
- prosti konec;
- prosti konec;
- predpisana prečna obremenitev na koncu;
- predpisan upogibni moment na koncu.



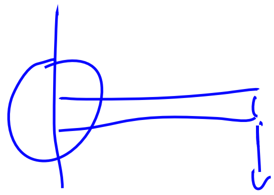
$$w(x=0)=0$$
$$w'(x=0)=0$$



$$w'(x=l) = -\operatorname{tg}\alpha$$

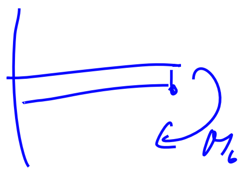
$$x=l\,;\quad M(x=l)=0 \iff w''(x=l)=0$$
$$Q(x=l)=0 \iff w'''(x=l)=0$$

$$\frac{dM}{dx}=Q \qquad M=-EI\,w''$$

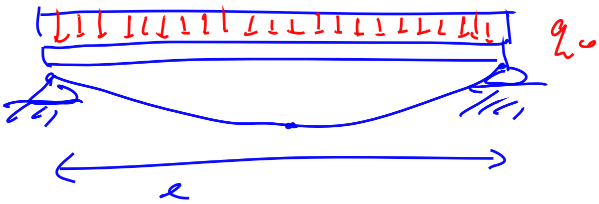


$$M=-EI\frac{d^2w}{dx^2} \Rightarrow -EI\frac{d^3w}{dx^3}=Q$$

$$w'''(x=l)=-\frac{Q}{EI} \qquad w''(x=l)=0$$



$$w'''(x=l)=0\,;\quad w''(x=l)=-\frac{M_0}{EI}$$



Upogib enostavno podprtega nosilca z linijsko obremenitvijo, določitev maksimalnega upogiba

$$w_{\max} = \frac{5q_0 l^4}{384EI};$$

$$EI \frac{d^4 w}{dx^4} = q_0$$

$$w'''' = \frac{q_0}{EI} \Rightarrow w''' = \frac{q_0}{EI} x + C_1$$

$$\to w'' = \frac{1}{2}\frac{q_0}{EI} x^2 + C_1 x + C_2$$

$$w' = \frac{1}{6}\frac{q_0}{EI} x^3 + \frac{1}{2} C_1 x^2 + C_2 x +$$

$$4|' = 4x^3$$

$$\to w = \frac{1}{24}\frac{q_0}{EI} x^4 + \frac{1}{6} C_1 x^3 + \frac{1}{2} C_2 x$$

$$w = \frac{\quad}{24EI} x^4 + D_1 x^3 + D_2 x^2 +$$

$$w(x=0) = 0 \Rightarrow C_4 = 0 \qquad w''(x=0 \quad = 0$$

$$w(x=l) = 0 \Rightarrow \frac{q_0}{24EI} l^4 + \frac{1}{6} \quad = 0$$

$$w''(x=l) = 0 \Rightarrow \frac{q_0}{2EI} l^2 + C_1$$

$$C_1 = -\frac{q_0 l}{2EI} \qquad C_3 = -\frac{q_0}{24EI} l^3 + 6 \frac{\circ}{2EI} l^3 =$$

$$= \frac{q_0 l^3}{12EI}\left(1 - \frac{1}{2}\right) = \frac{q_0 l^3}{24EI}$$

$$w = \frac{q_0}{EI}\left[\frac{1}{24}x^4 - \frac{1}{12} l x^3 + \frac{1}{24} l^3 x\right] =$$

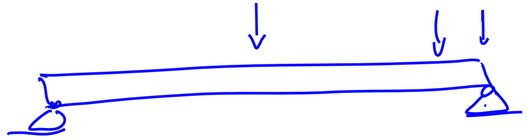
$$= \frac{q_0}{24EI} l^4 \left(\left(\frac{x}{l}\right)^4 - 2\left(\frac{x}{l}\right)^3 + \frac{x}{l}\right) \qquad \frac{x}{l} = \xi$$

$$w\left(\frac{l}{2}\right) = \frac{q_0 l^4}{24EI}\left(\left(\frac{1}{2}\right)^4 - 2\cdot\left(\frac{1}{2}\right)^3 + \frac{1}{2}\right)$$

$$\frac{1}{2}\left(\frac{1}{8} - \frac{1}{2} + 1\right) = \frac{1}{2}\,\frac{1 - 4 + 8}{8} = \frac{5}{16}$$

$$= \frac{5 q_0 l^4}{384 EI} = \frac{5 F l^3}{384 EI}$$

$$l \; F$$

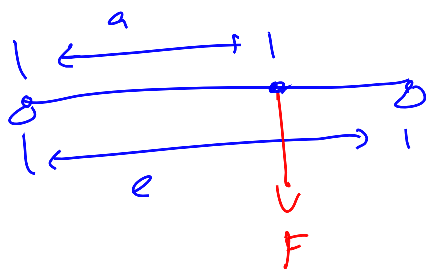


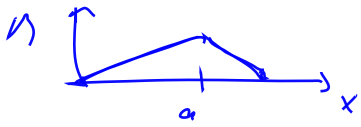
$\frac{5}{384} \leq \frac{1}{48}$

Upogib enostavno podprtega nosilca s točkovno obremenitvijo, določitev maksimalnega upogiba

$$w_{\max} = \frac{C\,Fl^3}{48EI}.$$

$-EI\frac{d^2w}{dx^2} = M$





$0 \leq x \leq a$; $a \leq x \leq l$

$EI w_l'''' = 0$ $EI w_d'''' = 0$

$w_l = C_1 x^3 + C_2 x^2 + C_3 x + C_4$ $w_d = D_1 x^3 + D_2 x^2 + D_3 x + D_4$

$w_l(0) = 0$; $w_l''(0) = 0$ $w_d(l) = 0$; $w_d''(l) = 0$

$C_4 = 0$; $C_2 = 0$

$w_l(a) = w_d(a)$ $w_l''(a) = w_d''(a)$

$w_l'(a) = w_d'(a)$ $w_l'''(a) = w_d'''(a)$

$w_l' = 3C_1 x^2 + 2C_2 x + C_3$

$w_l'' = 6C_1 x + 2C_2$

$w_d = D_1 (x-l)^3 + D_2 (x-l)^2 + D_3 (x-l) + D_4$

$-3D_1 x^2 l + D_2 x^2$

$w_d(l) = 0 \Rightarrow D_4 = 0$

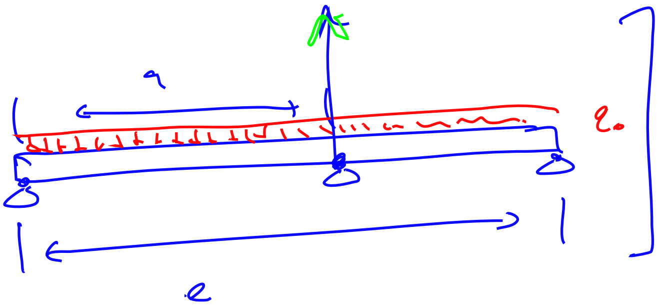
$w_d' = 3D_1 (x-l)^2 + 2D_2 (x-l) + D_3$

$w_d'' = 6D_1 (x-l) + 2D_2 \Rightarrow D_2 = 0$

$a = \frac{l}{2}$

Nosilci z večjim številom polji obremenitev.

Primer: Trotočkovno podprti nosilec.



$0 \le x \le a$ | $a \le x \le l$

w_l | w_d

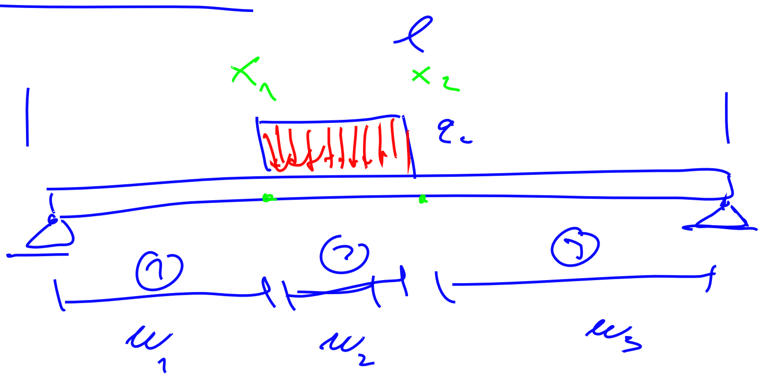
$$w_l = \frac{q_0}{24EI}x^4 + C_1 x^3 + C_2 x^2 + C_3 x + C_4$$

$$w_d = \frac{q_0}{24EI}x^4 + D_1 (x-l)^3 + D_2 (x-l)^2 + D_3 (x-l) + D_4$$

$C_2 = C_4 = 0$; $D_2 = D_4 = 0$ $\qquad w_d'''' = \frac{q_0}{EI}$

$w_l(x=a) = 0 \qquad w_d(x=a) = 0$

$w_l'(x=a) = w_d'(x=a) \qquad w_l''(x=a) = w_d''(x=a)$



$$w_1 = C_1 x^3 + C_2 x^2 + C_3 x + C_4 \quad ; \quad C_2 = C_4 = 0$$

$$w_3 = D_1 (x-l)^3 + D_2 (x-l)^2 + D_3 (x-l) + D_4 \quad ; \quad D_2 = D_4 = 0$$

$$w_2 = \frac{q_0}{24EI}x^4 + E_1 x^3 + E_2 x^2 + E_3 x + E_4$$

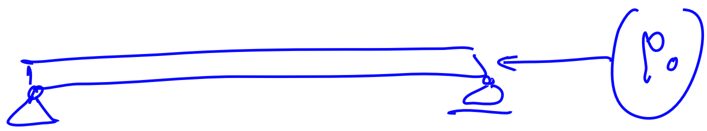
$w_1(x_1) = w_2(x_1)$; $w_1'(x_1) = w_2'(x_1)$; $w_1''(x_1) = w_2''(x_1)$

$w_1'''(x_1) = w_2'''(x_1)$ $\qquad$ Podobno za x_2

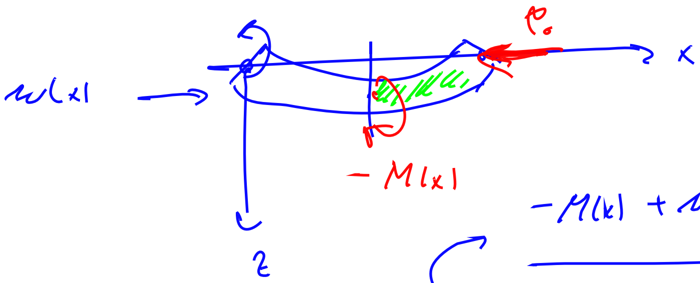
Metoda superpozicije.

Temperaturnii upogib nosilca, razlika temperature po prerezu, temperaturni moment.

Primer: konzolni nosilec, izračun upogiba za linerano razliko temperature po preseku.



Uklon nosilca.



$$-M(x) + w P_0 = 0$$

$$M(x) = -EI\, w''$$

$$w'' + \frac{P_0}{EI} w = 0$$

$$w(0) = 0; \quad w(l) = 0$$

$$w = A \cos \sqrt{\frac{P_0}{EI}}\, x + B \sin \sqrt{\frac{P_0}{EI}}\, x$$

$$w' = -A \sqrt{\;}\, \sin \sqrt{\;}\, x + B \sqrt{\;}\, \cos \sqrt{\;}\, x$$

$$w'' = -A \left(\frac{P_0}{EI}\right) \cos \sqrt{\;}\, x - B \left(\frac{P_0}{EI}\right) \sin \sqrt{\;}\, x$$

$$w(x=0) = A \Rightarrow A = 0$$

$$w(x=l) = B \sin \sqrt{\frac{P_0}{EI}}\, l = 0 \qquad B = 0$$

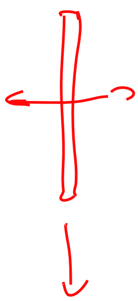
Izračun kritične obremenitve

$$P_c = \frac{\pi^2 EI}{l^2}.$$

$$\sqrt{\frac{P_0}{EI}}\, l = \pi$$

$$\frac{P_0}{EI} = \frac{\pi^2}{l^2}$$

$$P_c = \frac{\pi^2 EI}{l^2}$$



$P_0 < P_c$ ni uklona

$P_0 \geq P_c$ nastopi uklon