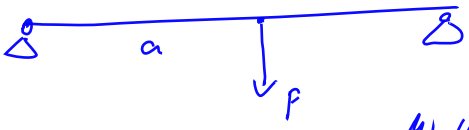
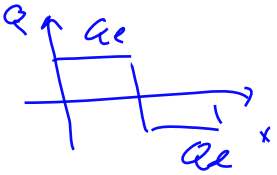
$M = \frac{EI}{R}$; $\frac{1}{R} = -\frac{d^2w}{dx^2}$; $\frac{d^2M}{dx^2} = -q(x) \Rightarrow \frac{d^2}{dx^2}\left(EI\frac{d^2w}{dx^2}\right) - q = 0$

$\frac{dQ}{dx} = -q$

Vaje 20. maj 2021



1. Za konzolno vpeti nosilec z enakomerno obremenitvijo q_0:
   - določi upogib nosilca;
   - določi maksimalen upogib.

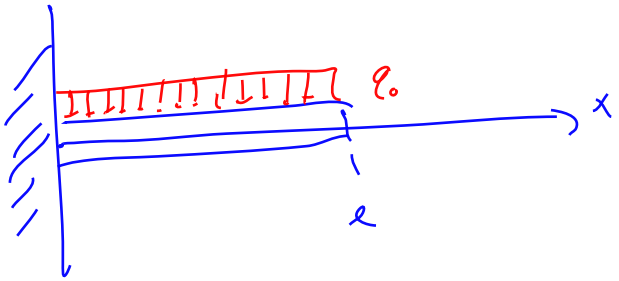


$w_l(a) = w_d(a)$

$w'_l(a) = w'_d(a)$

$w''_l(a) = w''_d(a) \Leftarrow$

$w'''_l(a) = w'''_d(a) \Leftarrow$



$\frac{dM}{dx} = Q \Rightarrow Q = -EI\frac{d^3w}{dx^3}$

$Q_l(a) - F = Q_d(a)$

$-EI\,w'''_l(a) - F = -EI\,w'''_d(a)$

$w'''' = \frac{q_0}{EI}$

$w''' = \frac{q_0}{EI}x + C_1$

$w'' = \frac{1}{2}\frac{q_0}{EI}x^2 + C_1 x + C_2$

$w' = \frac{1}{6}\frac{q_0}{EI}x^3 + \frac{1}{2}C_1 x^2 + C_2 x + C_3 \Leftarrow$

$w = \frac{1}{24}\frac{q_0}{EI}x^4 + \frac{1}{6}C_1 x^3 + \frac{1}{2}C_2 x^2 + C_3 x + C_4 \Leftarrow$

$w(0) = 0$
$w'(0) = 0$

$M(l) = 0,\ Q(l) = 0$

$w'''(l) = 0$

$w''(l) = 0$

$w(0) = 0 \Rightarrow C_4 = 0$

$w'(0) = 0 \Rightarrow C_3 = 0$

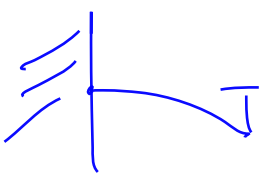
$\frac{q_0 l}{EI} + C_1 = 0 \Rightarrow C_1 = -\frac{q_0 l}{EI}$

$\frac{1}{2}\frac{q_0 l^2}{EI} + C_1 l + C_2 = 0$

$C_2 = -\frac{1}{2}\frac{q_0 l^2}{EI} - C_1 l = \frac{q_0 l^2}{EI}\left(-\frac{1}{2} + 1\right) = \frac{q_0 l^2}{2EI}$

$w = \frac{q_0}{24EI}x^4 - \frac{1}{6}\frac{q_0 l}{EI}x^3 + \frac{1}{2}\frac{q_0 l^2}{2EI}x^2 = \frac{q_0}{2EI}x^2\left(\frac{1}{12}x^2 - \frac{1}{3}lx + \frac{1}{2}l^2\right)$

$w = \frac{q_0 l^2}{2EI}x^2\left(\frac{1}{12}\left(\frac{x}{l}\right)^2 - \frac{1}{3}\frac{x}{l} + \frac{1}{2}\right)$



$f(x)$ na $[a, b]$; Kje $f(x)$ doseže ekstrem?

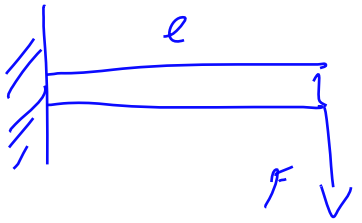
V točki, kjer je $f'(x)=0$ ali na krajiščih.

$$u_{max} = w(x=\ell) = \frac{q_0 \ell^2}{2EI} \ell^2 \left(\frac{1}{12} - \frac{1}{3} + \frac{1}{2} \right) = \frac{q_0 \ell^4}{8EI}$$

$$\frac{1-\xi+\xi}{12} = \frac{1}{4}$$

2. Za konzolno vpeti nosilec s točkovno obremenitvijo na koncu:

   - določi upogib nosilca;
   - določi maksimalen upogib in ga primerjaj z maksimalnim upogibom enekomerno obremenjenega nosilca.



$$EI\,w'''' = 0$$

$$w'''' = 0$$

$$w = C_1 x^3 + C_2 x^2 + C_3 x + C_4$$

$w(0)=0$, $w'(0)=0$

$w''(\ell)=0$ $\quad Q = -EI\,w'''$; $\quad w'''(\ell) = -\frac{F}{EI}$

$w(0)=0 \Rightarrow C_4 = 0$; $\quad w'(0)=0 \Rightarrow C_3 = 0$

$$w' = 3C_1 x^2 + 2C_2 x + C_3$$

$$w'' = 6C_1 x + 2C_2$$

$$w''' = 6C_1$$

$$w'''(\ell) = -\frac{F}{EI} = 6C_1 \Rightarrow C_1 = -\frac{F}{6EI}$$ ✓

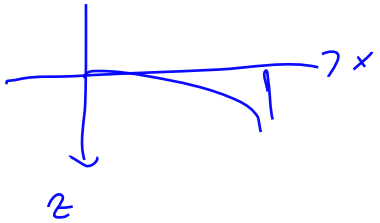
$$0 = w''(\ell) = 6C_1\ell + 2C_2 \Rightarrow C_2 = -\frac{3}{2}C_1\ell = \frac{F\ell}{4EI} = \frac{F\ell}{2EI}$$

$$w = -\frac{F}{6EI}x^3 + \frac{F\ell}{4EI}x^2 = \frac{F\ell}{2\cdot 6EI}\left(-x^3 + \frac{1}{2}\ell x^2\right) =$$

$$= \frac{F\ell^2}{12EI}x^2\left(-\frac{x}{3\ell} + \frac{1}{2}\right)$$

$$w_{max} = w(x=\ell) = \frac{F\ell^3}{12EI}\ell^2\left(-\frac{1}{3}+\frac{1}{2}\right)$$

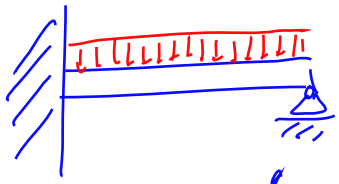
$$-\frac{2}{3}+\frac{3}{3} = \frac{1}{3}$$



$$w_{max} = \frac{F\ell^3}{3EI}$$

.

3. Konzolno vpeti nosilec z enakomerno obremenitvijo z gosoto q_0 je na koncu členkasto podprt. Določi upogib nosilca in silo podpore na koncu.



$$EIw'''' = q_0$$

$$w = \frac{q_0}{24EI}x^4 + C_1x^3 + C_2x^2 + C_3x + C_4$$

$w(0)=0,\ w'(0)=0$; $w(\ell)=0$; $w''(\ell)=0$

$C_3 = C_4 = 0$

$$w' = \frac{q_0}{6EI}x^3 + 3C_1x^2 + 2C_2x$$

$$w'' = \frac{q_0}{2EI}x^2 + 6C_1x + 2C_2$$

$$6C_1\ell + 2C_2 = -\frac{q_0\ell^2}{2EI}$$

$$C_1\ell^3 + C_2\ell^2 = -\frac{q_0\ell^4}{24EI} \qquad C_1\ell + C_2 = -\frac{q_0\ell^2}{24EI} \quad /(-2)$$

$$4C_1\ell = -\frac{q_0\ell^2}{EI}\left(\frac{1}{2} - \frac{1}{12}\right) = -\frac{5q_0\ell^2}{12EI} \Rightarrow C_1 = -\frac{5q_0\ell}{48EI}$$

$\frac{6-1}{12}$

$$C_2 = -C_1\ell - \frac{q_0\ell^2}{24EI} = \frac{q_0\ell^2}{EI}\left(+\frac{5}{48} - \frac{1}{24}\right) = \frac{q_0\ell^2}{48EI}(5-2)$$

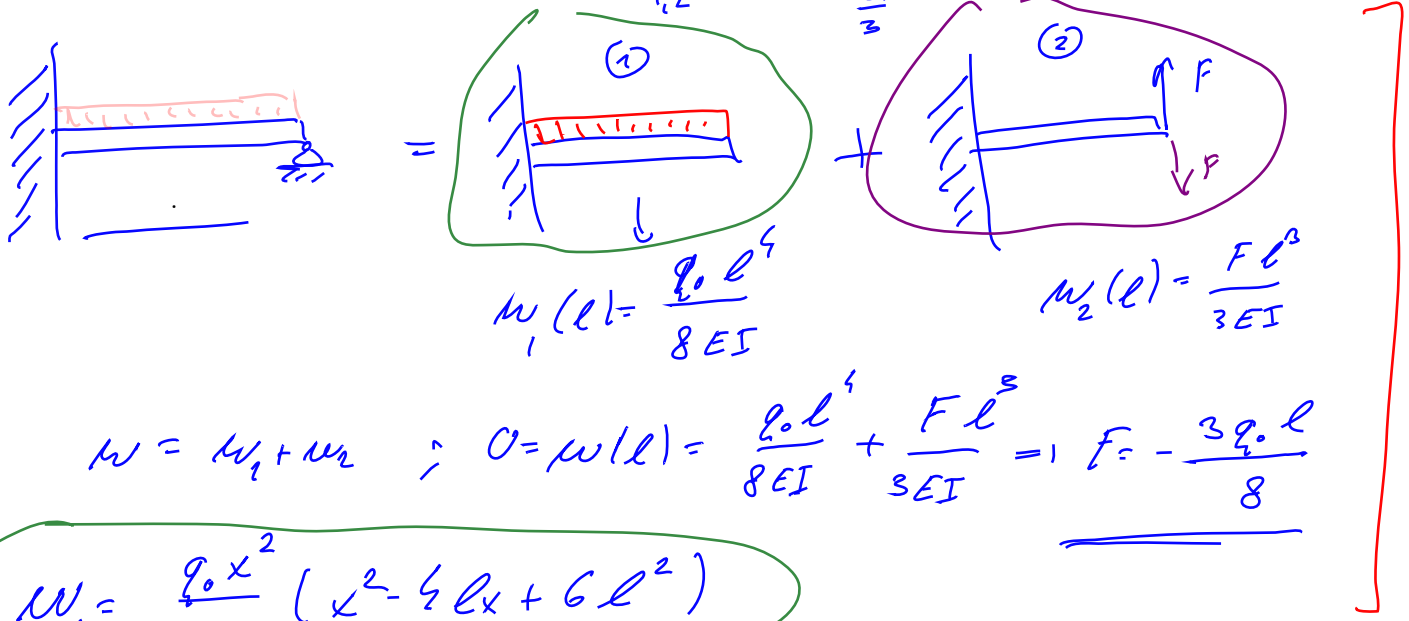
$$C_2 = \frac{3q_0\ell^2}{48EI} = \frac{q_0\ell^2}{16EI}$$

$$w = \frac{q_0}{24EI}x^4 - \frac{5q_0\ell}{48EI}x^3 + \frac{q_0\ell^2}{16EI}x^2 =$$

$$= \frac{q_0}{48EI}x^2\left(2x^2 - 5\ell x + 3\ell^2\right) = \frac{q_0\ell^2}{48EI}x^2\left(2\left(\frac{x}{\ell}\right)^2 - 5\frac{x}{\ell} + 3\right)$$

$$w' = \frac{q_0}{6EI}x^3 - \frac{5q_0\ell}{16EI}x^2 + \frac{q_0\ell^2}{8EI}x = \frac{q_0}{2EI}x\left(\frac{1}{3}x^2 - \frac{5}{8}\ell x + \frac{1}{4}\ell^2\right)$$

$$\frac{1}{3}x^2 - \frac{5}{8}\ell x + \frac{1}{4}\ell^2 = 0 \qquad x_{1,2} = \frac{\frac{5}{8}\ell \pm \sqrt{\frac{25}{64}\ell^2 - \frac{1}{3}\ell^2}}{\frac{2}{3}} \quad \dots$$



$$w_1(\ell) = \frac{q_0 \ell^4}{8EI} \qquad w_2(\ell) = \frac{F\ell^3}{3EI}$$

$$w = w_1 + w_2 \;;\; 0 = w(\ell) = \frac{q_0 \ell^4}{8EI} + \frac{F\ell^3}{3EI} \Rightarrow F = -\frac{3q_0\ell}{8}$$

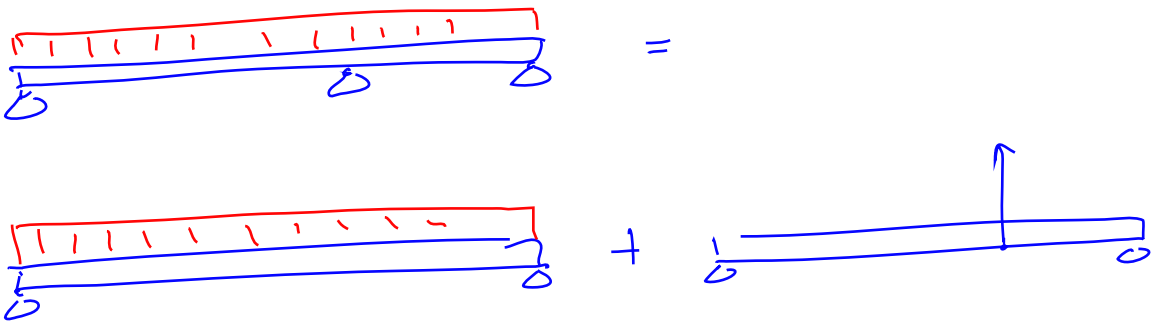
$$w_1 = \frac{q_0 x^2}{24EI}\left(x^2 - 4\ell x + 6\ell^2\right)$$

$$w_2 = \frac{F}{6EI}x^2(-x+3\ell) = -\frac{q_0\ell}{16EI}x^2(-x+3\ell)$$

$$w_1 + w_2 = \frac{q_0 x^2}{8EI}\left(\frac{1}{3}x^2 - \frac{4}{3}\ell x + 2\ell^2 + \frac{1}{2}x\ell - \frac{3}{2}\ell^2\right) =$$

$$= \frac{q_0 x^2}{8EI}\left(\frac{1}{3}x^2 - \frac{5}{6}\ell x + \frac{1}{2}\ell^2\right)$$

$$\frac{1}{2} - \frac{4}{3} = \frac{3-8}{6}$$



4. Enostavno podprti nosilec je na sredini obremenjen s točkovnim upogibnim momentom M_0.

   (a) Določi upogib nosilca.

   (b) Skiciraj upogib in določi maksimalni upogib.

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