Predavanje 26. maj 2021

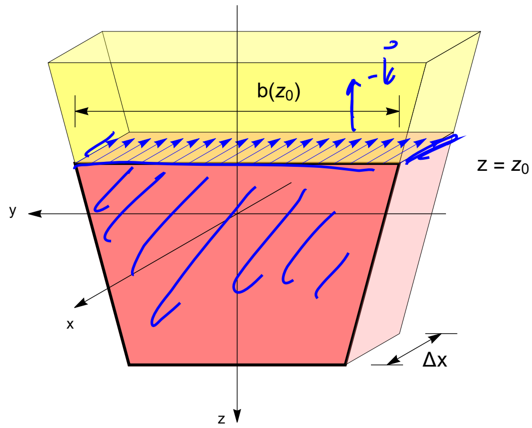
Temperaturni upogib nosilca, razlika temperature po prerezu, temperaturni moment.

Primer: konzolni nosilec, izračun upogiba za linerano razliko temperature po preseku.

Potek strižne napetosti

$\underline{t}\vec{n} = \vec{t}$; $\vec{n} = \vec{\imath}$

$\vec{k}(\underline{t}\vec{n}) = \tau$



$\vec{\imath}\cdot\underline{t}(-\vec{k}) = -\vec{k}\cdot\underline{t}\vec{\imath} = -\tau$

Slika 1: Presek nosilca med x in $x + \Delta x$.

$$\int_{A_0} \sigma(x+\Delta x)\,dA - \int_{A_0} \sigma(x)\,dA \underbrace{-\tau\,\Delta x\, b(z_0)} = 0 \quad \Big/ \frac{1}{\Delta x} \quad \lim_{\Delta x\to 0}$$

$$\tau\, b(z_0) = \lim_{\Delta x\to 0} \int_{A_0} \frac{\sigma(x+\Delta x)-\sigma(x)}{\Delta x}\,dA$$

$$\tau\, b(z_0) = \int_{A_0} \frac{d\sigma}{dx}\,dA \Rightarrow \tau(z_0) = \frac{1}{b(z_0)} \int_{A_0} \frac{d\sigma}{dx}\,dA$$

$$\sigma = \frac{M\bar{I}}{R} \ ; \quad \frac{d\sigma}{dx} = \frac{d}{dx}\left(\frac{M\bar{I}}{R}\right) = \frac{\bar{I}}{R}\frac{dM}{dx}$$

$$\sigma = \frac{M}{I}z \ ; \quad \frac{d\sigma}{dx} = \frac{z}{I}\frac{dM}{dx} = \frac{zQ}{I}$$

$$\tau(z_0) = \frac{1}{b(z_0)} \int_{A_0} \frac{Q}{I} z\,dA = \frac{Q}{I\,b(z_0)} \boxed{\int_{A_0} z\,dA}$$

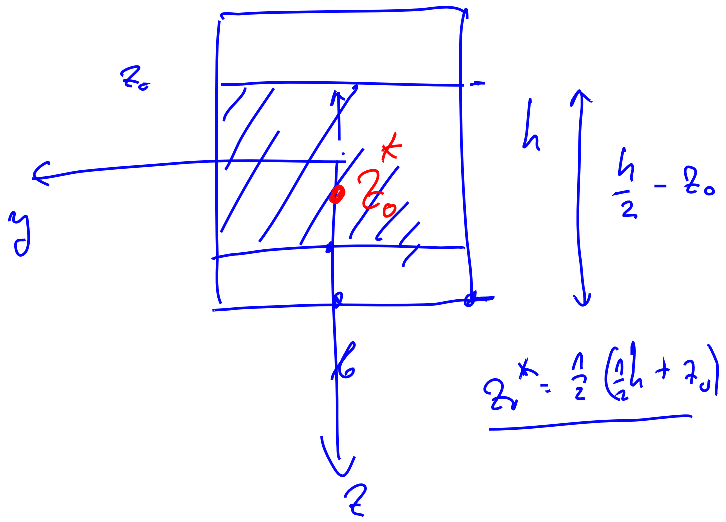
$$= \underline{S(z_0)}$$

$$z_* = \frac{1}{A}\int_A z\,dA$$

$\bar{\tau}(z_0) = \dfrac{Q\, S(z_0)}{I\, b(z_0)}$ $\boxed{\int_A \bar{\tau}(z)\, dA = Q}$ ✓

Linearni ploskovni moment(statični moment) $S(z) = \int_{A*} \zeta\, dA$.

Formula $\tau(z) = \frac{QS(z)}{Ib(z)}$.



$z_0^* = \frac{1}{2}\left(\frac{1}{2}h + z_0\right)$

Primer: strižna napetost na pravokotnem preseku:

$$b(z_0) = b$$

$$S(z_0) = \int_{A_0} z\,dA = A_0 \cdot z_0^* = b \cdot \left(\frac{h}{2} - z_0\right)\frac{1}{2}\left(\frac{h}{2} + z_0\right)$$

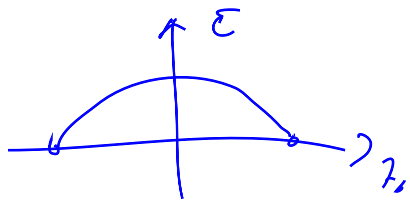
$$I = \frac{1}{12} \cdot b \cdot h^3$$

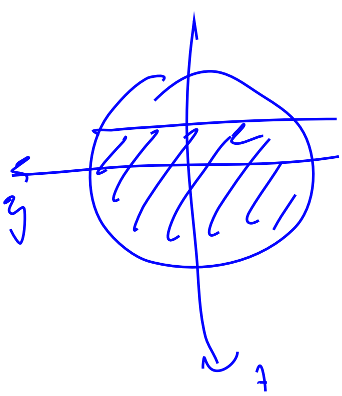
$$\tau(z_0) = \frac{12Q \cdot \cancel{b} \cdot \frac{1}{2}\left(\frac{h}{2} - z_0\right)\left(\frac{h}{2} + z\right)}{\cancel{b}\, h^3 \cdot b} = \frac{6Q}{h^3 b}\left(\frac{h}{2} + z_0\right)\left(\frac{h}{2} - z\right)$$

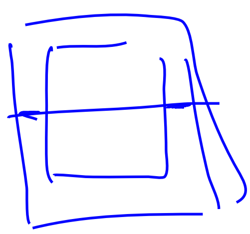
$$\tau(z_0) = \frac{6Q \cdot \left(\frac{h^2}{4} - z_0^2\right)}{h^3 b} = \frac{3}{2}\,\frac{Q\left(h^2 - 4z_0^2\right)}{h^3 b} = \frac{3Q\left(1 - 4\left(\frac{z_0}{h}\right)^2\right)}{2bh}$$

$$\tau(h/2) = 0 \; ; \quad \tau(-h/2) = 0$$

$$\tau(z_0 = 0) = \frac{3Q}{2bh} = \frac{3}{2}\frac{Q}{A} \; ; \quad A = bh$$

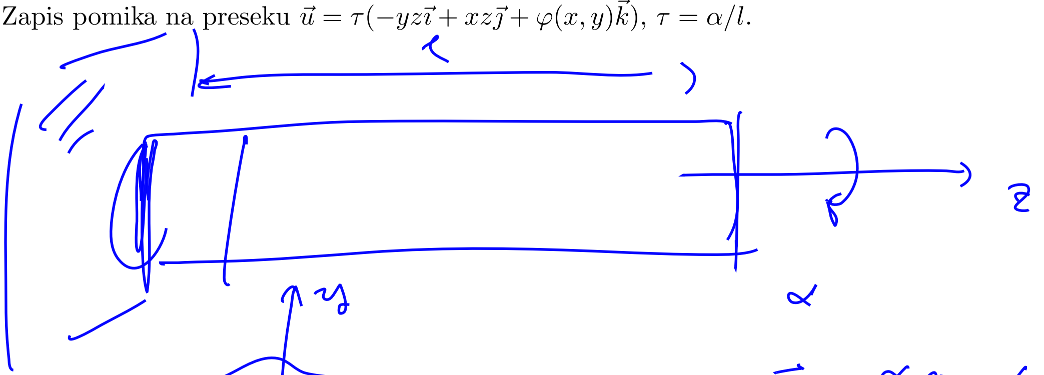


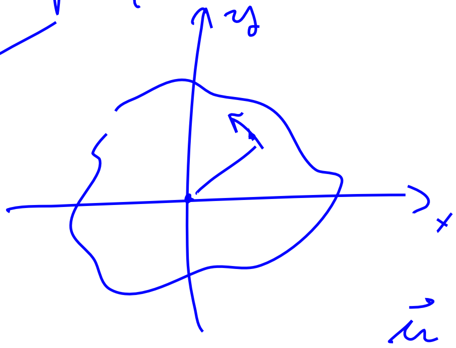




Torzija

Zapis pomika na preseku $\vec{u} = \tau(-yz\vec{i} + xz\vec{j} + \varphi(x,y)\vec{k})$, $\tau = \alpha/l$.





$$\boxed{\vec{u} = r\underline{\vec{e}_\varphi}\,\frac{\alpha}{l}z + \frac{\alpha}{l}\phi(x,y)\vec{k}}$$

$$\vec{u} \to \underline{e} \to \underline{t} \quad ; \quad \mathrm{div}\,\underline{t} = 0$$

$$\vec{e}_r = \cos\varphi\,\vec{i} + \sin\varphi\,\vec{j} \quad ; \quad x = r\cos\varphi \quad y = r\sin\varphi$$

$$\vec{e}_r = \frac{x}{r}\vec{i} + \frac{y}{r}\vec{j} \qquad \vec{e}_\varphi = -\sin\varphi\,\vec{i} + \cos\varphi\,\vec{j} = -\frac{y}{r}\vec{i} + \frac{x}{r}\vec{j}$$

$$\vec{u} = \frac{\alpha}{l}\left(-yz\vec{i} + xz\vec{j} + \phi(x,y)\vec{k}\right) \qquad r\vec{e}_\varphi = -y\vec{i} + x\vec{j}$$

$$u_1 = \frac{\alpha}{l}(-yz), \quad u_2 = \frac{\alpha}{l}xz, \quad u_3 = \frac{\alpha}{l}\phi$$

~~Osni pomik je rešitev naloge $\Delta\varphi = 0$ z robnim pogojem $\frac{\partial\varphi}{\partial n\,x} + \frac{\partial\varphi}{\partial n\,y} = yn_x - xn_y$.~~

$$\underline{e}; \quad e_{11} = \frac{\partial u_1}{\partial x} = 0; \quad e_{22} = \frac{\partial u_2}{\partial y} = 0; \quad e_{33} = \frac{\partial u_3}{\partial z} = 0$$

Torzija ohranja volumen.

$$e_{13} = \frac{1}{2}\left(\frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x}\right) = \frac{1}{2}\left(-\frac{\alpha}{l}y + \frac{\alpha}{l}\frac{\partial\phi}{\partial x}\right)$$

$$e_{23} = \frac{1}{2}\left(\frac{\partial u_2}{\partial z} + \frac{\partial u_3}{\partial y}\right) = \frac{1}{2}\left(\frac{\alpha}{l}x + \frac{\alpha}{l}\frac{\partial\phi}{\partial y}\right)$$

$$e_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x}\right) = \frac{1}{2}\left(-\frac{\alpha}{l}z + \frac{\alpha}{l}z\right) = 0$$

$$\underline{t} = 2\mu\,\underline{e} + \lambda\,\mathrm{Sl}\,\underline{e}\,\underline{I} = 2\mu\,\underline{e}$$

$t_{11} = t_{22} = t_{33} = t_{12} = 0$; $t_{13} = \mu\frac{\alpha}{\ell}\left(-y + \frac{\partial\phi}{\partial x}\right)$

$t_{23} = \mu\frac{\alpha}{\ell}\left(x + \frac{\partial\phi}{\partial y}\right)$

Omejitev na krožni presek s polmerom r_0.
Velja: če je presek krožni, je $\varphi = 0$.

$$\vec{0} = \operatorname{div}\underline{t} = \left(\frac{\partial t_{11}}{\partial x} + \frac{\partial t_{12}}{\partial y} + \frac{\partial t_{13}}{\partial z}\right)\vec{i} + \left(\frac{\partial t_{21}}{\partial x} + \frac{\partial t_{22}}{\partial y} + \frac{\partial t_{23}}{\partial z}\right)\vec{j} + \left(\frac{\partial t_{31}}{\partial x} + \frac{\partial t_{32}}{\partial y} + \frac{\partial t_{33}}{\partial z}\right)\vec{k} =$$

$$= 0\vec{i} + 0\vec{j} + \left(\mu\frac{\alpha}{\ell}\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right)\right)\vec{k}$$

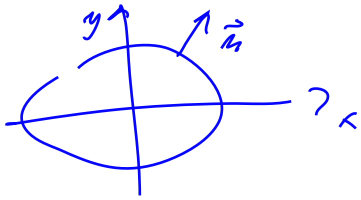
$$\boxed{\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0}$$

ϕ je harmonična funkcija;

Polarni moment $I = \int_A r^2\,dA = \frac{1}{2}\pi a^4$. Polarni moment tankostenskega obroča.

Laplaceov operator $\Delta\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}$: $\boxed{\Delta\phi = 0}$ ✓

Plašč je neobremenjen $\vec{t} = \underline{t}\cdot\vec{n} = \vec{0}$ na plašču



$$\vec{t} = \begin{bmatrix} 0 & 0 & t_{13} \\ 0 & 0 & t_{23} \\ t_{13} & t_{23} & 0 \end{bmatrix}\begin{bmatrix} n_1 \\ n_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ t_{13}n_1 + t_{23}n_2 \end{bmatrix}$$

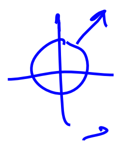
$$0 = t_{13}n_1 + t_{23}n_2 = \left(-y + \frac{\partial\phi}{\partial x}\right)n_1 + \left(x + \frac{\partial\phi}{\partial y}\right)n_2$$

Torzijski moment $M = \mu\tau I$, torzijska togost μI.

$$\boxed{\frac{\partial\phi}{\partial x}n_1 + \frac{\partial\phi}{\partial y}n_2 = y n_1 - x n_2}$$

Za krožni presek je $\phi = \text{konst.} = 0$

$\Delta\phi = 0$ ✓



$0 = y n_1 - x n_2 =$

$\vec{n} = \vec{e}_r = \frac{x}{r}\vec{i} + \frac{y}{r}\vec{j}$

$y\frac{x}{r} - x\frac{y}{r} = 0$ ✓

$\vec{n} = \vec{k}$

$$\vec{M} = \int_A d\vec{M} = \int_A \vec{r}\times\vec{t}\,dA = \int_A \vec{r}\times(\underline{t}\vec{n})\,dA$$

$$\underline{t}\vec{n} = t_{13}\vec{i} + t_{23}\vec{j} = \mu\frac{\alpha}{\ell}\left(-y\vec{i} + x\vec{j}\right)$$

$$\vec{r} \times \vec{t} = (x\vec{i} + y\vec{j}) \times \mu \frac{\alpha}{\ell}(-y\vec{i} + x\vec{j}) = \mu\frac{\alpha}{\ell}(x^2\vec{k} + y^2\vec{k}) = \mu\frac{\alpha}{\ell}(x^2+y^2)\vec{k}$$

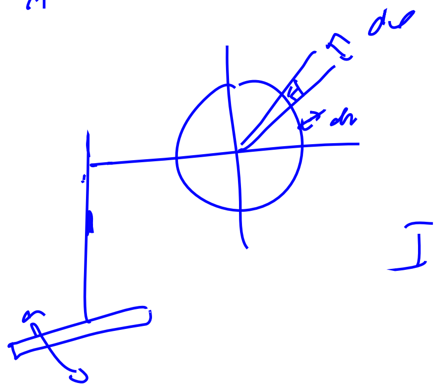
$$\vec{M} = \mu\frac{\alpha}{\ell}\int_A (x^2+y^2)\,dA\;\vec{k}$$

$$I = \int_A (x^2+y^2)\,dA = \int_A r^2\,dA = \int_A r^2 r\,d\varphi\,dr = \int_0^{2\pi} d\varphi \int_0^a r^3\,dr = 2\pi\frac{1}{4}a^4$$

Polární moment

$x = r\cos\varphi$

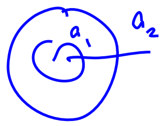
$y = r\sin\varphi$



$dA = r\,d\varphi \cdot dr$

$I = \frac{1}{2}\pi a^4$

$$\boxed{M = \mu\frac{\alpha}{\ell}\frac{1}{2}\pi a^4}$$



$$I = \int_A r^2\,dA = \int_{A_2} r^2\,dA - \int_{A_1} r^2\,dA$$

$$I = \frac{1}{2}\pi(a_2^4 - a_1^4) =$$

$a_1 = a_2 - t$

$a_2 = a_1 + t$

$$= \frac{1}{2}\pi\left(a_2^4 - (a_2 - t)^4\right) = \frac{1}{2}\pi 4 a_2^3 t = 2\pi a_2^3 t \doteq 2\pi a_1^3 t$$

$$a_2^4 - 4a_2^3 t + \underbrace{6a_2^2t^2 - 4at^3 + t^4}$$

$$\frac{\alpha}{\ell} = \vartheta' = \frac{d\vartheta}{dx}$$