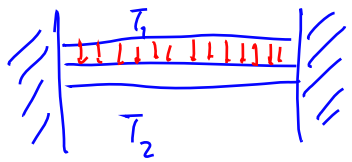
Vaje 27. maj 2021

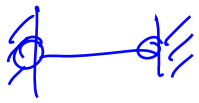
1. Izračunaj termalni upogib nosilca, ki je konzolno vpet na obeh krajiščih.



$\frac{d^2T}{dz^2} = 0$ $\quad T_2 > T_1$

$\Downarrow$

$T = \frac{\Delta T}{h} z + T_0$



$w'' = -\frac{M + M_T}{EI}$ $\quad M_T = \alpha \frac{\Delta T}{h} EI$ $\quad$ konst. $\quad \frac{dq}{dx} = -q_0$

$w''' = -\frac{1}{EI}(M' + M_T) = -\frac{1}{EI} Q$; $\quad w'''' = -\frac{1}{EI}(-q_0) = \frac{q_0}{EI}$

$w'''' = \frac{q_0}{EI}$ $\quad (x^4)''' = (4x^3)'' = (12x^2)' = 24x$

$\rightarrow w = \frac{q_0}{24EI} x^4 + C_1 x^3 + C_2 x^2 + C_3 x + C_4$

$w(0) = 0$; $w'(0) = 0$ $\quad | \quad w(l) = 0$, $w'(l) = 0$

$w(0) = 0 \Rightarrow C_4 = 0$

$w' = \frac{q_0}{6EI} x^3 + 3C_1 x^2 + 2C_2 x + C_3$; $\quad w'(0) = 0 \Rightarrow C_3 = 0$

$w(l) = 0 \Rightarrow \frac{q_0}{24EI} l^4 + C_1 l^3 + C_2 l^2 = 0 \quad / \cdot 2 \quad / \cdot l$

$w'(l) = 0 \Rightarrow \frac{q_0}{6EI} l^3 + 3C_1 l^2 + 2C_2 l = 0$

$\frac{q_0}{EI} l^2 \left(-\frac{1}{12} + \frac{1}{6}\right) + C_1 l = 0$ $\quad \left(\frac{1}{12}\right)$

$\Rightarrow C_1 = -\frac{q_0 l}{12EI}$

$\frac{q_0}{24EI} l^2 - \frac{q_0 l^2}{12EI} + C_2 = 0 \Rightarrow C_2 = \frac{q_0 l^2}{24EI}$

$w = \frac{q_0}{24EI} x^4 - \frac{q_0 l}{12EI} x^3 + \frac{q_0 l^2}{24EI} x^2 = \frac{q_0}{24EI} x^2 (x^2 - 2xl + l^2)$

$w = \frac{q_0}{24EI} x^2 (x-l)^2$

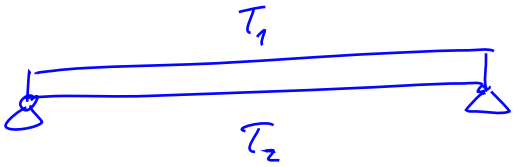
$q_0 = 1 \qquad w = 0$

$$w'' = -\frac{M + M_T}{EI} \qquad M_T = \alpha\rho EI$$

$$0 = -\frac{M + M_T}{EI} = -\frac{M}{EI} - \alpha\rho$$

$$\boxed{M = \alpha\rho EI}$$

2. Izračunaj termalni upogib enostavno podprtega nosilca.



$T_2 > T_1$

$w'' = -\frac{M + M_T}{EI}$; $w'''' = 0$ nosilec ni obremenjen

$w = C_1 x^3 + C_2 x^2 + C_3 x + C_4$

$w(0) = 0$; $w(l) = 0$

$\varphi(0) = 0$; $\varphi(l) = 0$ $\Rightarrow$ $w''(0) = -\frac{M_T}{EI} = -\alpha p$

$w''(l) = -\alpha p$

$w(0) = 0 \Rightarrow C_4 = 0$ $\qquad w' = 3C_1 x^2 + 2C_2 x + C_3$

$w'' = 6C_1 x + 2C_2$

$w''(0) = -\alpha p \Rightarrow 2C_2 = -\alpha p \Rightarrow C_2 = -\frac{1}{2}\alpha p$

$w(l) = 0$ $\qquad C_1 l^3 + C_2 l^2 + C_3 l + C_4 = 0$ ←

$w''(l) = -\alpha p$ $\qquad 6C_1 l + 2C_2 = -\alpha p$

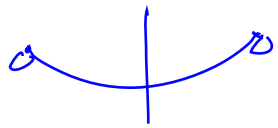
$6C_1 l - \alpha p = -\alpha p \Rightarrow C_1 = 0$

$-\frac{1}{2}\alpha p l^2 + C_3 l = 0 \Rightarrow C_3 = \frac{1}{2}\alpha p l$

$w = -\frac{1}{2}\alpha p x^2 + \frac{1}{2}\alpha p l x = \frac{1}{2}\alpha p x(l - x)$ $\qquad 0 < x < l$

$w' = 0$

$w \geq 0$

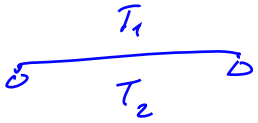


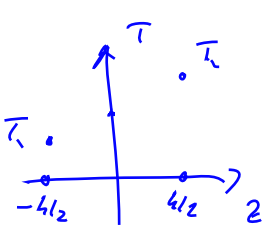
$p > 0$

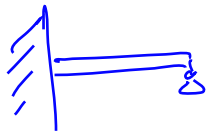
$w' = \frac{1}{2}\alpha p (l - 2x)$ $\qquad x = \frac{l}{2}$

$w_{max} = \frac{1}{2}\alpha p \frac{l}{2} \cdot \frac{l}{2} = \frac{1}{8}\alpha p l^2$

$T = \beta z + T_1$

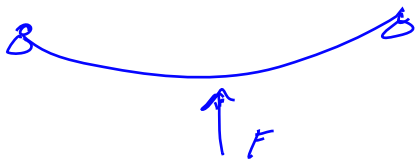






$\beta = \frac{T_2 - T_1}{h}$

$$w_{max} = \frac{1}{8}\alpha\,\frac{T_2 - T_1}{h}\,l^2$$



$w = w_1 + w_2$; $w_1 = \frac{1}{2}\alpha\rho x(l-x)$

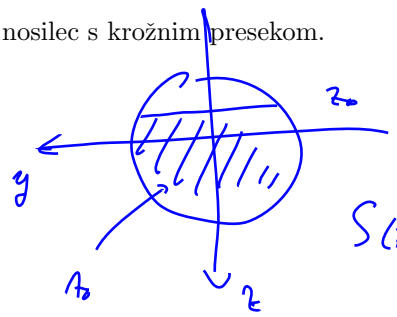
w_2 ; rezite za tačkasto obremenitev pri $x = \frac{l}{2}$

Določi F, da je $w(l/2) = 0$ $\qquad w_{2\,max} = \frac{Fl^3}{48EI}$ ⇓

$$0 = \frac{1}{8}\alpha\rho l^2 + \frac{Fl^3}{48EI} \Rightarrow F = -\frac{\alpha\rho 48EI}{8\,l} = -\frac{6\alpha\rho EI}{e}$$

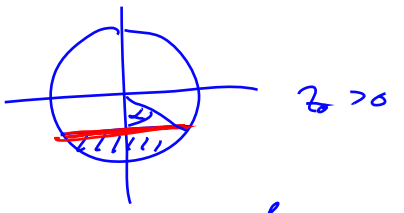
3. Določi potek strižne napetosti za prečno obremenjen nosilec s krožnim presekom.

$$\tau(z_0) = \frac{Q\, S(z_0)}{I\, b(z_0)}$$



$$S(z_0) = \int_{A_0} z\, dA$$

$$S(z_0) = A(z_0)\, z_0^*$$



$$A(z_0) = \frac{r^2}{2}(2\alpha - \sin 2\alpha)$$

$$z_0^* = \frac{4r\sin^3\alpha}{3(2\alpha - \sin 2\alpha)}$$

$$\sin\alpha = \frac{b}{2r} \Rightarrow \qquad \cos\alpha = \frac{z_0}{r}$$

$$S(z_0) = \frac{r^2}{2}\cdot\frac{4r\sin^3\alpha}{3} = \frac{2}{3}r^3\sin^3\alpha \qquad b(z_0)$$

$$b = 2r\sin\alpha \qquad I = \frac{1}{4}\pi r^4$$

$z_0 \geq 0$

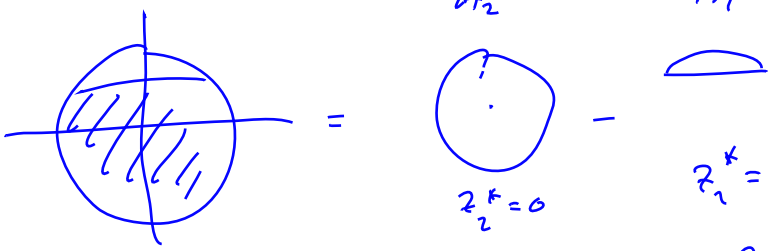
$$\tau(z_0) = \frac{4\cdot Q\cdot 2r^3\sin^3\alpha}{3\pi r^4\cdot 2r\sin\alpha} =$$

$$= \frac{4Q\sin^2\alpha}{3\pi r^2} = \frac{4}{3}\left(\frac{Q}{A}\right)\left(1-\left(\frac{z_0}{r}\right)^2\right)$$

$$\sin^2\alpha = 1-\cos^2\alpha = 1-\left(\frac{z_0}{r}\right)^2$$

A

$z_0 < 0$



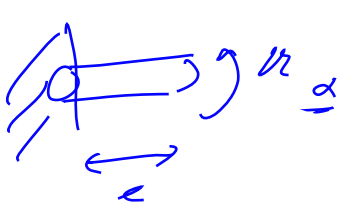
$$z_1^* = -z_0^*$$

$$z_0^* = \frac{1}{A_2 - A_1}\left(A_2 z_2^* - A_1 z_1^*\right) = \frac{A_1 z_0^*}{A_2 - A_1}$$

$$S(z_0^*) = (A_2 - A_1)\, z_0^* = A_1 z_0^*$$

$$\tau(z_0) = \frac{4}{3}\frac{Q}{A}\left(1-\left(\frac{z_0}{r}\right)^2\right) \quad \text{velja za} \quad -r \leq z_0 \leq r$$

Torzija $M = \frac{1}{2} \mu \frac{\alpha}{\ell} \pi r^4$



$M = 2\,kNm$

Določi r, da napetost v palici ne bo presegla $\sigma_0 = 180\,MPa$

$$\tau_{13} = \mu \frac{\alpha}{\ell}\left(-y + \frac{\partial \phi}{\partial x}\right) \; ; \quad \phi = 0$$

$$\tau_{23} = \mu \frac{\alpha}{\ell}\left(x + \frac{\partial \phi}{\partial y}\right)$$

$$\mu \frac{\alpha}{\ell} r < \sigma_0$$

$$\frac{\alpha}{\ell} = \frac{2M}{\mu \pi r^4}$$

$$r < \frac{\sigma_0}{\mu} \frac{\ell}{\alpha} = \frac{\sigma_0}{\mu} \cdot \frac{\mu \pi r^4}{2M}$$

$$\frac{2M}{\pi \sigma_0} < r^3 \; ; \quad r > \left(\frac{2M}{\pi \sigma_0}\right)^{1/3}$$

$$r > \left(\frac{2 \cdot 2\,10^3\,Nm^3}{\pi\, 180 \cdot 10^6 \cdot N}\right)^{1/3} = \left(\frac{4}{180\,\pi}\,10^{-3}\right)^{1/3}_{m} = \left(\frac{1}{45\pi}\right)^{1/3} \cdot dm$$

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