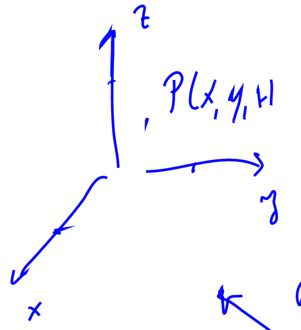
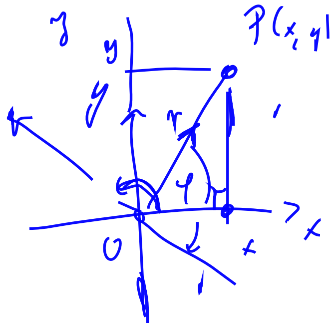


Predavanje 24. februar 2021



Polarni koordinatni sistem (PKS)

(r,φ) $\qquad r \ge 0 \qquad 0 \le \varphi < 2\pi$

$x = r\cos\varphi$

$y = r\sin\varphi$

$P = O(0,0)$

$r = 0$

φ poljuben

$x^2 + y^2 = (r\cos\varphi)^2 + (r\sin\varphi)^2 =$

$= r^2(\underline{\cos^2\varphi + \sin^2\varphi}) = r^2 \qquad \underline{r = \sqrt{x^2+y^2}}$

$\operatorname{tg}\varphi = \frac{y}{x} \Rightarrow \varphi = \underline{\operatorname{arctg}} \frac{y}{x} \qquad x > 0, \qquad x = 0 \quad \varphi = \frac{\pi}{2}$

ali

$\underline{x < 0 \qquad \varphi = \operatorname{arctg}\frac{y}{x} + \pi} \qquad \varphi = \frac{3\pi}{2}$

$\varphi = \frac{\pi}{2} \quad (x=0,\ y>0)$

$\varphi = \frac{3\pi}{2} \quad (x=0,\ y<0)$

Bazna vektorja

$\vec{e}_r = \cos\varphi\vec{\imath} + \sin\varphi\vec{\imath}, \qquad \vec{e}_\varphi = = -\sin\varphi\vec{\imath} + \cos\varphi\vec{\imath}.$

$\vec{OP} = \vec{r} = x\vec{i} + y\vec{j} = r\cos\varphi\,\vec{i} + r\sin\varphi\,\vec{j} = r(\cos\varphi\,\vec{i} + \sin\varphi\,\vec{j})$

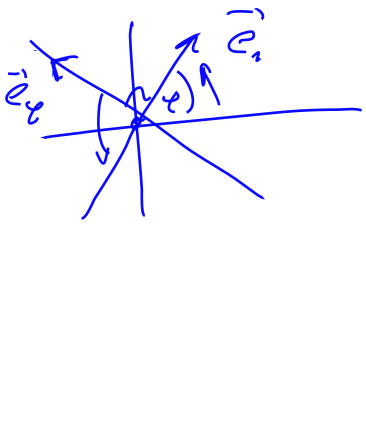
$\vec{e}_r = \cos\varphi\,\vec{i} + \sin\varphi\,\vec{j} \qquad |\vec{e}_r| = \sqrt{\cos^2\varphi + \sin^2\varphi} = 1$

$\vec{e}_r$ radialni bazni vektor $\qquad \boxed{\vec{r} = r\,\vec{e}_r}$

$\vec{e}_\varphi = -\sin\varphi\,\vec{i} + \cos\varphi\,\vec{j} \qquad |\vec{e}_\varphi| = 1$

obodni bazni vektor.

$\vec{e}_r \cdot \vec{e}_\varphi = -\cos\varphi\sin\varphi + \sin\varphi\cos\varphi = 0 \qquad \vec{e}_r \perp \vec{e}_\varphi$



Velja:

$$\boxed{\frac{\partial \vec{e}_r}{\partial \varphi} = \vec{e}_\varphi \qquad \frac{\partial \vec{e}_\varphi}{\partial \varphi} = -\vec{e}_r.}$$

$$\frac{d\vec{e}_r}{d\varphi} = \frac{d}{d\varphi}\left(\cos\varphi\,\vec{i} + \sin\varphi\,\vec{j}\right) = \frac{d\cos\varphi}{d\varphi}\vec{i} + \frac{d\sin\varphi}{d\varphi}\vec{j} =$$

$$= -\sin\varphi\,\vec{i} + \cos\varphi\,\vec{j} = \vec{e}_\varphi$$

$$\frac{d\vec{e}_\varphi}{d\varphi} = \frac{d}{d\varphi}\left(-\sin\varphi\,\vec{i} + \cos\varphi\,\vec{j}\right) = \frac{d}{d\varphi}(-\sin\varphi)\,\vec{i} + \frac{d}{d\varphi}\cos\varphi\,\vec{j} =$$

$$= -\cos\varphi\,\vec{i} - \sin\varphi\,\vec{j} = -\vec{e}_r$$

$$r = r(t),\ \varphi = \varphi(t)$$

Kinematika v polarnem koordinatnem sistemu

Vektor hitrosti

$$\boxed{\vec{v} = \dot{r}\,\vec{e}_r + r\dot{\varphi}\,\vec{e}_\varphi.}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(r\,\vec{e}_r) = \frac{dr}{dt}\vec{e}_r + r\frac{d\vec{e}_r}{dt} = \dot{r}\,\vec{e}_r + r\dot{\varphi}\,\vec{e}_\varphi$$

$$\vec{e}_r = \cos\varphi(t)\,\vec{i} + \sin\varphi(t)\,\vec{j}$$

$$\frac{d\vec{e}_r}{dt} = \frac{d\vec{e}_r}{d\varphi}\frac{d\varphi}{dt} = \vec{e}_\varphi\,\dot{\varphi}$$

$$h(t) = f(g(t))\ ;\quad \dot{h} = f'(g(t))\,\dot{g}$$

$\dot{r}\,\vec{e}_r$ vektor radialne hitrosti ; $\dot{r}$ radialna hitrost

$r\dot{\varphi}$ obodna hitrost $\qquad \dot{\varphi} = \frac{d\varphi}{dt}$ kotna hitrost

$$v^2 = |\vec{v}|^2 = \vec{v}\cdot\vec{v} = (\dot{r}\,\vec{e}_r + r\dot{\varphi}\,\vec{e}_\varphi)\cdot(\dot{r}\,\vec{e}_r + r\dot{\varphi}\,\vec{e}_\varphi) = \dot{r}^2 + r^2\dot{\varphi}^2$$

$$v = \sqrt{\dot{r}^2 + (r\dot{\varphi})^2}$$

$\dot{r}$ radialna hitrost, $r\dot{\varphi}$ obodna hitrost.

Vektor pospeška

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\dot{r}\,\vec{e}_r + r\dot{\varphi}\,\vec{e}_\varphi\right) =$$

$$= \ddot{r}\,\vec{e}_r + \dot{r}\underbrace{\frac{d\vec{e}_r}{dt}}_{\dot{\varphi}\vec{e}_\varphi} + \dot{r}\dot{\varphi}\,\vec{e}_\varphi + r\ddot{\varphi}\,\vec{e}_\varphi + r\dot{\varphi}\frac{d\vec{e}_\varphi}{dt} =$$

$$\frac{d\vec{e}_\varphi}{dt} = \frac{d\vec{e}_\varphi}{d\varphi}\cdot\dot{\varphi} = -\vec{e}_r\,\dot{\varphi}$$

$$= \underbrace{\left(\ddot{r} - r\dot{\varphi}^2\right)\vec{e}_r}_{\vec{a}_r} + \underbrace{\left(r\ddot{\varphi} + 2\dot{r}\dot{\varphi}\right)\vec{e}_\varphi}_{\vec{a}_\varphi}$$

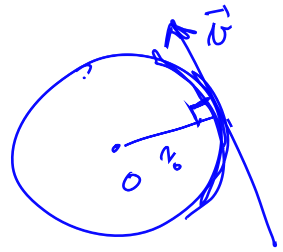
radialni pospešek   obodni pospešek

$$a_r = \ddot{r} - r\dot{\varphi}^2$$

$$\vec{a} = \left(\cancel{\ddot{r}} - r\dot{\varphi}^2\right)\vec{e}_r + \left(r\ddot{\varphi} + 2\cancel{\dot{r}}\dot{\varphi}\right)\vec{e}_\varphi$$

Radialni, obodni pospešek.

tir leži na krožnici



Kroženje

Enakomerno kroženje, kotna hitrost.

$r = r_0 \Rightarrow \dot r = 0, \ \ddot r = 0$ $\qquad \vec v = r \dot\varphi \vec e_\varphi$

$\vec a = -r_0 \dot\varphi^2 \vec e_r + r_0 \ddot\varphi \vec e_\varphi$

$\ddot\varphi = 0 \qquad \dot\varphi =$ konst.

To je enakomerno kroženje.

1) kotna hitrost je konstantna $\dot\varphi = \omega$ konst.

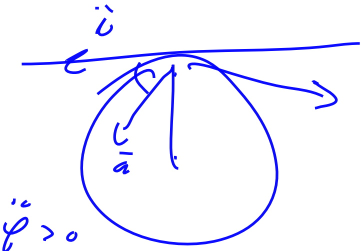
2) $\vec a$ kaže natanko proti središču krožnice

1) $\Leftrightarrow$ 2)

Neakomerno kroženje, kotni pospešek. $\quad \dot\varphi$ ni konst $\quad \ddot\varphi = \alpha$

kotni pospešek

$\vec a = -r_0 \dot\varphi^2 \vec e_r + r_0 \ddot\varphi \vec e_\varphi$



$\vec v \cdot \vec a = r_0 \dot\varphi \vec e_\varphi \cdot \vec a = r_0^2 \dot\varphi \ddot\varphi$

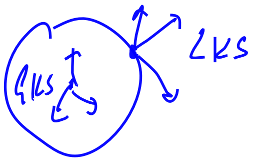
$\vec v \cdot \vec a > 0$ pospeševanje $\Leftrightarrow \dot\varphi \ddot\varphi > 0$

zaviranje

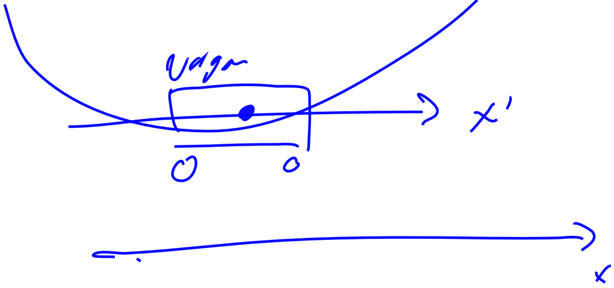
$\vec v \cdot \vec a < 0$

$\dot\varphi \ddot\varphi < 0$

Kroženje je enakomerno natanko tedaj, ko vektor pospeška kaže proti središču kroženja.



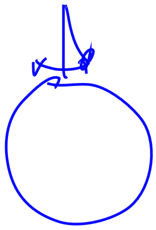
Dinamika



Newtonovi zakoni

1. Koordinatni sistem(KS) je inercialen (IKS) natanko tedaj, ko se prosta materialna točka giblje premočrtno s konstantno brzino ali pa miruje.
2. V IKS velja Newtonova enačba $m\vec{a} = \vec{F}$.
3. Zakon akcije in reakcije $\vec{F}_{ij} = -\vec{F}_{ji}$.

$\vec{a} = \ddot{\vec{r}}$



$\vec{F}_{ij}$ sila P_i na P_j

$\vec{F}_{ji}$ sila P_j na P_i

$$\boxed{m\ddot{\vec{r}} = \vec{F}}$$

$\vec{c}$ konst;

$\vec{v}$ -II-

$m(\vec{r} + \vec{c})^{\cdot\cdot} = \vec{F}$

$m(\vec{r} + \vec{v}t)^{\cdot\cdot} = \vec{F}$

Gibanje je natanko določeno z Newtonovo enačbo in začetnimi pogoji.

$$\left.\begin{aligned}\vec{r}(t=t_0) &= \vec{r}_0\\ \dot{\vec{r}}(t=t_0) &= \vec{v}_0\end{aligned}\right\}$$ začetni pogoji

$\vec{F} = \vec{0}$

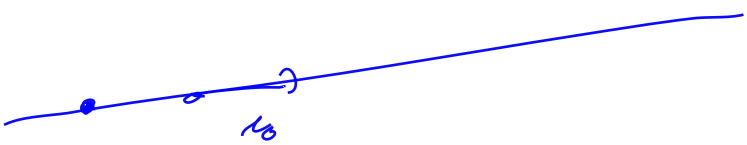
$m\ddot{\vec{r}} = \vec{0}$

$\ddot{\vec{r}} = \vec{0} \Rightarrow \dot{\vec{r}} = \text{konst.} = \vec{v}_0$

$\vec{r} = \vec{v}_0 t + \vec{r}_0$

$\vec{r}(t=0) = \vec{r}_0$; $\dot{\vec{r}}(t=0) = \vec{v}_0$

Enakomerno gibanje



Sistem materialnih točk

Razdelitev sil na zunanje in notranje. Rezultanta notranjih sil je enaka nič; $\sum_{i=1}^{N}\sum_{j=1}^{N}\vec{F}_{ij}=\vec{0}$.

m_i; P_i; $\vec{r}_i = \vec{OP_i}$; $\vec{v}_i = \dot{\vec{r}}_i$, $\vec{a}_i = \ddot{\vec{r}}_i$ $\quad i=1,\dots,N$

$m_i\vec{a}_i$ = rezultanta vseh sil na P_i $= \vec{F}_i + \sum_{j=1, j\neq i}^{N}\vec{F}_{ji}$

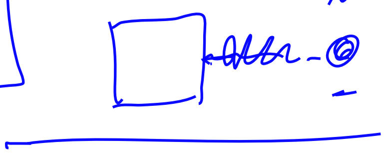
rezultanta zunanjih sil na P_i

rezultanta notranjih sil

$$\boxed{m\vec{a}_i = \vec{F}_i + \sum_{j=1}^{N}\vec{F}_{ji}}$$

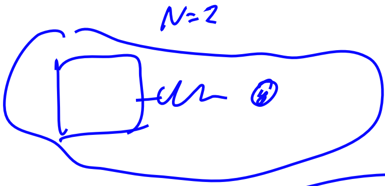
$i=1,\dots,N$

$N=1$



$\vec{F}_{ii} = -\vec{F}_{ii} \Rightarrow \vec{F}_{ii} = \vec{0}$

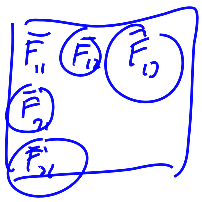
$N=2$



$$\sum_{i=1}^{N} m\vec{a}_i = \sum_{i=1}^{N}\vec{F}_i + \sum_{i=1}^{N}\sum_{j=1}^{N}\vec{F}_{ji}$$

Rezultanta vseh notranjih sil

$\vec{F}_{ji} = -\vec{F}_{ij}$



Rezultanta zunanjih sil

$$\sum_{i=1}^{N} m_i\vec{a}_i = \sum_{i=1}^{N}\vec{F}_i.$$

$$m\vec{a}_* = \sum_{i=1}^{N}\vec{F}_i = \vec{F}$$

$\vec{r}_* = \frac{1}{m}\sum_{i=1}^{N} m_i\vec{r}_i$ $\qquad m = \sum_{i=1}^{N} m_i$

$\vec{a}_i$

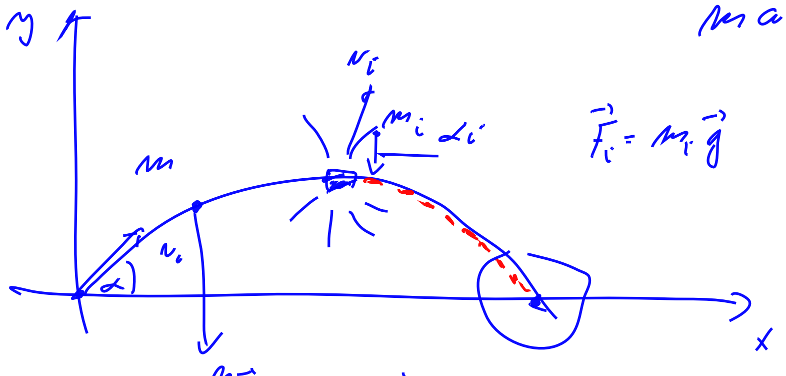
$$\vec v_* = \dot{\vec r}_* = \frac{1}{m}\sum_{i=1}^{N} m_i \dot{\vec r}_i \qquad \vec a_* = \frac{1}{m}\sum_{i=1}^{N} m_i \ddot{\vec r}_i$$

$$\sum_{i=1}^{N} m_i \vec a_i = m \vec a_*$$

Masno središče

Masno središče. Enačba gibanja masnega središča $m\vec{a}_* = \vec{F}$. Primer: poševni met po eksploziji.

$$m\vec a_* = \sum_{i=1}^{N} \vec F_i = \vec g \sum_{i=1}^{N} m_i = m\vec g$$

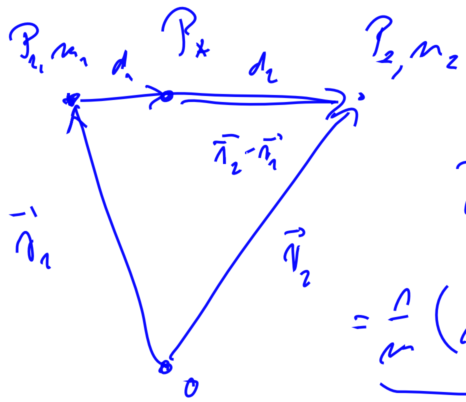


$$\vec F_i = m_i \vec g$$

$$m\vec a_* = m\vec g$$

$$m\vec a = m\vec g$$

Primer: masno središče dveh točk leži na njuni zveznici in jo deli v obratnem razmerju njunih mas.



$$m = m_1 + m_2$$

$$\vec r_* = \frac{1}{m}\left(m_1 \vec r_1 + m_2 \vec r_2\right) =$$

$$= \frac{1}{m}\left(m_1 \vec r_1 + m_2 \vec r_1 - m_2 \vec r_1 + m_2 \vec r_2\right) =$$

$$= \frac{m_1 + m_2}{m}\vec r_1 + \frac{m_2}{m_1 + m_2}\left(\vec r_2 - \vec r_1\right) =$$

$$= \vec r_1 + \frac{m_2}{m_1 + m_2}\left(\vec r_2 - \vec r_1\right)$$

$$m_1 = m_2 \Rightarrow \vec r_* = \vec r_1 + \frac{1}{2}\left(\vec r_2 - \vec r_1\right)$$

$$m_2 > m_1 \qquad \frac{m_2}{m_1 + m_2} > \frac{1}{2}$$

$$d_1 = \left|\frac{m_2}{m_1 + m_2}\left(\vec r_2 - \vec r_1\right)\right| = \frac{m_2}{m_1 + m_2}\left|\vec r_2 - \vec r_1\right|$$

$$d_2 = \left|\vec r_2 - \vec r_1\right| - d_1 = \left|\vec r_2 - \vec r_1\right|\left(1 - \frac{m_2}{m_1 + m_2}\right)$$

$$d_2 = \frac{m_1}{m_1 + m_2}\left|\vec r_2 - \vec r_1\right|$$

$$\frac{d_1}{d_2} = \frac{m_2 \cancel{\left|\vec r_2 - \vec r_1\right|} \cancel{\left(m_1 + m_2\right)}}{\cancel{\left(m_1 + m_2\right)} m_1 \cancel{\left|\vec r_2 - \vec r_1\right|}}$$

$$\frac{d_1}{d_2} = \frac{m_2}{m_1} \qquad m_1 d_1 = m_2 d_2$$

Zapis masnega središča sistema kot masno središče dveh masnih središč njunih podsistemov.

$$P_i,\ i = 1, \dots, N \qquad P_{i'}:\ i' \in I_1 \qquad P_{i''}:\ i'' \in I_2$$

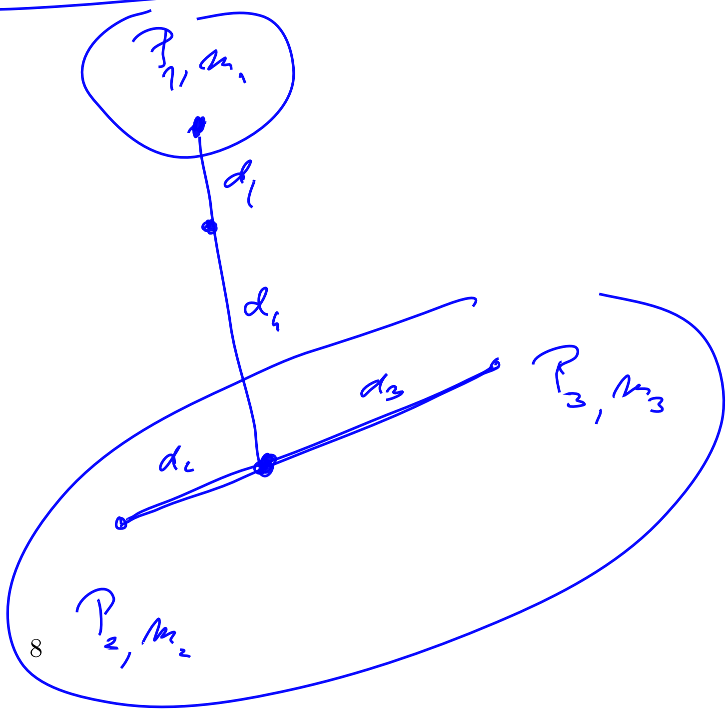
$$\underline{I_1} \cup I_2 = \underline{\{1, \dots, N\}} \qquad \sum_{i' \in I_1} m_{i'} = \hat{m}_1 \qquad \sum_{i'' \in I_2} m_{i''} = \hat{m}_2$$

$$\vec{r}^* = \frac{1}{m} \sum_{i=1}^{N} m_i \vec{r}_i = \frac{1}{\hat{m}_1 + \hat{m}_2} \Big(\underbrace{\sum_{i \in I_1} m_i \vec{r}_i} + \sum_{i \in I_2} m_i \vec{r}_i \Big) =$$

$$= \frac{1}{\hat{m}_1 + \hat{m}_2} \Big(\hat{m}_1 \cdot \underbrace{\frac{1}{\hat{m}_1} \sum_{I_1} m_i \vec{r}_i}_{\vec{r}_1^*} + \hat{m}_2 \cdot \underbrace{\frac{1}{\hat{m}_2} \sum_{I_2} m_i \vec{r}_i}_{\vec{r}_2^*} \Big)$$

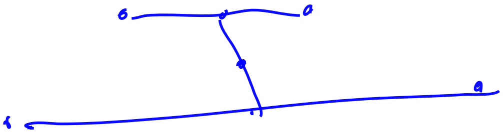
$$\boxed{\vec{r}_* = \frac{1}{m} \sum_{i=1}^{N} m_i \vec{r}_i = \frac{1}{\hat{m}_1 + \hat{m}_2} (\hat{m}_1 \vec{r}_1^* + \hat{m}_2 \vec{r}_2^*)}$$

Primer: masno središče sistema treh točk.



$$\frac{d_1}{d_4} = \frac{m_2 + m_3}{m_1}$$

$$\frac{d_2}{d_3} = \frac{m_3}{m_2}$$

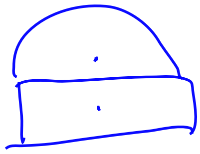


Masno središče likov in teles.

površinsk. gostota

$A;\ m \qquad |A|$ ploščina lika $\qquad m = \rho_0 |A|$

$$\boxed{\vec{r}_* = \frac{1}{m}\int_A \vec{r}\,\rho\, dA}$$



$$\Delta m = \rho|\Delta A| \qquad m = \sum \rho |\Delta A|$$

$$m = \int_A \rho\, dA$$

$V; \qquad \vec{r}_* = \frac{1}{m}\int_V \vec{r}\rho\, dV$

Masno središče trikotnika.