$$[\vec\alpha] = \frac{m}{s^2} \qquad [a_0] = \frac{m}{s^2}\ ; \ [b_0] = \frac{m}{s}$$

$$[\alpha t^2] = m \qquad [\beta t] = m$$

Vaje 25. februar 2021

1. Gibanje točke je dano z $\vec{r}(t) = \vec{\alpha}t^2 + \vec{\beta}t + \vec{\gamma}$, kjer je $\vec{\alpha} = a_0(\vec{\imath} + 2\vec{\jmath} + 3\vec{k})$, $\vec{\beta} = b_0(-3\vec{\imath} - 2\vec{\jmath} - \vec{k})$. Tu sta a_0 in b_0 pozitivni konstanti. Določi pogoj na t, da bo gibanje pospešeno.

pospešuamo.

$$\vec{v}\cdot\vec{a} \geq 0$$

$$\vec{v} = \dot{\vec{r}} = \frac{d}{dt}(\vec\alpha t^2 + \vec\beta t + \vec\gamma) = 2t\vec\alpha + \vec\beta$$

$$\vec{a} = \dot{\vec{v}} = \frac{d}{dt}(2t\vec\alpha + \vec\beta) = 2\vec\alpha$$

$$\frac{d}{dt}(\vec\alpha t^2 + \vec\beta t + \vec\gamma) = \underbrace{\frac{d}{dt}(\vec\alpha t^2)}_{\vec\alpha \frac{dt^2}{dt} = 2\vec\alpha t} + \underbrace{\frac{d}{dt}(\vec\beta t)}_{\vec\beta \frac{dt}{dt} = \vec\beta} + \underbrace{\frac{d}{dt}(\vec\gamma)}_{\vec 0}$$

$$\vec{v}\cdot\vec{a} = (2t\vec\alpha + \vec\beta)\cdot 2\vec\alpha = 4t\,\vec\alpha\cdot\vec\alpha + 2\vec\alpha\cdot\vec\beta =$$

$$= 2(2t|\vec\alpha|^2 + \vec\alpha\cdot\vec\beta)$$

$$\vec{v}\cdot\vec{a} \geq 0 \Rightarrow 2\cancel{2}t|\vec\alpha|^2 + \cancel{2}\vec\alpha\cdot\vec\beta \geq 0$$

$$2t|\vec\alpha|^2 \geq -\vec\alpha\cdot\vec\beta \iff t \geq -\frac{\vec\alpha\cdot\vec\beta}{2|\vec\alpha|^2}$$

$$\vec\alpha = a_0(\vec\imath + 2\vec\jmath + 3\vec k)$$

$$\vec\beta = b_0(-3\vec\imath - 2\vec\jmath - \vec k)$$

$$t \geq -\frac{-10a_0b_0}{14a_0^2} = \frac{5b_0}{2\cdot 7a_0}$$

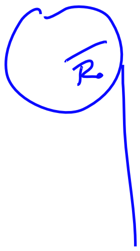
$$|\vec\alpha|^2 = a_0^2(1+4+9) = 14a_0^2$$

$$\vec\alpha\cdot\vec\beta = a_0b_0(-3-4-3) = -a_0b_0\,10$$

$$[t] = \left[\frac{b_0}{a_0}\right] = \frac{m\,s^2}{s\,m} = s$$

$$t \geq \frac{5b_0}{14a_0}$$

2. V času $t = 0$ vklopimo stroj, ki poganja valj s polmerom R_0 na katerega se navija vrvica. Valj v času t_1 doseže polno število obratov o_1. Izračunaj koliko vrvice se navije do časa t_1. Tu privzami, da se do časa t_1 valj vrti enakomerno pospešeno.



$$\frac{d\omega}{dt} = \alpha$$

ω kotna hitrost

α kotni pospešek = konst.

$$\omega = \alpha t + \omega_0$$

$$\omega(t=0) = 0 \qquad 0 = \omega_0$$

$$\omega(t=t_1) = o_1 \qquad o_1 = \alpha t_1 \qquad \alpha = \frac{o_1}{t_1}$$

$$\omega = \dot{\varphi} = \frac{d\varphi}{dt}$$

$$\frac{d\varphi}{dt} = \frac{o_1}{t_1} t \qquad \frac{d}{dt} t^2 = 2t \qquad \frac{d}{dt}\left(\frac{1}{2} t^2\right) = t$$

$$\varphi = \frac{1}{2} t^2 \frac{o_1}{t_1} + \varphi_0 \qquad \varphi(t=0) = 0 \Rightarrow \varphi_0 = 0$$

$$\varphi(t_1) = \frac{1}{2} t_1^2 \frac{o_1}{t_1} = \frac{1}{2} t_1 o_1$$

$$l = R_0 \varphi(t_1) = \frac{1}{2} R_0 t_1 o_1 = \frac{1}{2} \cdot \frac{1}{10} m \cdot 60 s \cdot 100\pi \frac{1}{s} = 300 m$$

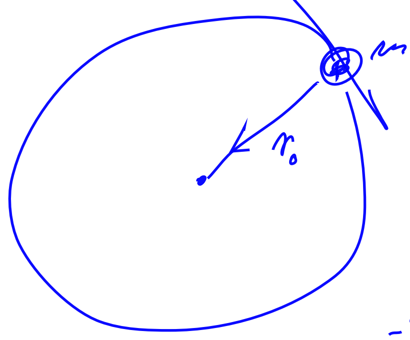
Podatki: $t_1 = 60 s$; $o_1 = 3000$ obratov na minuto

$R_0 = 0{,}1 m$

$o_1 = 3000\ o/min$

$$\frac{o}{min} = \frac{2\pi}{60 s} \qquad o_1 = 3000 \frac{2\pi}{60 s} = 2\pi \cdot 50 \frac{1}{s}$$

3. Kamen z maso $1kg$, ki je pripet na vrvico dolžine r_0, enakomerno kroži s kotno hitrostjo ω. Določi silo, ki jo mora vzdržati vrvica, da se ne strga. Izračunaj za konkretne vrednosti $r_0 = 1\mathrm{m}$, $\omega = 300\mathrm{o/min}$.



$$\vec a = -r_0 \dot\varphi^2 \vec e_r + \cancel{r_0 \overset{=0}{\ddot\varphi} \vec e_\varphi}$$

$$\dot\varphi = \omega$$

$$\vec a = -r_0 \dot\varphi^2 \vec e_r \qquad \vec F = m \vec a$$

$$F = m r_0 \omega^2 = 1\,kg \cdot 1\,m \cdot \left(300 \cdot \frac{2\pi \cdot s}{60\, s}\right)^2 =$$

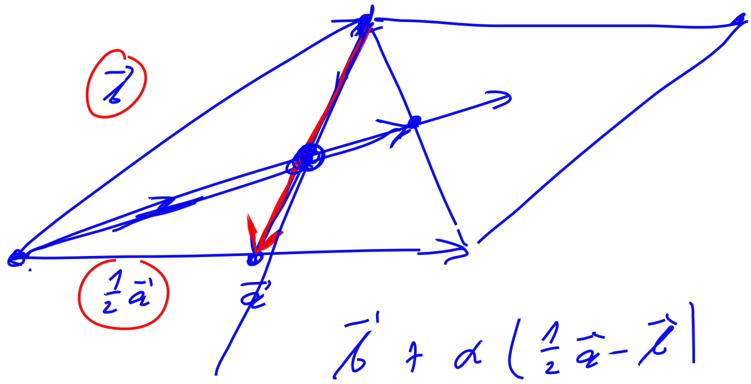
$$= 100\pi^2 \, kg\,m/s^2 \doteq 1000\,N$$

$$\vec F = m\vec a \qquad [\vec F] = [m \vec a] = kg\,m/s^2 = N$$

1kg

$\vec g = 9.8\, m/s^2$ 9.8 N

4. Masno središče trikotnika.



$\frac12(\vec a+\vec b)$

$\vec b + \alpha\left(\frac12 \vec a - \vec b\right) \quad /\cdot \vec a \quad /\cdot \vec b$

$$\boxed{\vec b + \alpha\left(\frac12 \vec a - \vec b\right) = \beta\,\frac12(\vec a + \vec b)}$$

$$\vec a\cdot\vec b + \alpha\left(\frac12|\vec a|^2 - \vec a\cdot\vec b\right) = \frac12\beta\left(|\vec a|^2 + \vec a\cdot\vec b\right)$$

$$|\vec b|^2 + \alpha\left(\frac12\vec a\cdot\vec b - |\vec b|^2\right) = \frac12\beta\left(\vec a\cdot\vec b + |\vec b|^2\right)$$

$$\left(\frac12|\vec a|^2 - \vec a\cdot\vec b\right)\alpha - \frac12\beta\left(|\vec a|^2+\vec a\cdot\vec b\right) = -\vec a\cdot\vec b \quad \Big]\cdot\left(\frac12\vec a\cdot\vec b - |\vec b|^2\right)$$

$$\left(\frac12\vec a\cdot\vec b - |\vec b|^2\right)\alpha - \frac12\beta\left(\vec a\cdot\vec b+|\vec b|^2\right) = -|\vec b|^2 \quad \Big]\cdot -\left(\frac12|\vec a|^2-\vec a\cdot\vec b\right) \quad \Big\} +$$

$$+\frac12\beta\left[\left(|\vec a|^2+\vec a\cdot\vec b\right)\cdot\left(\frac12\vec a\cdot\vec b - |\vec b|^2\right) - \left(\vec a\cdot\vec b + |\vec b|^2\right)\left(\frac12|\vec a|^2 - \vec a\cdot\vec b\right)\right] =$$

$$= +\left[\vec a\cdot\vec b\left(\frac12\vec a\cdot\vec b - |\vec b|^2\right) - |\vec b|^2\left(\frac12|\vec a|^2 - \vec a\cdot\vec b\right)\right]$$

$$\frac12\beta\Big[\cancel{\frac12|\vec a|^2\,\vec a\cdot\vec b} - |\vec a|^2|\vec b|^2 + \frac12(\vec a\cdot\vec b)^2 - \cancel{(\vec a\cdot\vec b)|\vec b|^2}$$
$$-\cancel{(\vec a\cdot\vec b)\frac12|\vec a|^2} + (\vec a\cdot\vec b)^2 - |\vec b|^2\frac12|\vec a|^2 + \cancel{|\vec b|^2\,\vec a\cdot\vec b}\Big] =$$

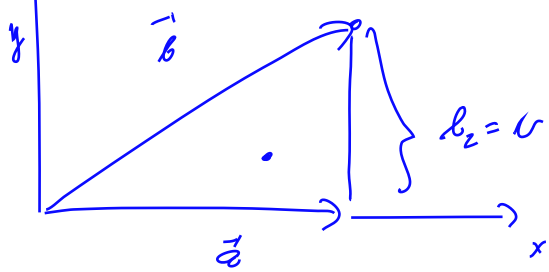
$$= \left[\frac12(\vec a\cdot\vec b)^2 - \cancel{(\vec a\cdot\vec b)|\vec b|^2} - |\vec b|^2\frac12|\vec a|^2 + \cancel{|\vec b|^2(\vec a\cdot\vec b)}\right]$$

$$\frac12\beta\left[-\frac32|\vec a|^2|\vec b|^2 + \frac32(\vec a\cdot\vec b)^2\right] = \left[\frac12(\vec a\cdot\vec b)^2 - \frac12|\vec a|^2|\vec b|^2\right]$$

$$\cancel{3\left[-\frac12|\vec a|^2|\vec b|^2 + \frac12(\vec a\cdot\vec b)^2\right]}$$

$$\frac32\beta = 1 \Rightarrow \beta = \frac23$$

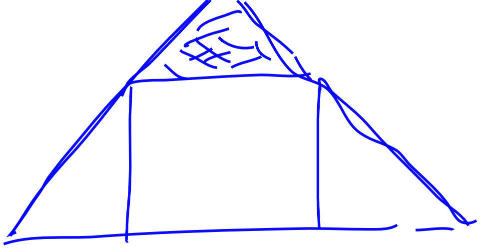
$$\vec r_* = \beta\,\frac12(\vec a + \vec b) = \frac13(\vec a + \vec b)$$



$$\vec a = a_1\vec i\,; \quad \vec b = b_1\vec i + b_2\vec j$$

$$\vec r_* = \frac13(a_1+b_1)\,\vec i + \frac13 b_2$$

$$\boxed{y_* = \frac13 v}$$

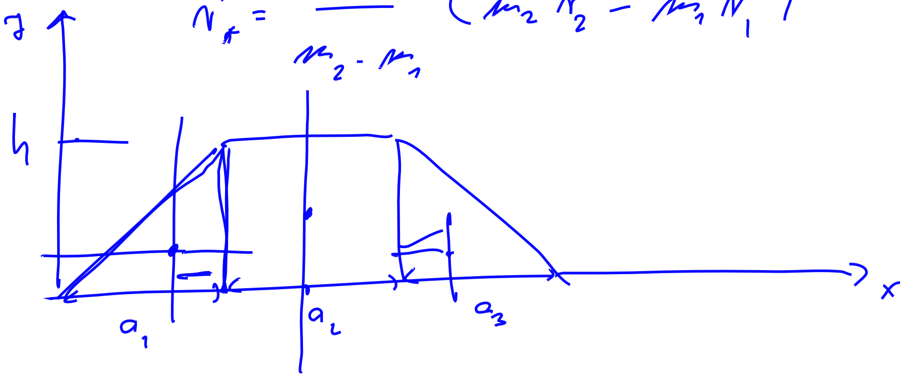


5. Izračunaj masno središče:

- trapeza, kot unijo dveh trikotnikov in pravokotnika;
- trapeza, kot razliko dveh trikotnikov.

$$\vec{r}_* = \frac{1}{m_2 - m_1}\left(m_2 r_2^* - m_1 r_1^*\right)$$

$$A = A_2 - A_1$$



	A	x_*	y_*
$\triangle$	$\frac{1}{2}a_1 h$	$\frac{2}{3}a_1$	$\frac{1}{3}h$
$\square$	$a_2 h$	$a_1 + \frac{1}{2}a_2$	$\frac{1}{2}h$
$\triangle$	$\frac{1}{2}a_3 h$	$a_1 + a_2 + \frac{1}{3}a_3$	$\frac{1}{3}h$

$$\vec{r}_* = \frac{1}{\underbrace{A_1 + A_2 + A_3}}\left(A_1 \vec{r}_1^* + A_2 \vec{r}_2^* + A_3 \vec{r}_3^*\right)$$

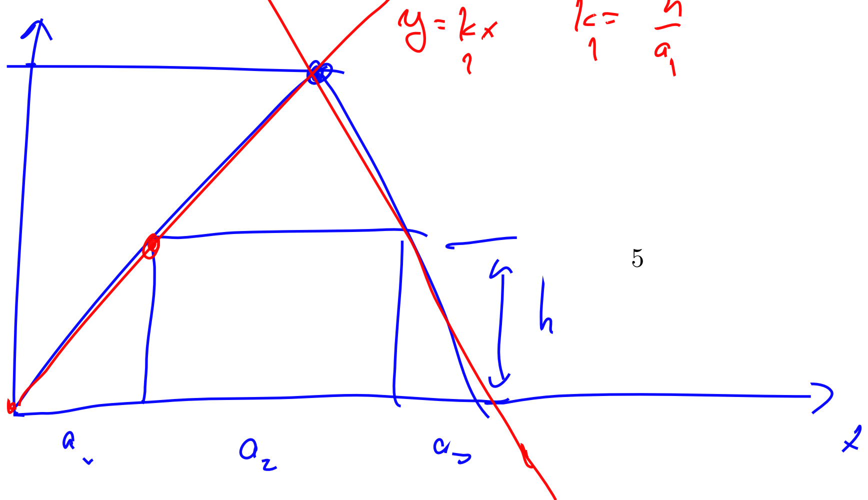
$$A_1 + A_2 + A_3 = \frac{1}{2}a_1 h + a_2 h + \frac{1}{2}a_3 h$$

$$y_* = \frac{1}{h\left(\frac{1}{2}a_1 + a_2 + \frac{1}{2}a_3\right)}\left(\frac{1}{2}a_1 h \cdot \frac{1}{3}h + a_2 h \cdot \frac{1}{2}h + \frac{1}{2}a_3 h \cdot \frac{1}{3}h\right) =$$

$$= \frac{h\left(\frac{1}{6}a_1 + \frac{1}{2}a_2 + \frac{1}{6}a_3\right)}{\frac{1}{2}a_1 + a_2 + \frac{1}{2}a_3} = \frac{a_1 + 3a_2 + a_3}{3a_1 + 6a_2 + 3a_3} \cdot h$$

$a_1 = a_3 \Rightarrow x_* = a_1 + \frac{1}{2}a_2$ DN

$y = k_1 x$ $\quad k_1 = \frac{h}{a_1}$



$$k_1 x = k_2 x + n_2$$

$$x = \frac{n_2}{k_1 - k_2}$$

$$y = k_2 x + n_2 \qquad 0 = k_2(a_1 + a_2 + a_3) + n_2$$

$$k_2 = -\frac{h}{a_3}$$