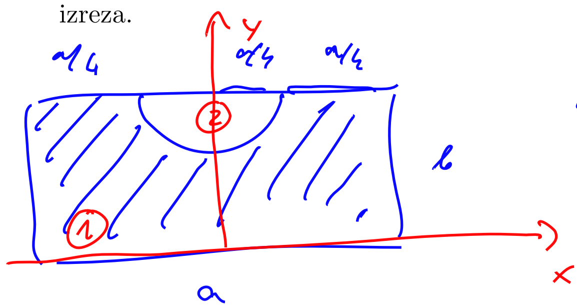
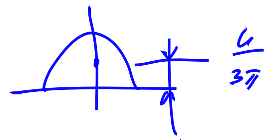
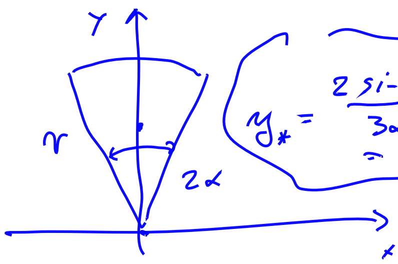
Vaje 4. marec 2021

1. Določi masno središče homogenega pravokotnika $a \times b$ s polkrožnim izrezom na sliki, če veš, da ima masno središče krožnega izreza s kotom 2α koordinati $(0, \frac{2\sin\alpha}{3\alpha})$, glej skico krožnega izreza.



$$\vec r_* = \frac{1}{A_1 - A_2}\left(A_1 \vec r_1^{\,*} - A_2 \vec r_2^{\,*}\right)$$





$$y_* = \frac{2\sin\alpha}{3\alpha} r$$

$2\alpha = \pi$

$\alpha = \frac{\pi}{2}$

$$y_* = \frac{2 \cdot a}{3 \cdot \frac{\pi}{2} \cdot 4} = \frac{4a}{3\pi 4} = \frac{a}{3\pi}$$

	A	y
(rectangle)	ab	$b/2$
(semicircle)	$\frac{1}{2}\pi\frac{a^2}{16}$	$b - \frac{a}{3\pi}$

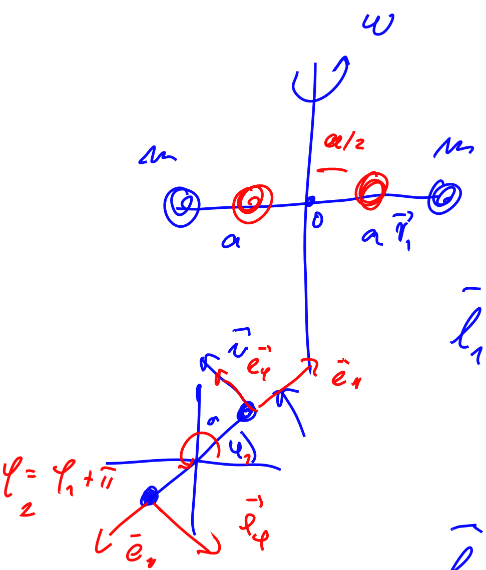
$x_* = 0$

$r = \frac{a}{4}$

$$y_* = \frac{1}{ab - \frac{\pi a^2}{32}}\left(ab \cdot \frac{b}{2} - \frac{\pi a^2}{32}\left(b - \frac{a}{3\pi}\right)\right) =$$

$$= \frac{1}{\cancel{a}(b - \pi a/32)}\left(\frac{1}{2}\cancel{a}b^2 - \frac{1}{32}\pi a^{\cancel{2}} b + \frac{a^{2\,\cancel{3}}}{96}\right)$$

2. Ohranitev vrtilne količine. Na os je pravokotno pritrjena dvostranska ročica dolžine $2a$ z masama m na obeh koncih, glej skico. Za koliko se spremeni kotna hitrost vrtenja osi, če se dolžina ročice skrči na a.



$\boxed{L_0 = L_1}$ zakon o ohranitvi vrtilne količine

$\vec L = \vec l_1 + \vec l_2$; $\vec L(0)$

$\vec l_1 = \vec r_1 \times m \vec v_1 = a^2 \dot\varphi\, \vec e_r \times \vec e_\varphi = a\dot\varphi\, \vec k = a^2\omega\, \vec k$

$\vec r_1 = a\,\vec e_r$ $\quad \vec v_1 = a\dot\varphi\, \vec e_\varphi$ $\quad \dot\varphi = \omega$

$\vec e_r \times \vec e_\varphi = \vec k$

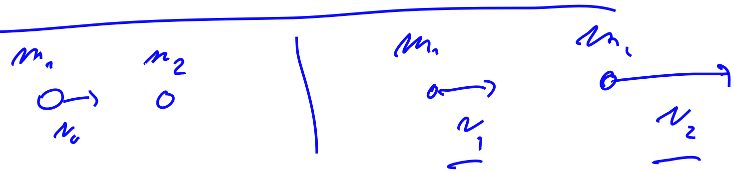
$\vec l_2 = \vec r_2 \times m \vec v_2 = a^2\dot\varphi\, \vec e_r \times \vec e_\varphi = a^2\omega\, \vec k$

$\vec L = 2a^2\omega\, \vec k$

$\vec L_0 = 2a^2\omega_0\, \vec k$ $\qquad \vec L_1 = 2\left(\frac{a}{2}\right)^2 \omega_1\, \vec k$

$\vec L_0 = \vec L_1 \Rightarrow 2a^2\omega_0 = 2\,\frac{a^2}{4}\,\omega_1 \Rightarrow \boxed{\omega_1 = 4\omega_0}$

Zk



$m_1 v_0 = m_1 v_1 + m_2 v_2$

Trk je elastičen, če se ohranja energija. (kinetična energija) $\frac{1}{2} m v^2$

$\frac{1}{2} m_1 v_0^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$\frac{m_2}{m_1} = \mu$ $\qquad v_0 = v_1 + \mu v_2 \Rightarrow v_2 = (v_0 - v_1)/\mu$

$v_0^2 = v_1^2 + \mu v_2^2$

$v_0^2 = v_1^2 + \mu\left(\frac{v_0 - v_1}{\mu}\right)^2 = v_1^2 + \frac{1}{\mu}\left(v_0^2 - 2v_0 v_1 + v_1^2\right)$

$\left(1 + \frac{1}{\mu}\right) v_1^2 + \left(-\frac{2v_0}{\mu}\right) v_1 + \left(\frac{1}{\mu} v_0^2 - v_0^2\right) = 0$

$$ax^2 + bx + c = 0 \qquad x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$v_{1,a,b} = \frac{\frac{2v_0}{\mu} \pm \sqrt{\frac{4v_0^2}{\mu^2} - 4\left(1+\frac{1}{\mu}\right)v_0^2\left(\frac{1}{\mu}-1\right)}}{2\left(1+\frac{1}{\mu}\right)} = \frac{v_0 \pm v_0\sqrt{1-\mu^2\left(1+\frac{1}{\mu}\right)\left(\frac{1}{\mu}-1\right)}}{\mu+1}$$

3. Za podani ravninski sistem sil s prijemališči v ogliščih enakostraničnega trikotnika izračunaj rezultanto sil in navorov glede na dani pol.

$$= v_0 \frac{1 \pm \sqrt{1+\mu^2-1}}{1+\mu} = v_0 \frac{1\pm\mu}{1+\mu}$$

$$\mu^2\left(1+\frac{1}{\mu}\right)\left(1-\frac{1}{\mu}\right)$$

$$(\mu+1)(\mu-1) = \mu^2 - 1$$

$$m_1 = m_2 \Rightarrow \mu = 1$$

$$v_{1,a} = v_0 \frac{2}{2} = v_0 \qquad v_{2,a} = 0$$

$$v_{1,b} = 0 \qquad v_{2,b} = v_0$$

$$v_2 = \frac{1}{\mu}(v_0 - v_1) = v_0 - v_1$$

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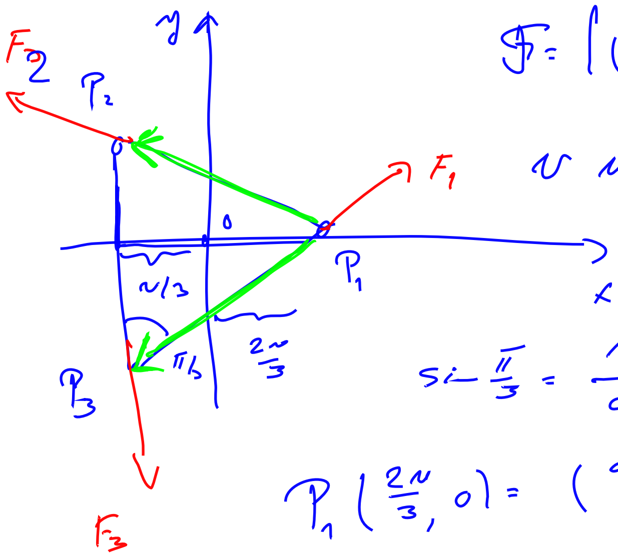
$$v_1 = 0, \quad v_2 = v_0$$

$$v_2 \geq v_1$$



$$\mathcal{F} = \{(P_1, \vec{F}_1), (P_2, \vec{F}_2), (P_3, \vec{F}_3)\}$$

v višini trikotnika

$$\sin\frac{\pi}{3} = \frac{v}{a} \Rightarrow v = a \cdot \sin\frac{\pi}{3} = \frac{a\sqrt{3}}{2}$$

$$P_1\left(\frac{2v}{3}, 0\right) = \left(\frac{a\sqrt{3}}{3}, 0\right) = P_1\left(\frac{a}{\sqrt{3}}, 0\right)$$

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\to P_2\left(-\frac{v}{3}, \frac{a}{2}\right) = P_2\left(-\frac{a}{2\sqrt{3}}, \frac{a}{2}\right) \qquad \frac{v}{3} = \frac{a\sqrt{3}}{2\cdot 3} = \frac{a\sqrt{3}}{2\sqrt{3}\sqrt{3}}$$

$$P_3\left(-\frac{a}{2\sqrt{3}}, -\frac{a}{2}\right)$$

$$\vec{F}_1 = \frac{\overrightarrow{P_3P_1}}{|P_3P_1|} \cdot F_1 = \frac{1}{a}(P_1 - P_3)F_1 = \frac{1}{a}\left[\left(\frac{a}{\sqrt{3}} + \frac{a}{2\sqrt{3}}\right)\vec{i} + \left(0 + \frac{a}{2}\right)\vec{j}\right]F_1$$

$$= \left(\frac{1}{\sqrt{3}}\frac{3}{2}\vec{i} + \frac{1}{2}\vec{j}\right)F_1 = \frac{1}{2}\left(\sqrt{3}\,\vec{i} + \vec{j}\right)F_1$$

$$\vec{F}_2 = \frac{\overrightarrow{P_1P_2}}{|P_1P_2|} F_2 = \frac{1}{a}(P_2 - P_1)F_2 = \frac{1}{a}\left[\left(-\frac{a}{2\sqrt{3}} - \frac{a}{\sqrt{3}}\right)\vec{i} + \left(\frac{a}{2} - 0\right)\vec{j}\right]F_2 =$$

$$= \left[-\left(\frac12+1\right)\frac{1}{\sqrt3}\vec i + \frac12 \vec j\right] F_2 = \left(-\frac{\sqrt3}{2}\vec i + \frac12 \vec j\right)F_2 = \frac12\left(-\sqrt3\vec i + \vec j\right)F_2$$

$$\vec F_3 = \frac{\overrightarrow{P_2P_3}}{|\overrightarrow{P_2P_3}|}F_3 = \frac1a(P_3-P_2)F_3 = \frac1a\left(\left(-\frac{a}{2\sqrt3}+\frac{a}{2\sqrt3}\right)\vec i + \left(-\frac a2 - \frac a2\right)\vec j\right) F_3 =$$

$$= -\vec j\, F_3$$

$$\vec R(\mathcal F) = \vec F_1+\vec F_2+\vec F_3 = \frac12\left(\sqrt3\,\vec i+\vec j\right)F_1 + \frac12\left(-\sqrt3\,\vec i+\vec j\right)F_2 - \vec j F_3 =$$

$$= \frac12\sqrt3\left(F_1-F_2\right)\vec i + \left(\frac12F_1+\frac12F_2-F_3\right)\vec j$$

$$\vec N(\mathcal F, P_1) = \underbrace{\overrightarrow{P_1P_1}}_{=\vec 0}\times\vec F_1 + \overrightarrow{P_1P_2}\times\vec F_2 + \overrightarrow{P_1P_3}\times \vec F_3 =$$

$$= (P_2-P_1)\times\frac12\left(-\sqrt3\,\vec i+\vec j\right)F_2 + (P_3-P_1)\times(-\vec j)F_3 =$$

$$= \left(\left(-\frac{a}{2\sqrt3}-\frac{a}{\sqrt3}\right)\vec i + \left(\frac a2 - 0\right)\vec j\right)\times\frac12\left(-\sqrt3\,\vec i+\vec j\right)F_2 +$$

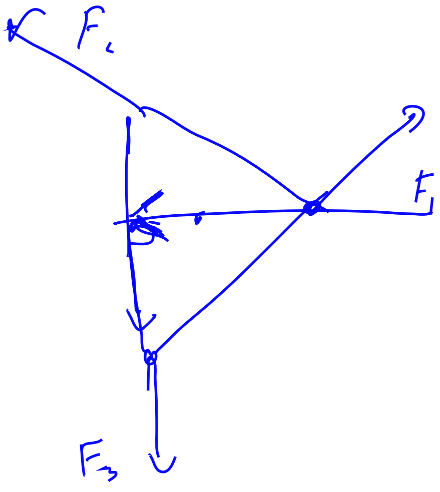
$$\left(\left(-\frac{a}{2\sqrt3}-\frac{a}{\sqrt3}\right)\vec i + \left(-\frac a2 - 0\right)\vec j\right)\times\left(-\vec j F_3\right) =$$

$$= a\left(\frac{1}{\sqrt3}\cdot\left(-\frac32\right)\vec i + \frac12\vec j\right)\times\frac12\left(-\sqrt3\,\vec i+\vec j\right)F_2 +$$

$$a\left(\frac{1}{\sqrt3}\left(-\frac32\right)\vec i - \frac12\vec j\right)\times\left(-\vec j F_3\right) =$$

$$\frac14 a\left[\underbrace{\left(-\sqrt3\,\vec i+\vec j\right)\times\left(-\sqrt3+\vec j\right)}_{=\vec 0}F_2 + \left(-\sqrt3\,\vec i-\vec j\right)\times\left(-2\vec j\right)F_3\right]$$

$$= \frac14 a\, 2\sqrt3\, F_3\vec k = \frac12 a\sqrt3\, F_3\, \vec k$$



$$\overrightarrow{P_1P_3}\times\vec F_3 = \vec k\left(F_3\cdot\frac12 a\sqrt3\right)$$

Kdaj je ta sistem sil navmnešen ?

$\vec{R}(\mathcal{F}) = \vec{0}$, $\vec{N}(\mathcal{F}, P_1) = \vec{0}$ $\Downarrow$ $F_3 = 0$

$\underline{F_1 = F_2}$ $\quad \frac{1}{2}(F_1 + F_2) - F_3 = 0$

$\underline{F_1 + F_2 = 0}$

4. Za podani prostorski sistem sil s prijemališči v ogliščih kvadra izračunaj rezultanto sil, navorov, njegovo invariato.

$\boxed{\underline{F_1 = F_2}}$

$$\vec{N}(\mathcal{F}, C) = \underline{\vec{OP}_1 \times \vec{R}(\mathcal{F})} + \vec{N}(\mathcal{F}, P_1)$$

$$\frac{a}{\sqrt{3}}\vec{i} \times \left(\frac{1}{2}\sqrt{3}(F_1 - F_2)\vec{i} + \left(\frac{1}{2}(F_1 + F_2) - F_3\right)\vec{j}\right) =$$

$$\boxed{\frac{a}{\sqrt{3}}\left(\frac{1}{2}(F_1 + F_2) - F_3\right)\vec{k}}$$