Vaje 11. marec 2021

1. Dan je ravninski sistem sil $\mathcal{F} = \{(P_1, \vec{F}_1), (P_2, \vec{F}_2), (P_3, \vec{F}_3), (P_4, \vec{F}_4)\}$, kjer imajo prijemališča sil koordinate $P_1 = (0, 2a)$, $P_2 = (a, -2a)$, $P_3 = (2a, -a)$, $P_4 = (-2a, -2a)$, sile pa so $\vec{F}_1 = F_0(-2\vec{\imath} - \vec{\jmath})$, $\vec{F}_2 = F_0(3\vec{\imath} - 2\vec{\jmath})$, $\vec{F}_3 = 2F_0\vec{\imath}$, $\vec{F}_4 = F_0(-2\vec{\imath} - 2\vec{\jmath})$.
   - (a) Izračunaj rezultanto sistema sil.
   - (b) Izračunaj rezultanto navora sistema sil glede na pol v koordinatnem izhodišču.
   - (c) Izračunaj invarianto sistema sil.
   - (d) Določi os sistema.

a) $\vec{R}(\mathcal{F}) = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = F_0\left(\vec{\imath}(-2+3+2-2) + \vec{\jmath}(-1-2+0-2)\right) =$

$= \underline{\underline{F_0(\vec{\imath} - 5\vec{\jmath})}}$

b) $O(0,0)$ $\quad \vec{OP}_1 = 2a\vec{\jmath}$, $\vec{OP}_2 = a\vec{\imath} - 2a\vec{\jmath}$, $\vec{OP}_3 = 2a\vec{\imath} - a\vec{\jmath}$, $\vec{OP}_4 = -2a\vec{\imath} - 2a\vec{\jmath}$

$$\vec{N}(\mathcal{F}, O) = \sum_{i=1}^{4} \vec{OP}_i \times \vec{F}_i =$$

$$= aF_0\left[2\vec{\jmath} \times (-2\vec{\imath} - \vec{\jmath}) + (\vec{\imath} - 2\vec{\jmath}) \times (3\vec{\imath} - 2\vec{\jmath}) + (2\vec{\imath} - \vec{\jmath}) \times 2\vec{\imath} + (-2\vec{\imath} - 2\vec{\jmath}) \times (-2\vec{\imath} - 2\vec{\jmath})\right] =$$

$\vec{\imath} \times \vec{\jmath} = \vec{k}$

$\vec{\jmath} \times \vec{\imath} = -\vec{k}$

$$= aF_0\vec{k}\left[4 + (-2+6) + 2\right] = \underline{\underline{10aF_0\vec{k}}}$$

c) $$I(\mathcal{F}) = \vec{R}(\mathcal{F}) \cdot \vec{N}(\mathcal{F}, O) = F_0(\vec{\imath} - 5\vec{\jmath}) \cdot 10aF_0\vec{k} = 0$$

Če je sistem sil ravninski, je njegova invarianta enaka nič.

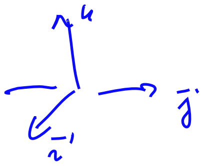
$\mathcal{F}$ ima skupno prijemališče.

$$\vec{OP}_0 = \frac{\vec{R} \times \vec{N}(\mathcal{F}, O)}{|\vec{R}|^2} = \frac{F_0(\vec{\imath} - 5\vec{\jmath}) \times 10aF_0\vec{k}}{F_0^2\sqrt{1+25}} =$$

$$= \frac{10}{\sqrt{26}}a(\vec{\imath} - 5\vec{\jmath}) \times \vec{k} = \frac{10}{\sqrt{26}}a(-\vec{\jmath} - 5\vec{\imath}) = \underline{\underline{-\frac{10}{\sqrt{26}}a(5\vec{\imath} + \vec{\jmath})}}$$

$$\vec{i} \times \vec{k} = -\vec{j}$$

$$\vec{j} \times \vec{k} = \vec{i}$$



$$\vec{R} = (\vec{i} - 5\vec{j})F_0$$

$$|\vec{R}|^2 = \vec{R} \cdot \vec{R} = F_0^2(\vec{i} - 5\vec{j}) \cdot (\vec{i} - 5\vec{j}) = F_0^2(1 + (-5)(-5)) = $$

$$= 26F_0^2$$

$$\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}; \quad |\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

2. Podan je prostorski sistem sil $\vec{F}_1 = \vec{\imath} - \vec{\jmath}$, $\vec{F}_2 = 2\vec{\imath} - \vec{\jmath} + \vec{k}$, $\vec{F}_3 = -\vec{\imath} + 2\vec{\jmath} + \vec{k}$, s prijemališči v točkah $P_1(1,2,1)$, $P_2(-1,0,1)$, $P_3(1,-1,0)$.

   (a) Izračunaj rezultanto sistema sil.

   (b) Izračunaj rezultanto navora sistema sil glede na pol v koordinatnem izhodišču.

   (c) Izračunaj invarianto sistema sil.

   (d) Določi os sistema.

$$\vec{R}(\mathcal{F}) = \vec{\imath}(1+2-1) + \vec{\jmath}(-1-1+2) + \vec{k}(0+1+1) = 2\vec{\imath} + 2\vec{k}$$

$$\vec{N}(\mathcal{F}, 0) = (\vec{\imath} + 2\vec{\jmath} + \vec{k}) \times (\vec{\imath} - \vec{\jmath}) + (-\vec{\imath} + \vec{k}) \times (2\vec{\imath} - \vec{\jmath} + \vec{k})$$
$$+ (\vec{\imath} - \vec{\jmath}) \times (-\vec{\imath} + 2\vec{\jmath} + \vec{k}) =$$
$$= (-2\vec{k} + \vec{\jmath}) + (-\vec{k} + \vec{\imath}) + (\vec{k} + \vec{\jmath}) + (2\vec{\jmath} + \vec{\imath}) +$$
$$(2\vec{k} - \vec{\jmath}) + (-\vec{k} - \vec{\imath}) = +\vec{\imath} + 3\vec{\jmath} - \vec{k}$$

c) $I(\mathcal{F}) = \vec{R}(\mathcal{F}) \cdot \vec{N}(\mathcal{F}, 0) = (2\vec{\imath} + 2\vec{k}) \cdot (\vec{\imath} + 3\vec{\jmath} - \vec{k}) = 2 - 2 = 0$

Sistem ima skupno prijemališče.

$$\vec{a} \cdot \vec{b} = (a_1\vec{\imath} + a_2\vec{\jmath} + a_3\vec{k}) \cdot (b_1\vec{\imath} + b_2\vec{\jmath} + b_3\vec{k}) =$$
$$= a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$\vec{OP_0} = \frac{\vec{R} \times \vec{N}(\mathcal{F}, 0)}{|\vec{R}|^2} \qquad |\vec{R}|^2 = |2(\vec{\imath} + \vec{k})|^2 = 4 \cdot 2 = 8$$

$$|\vec{\imath} + \vec{k}| = \sqrt{1+1} = \sqrt{2}$$

$$= \frac{1}{8}\, 2(\vec{\imath} + \vec{k}) \times (\vec{\imath} + 3\vec{\jmath} - \vec{k}) = \frac{1}{4}(3\vec{k} + \vec{\jmath} + \vec{\jmath} - 3\vec{\imath}) =$$
$$= \frac{1}{4}(-3\vec{\imath} + 2\vec{\jmath} + 3\vec{k})$$

$\vec{N}(\mathcal{F}, P_0) = \vec{0}$, ker je P_0 skupno prijemališče.

$$\vec{N}(\mathcal{F}, P_0) = \vec{P_0 O} \times \vec{R} + \vec{N}(\mathcal{F}, 0) = -\vec{\imath} - 3\vec{\jmath} + \vec{k} + (\vec{\imath} + 3\vec{\jmath} - \vec{k})$$
$$= \vec{0} \checkmark$$

$$\vec{P_0 O} \times \vec{R} = -\frac{1}{4}\left(-3\vec{i} + 2\vec{j} + 3\vec{k}\right) \times 2\left(\vec{i} + \vec{k}\right) =$$

$$= -\frac{1}{2}\left(-2\vec{k} + 3\vec{j} + 3\vec{j} + 2\vec{i}\right) = -\frac{1}{2}\left(2\vec{i} + 6\vec{j} - 2\vec{k}\right) =$$

$$= -\vec{i} - 3\vec{j} + \vec{k}$$

3. Podan je sistem sil $\vec{F}_1 = F_0(3\vec{\imath} + 2\vec{\jmath} + \vec{k})$, $\vec{F}_2 = F_0(-3\vec{\jmath} + \vec{k})$, $\vec{F}_3 = F_0(-2\vec{\imath} + 3\vec{\jmath} - 3\vec{k})$ s prijemališči v točkah $P_1(a, 0, a)$, $P_2(2a, 2a, -2a)$, $P_3(-a, 0, 0)$. Dodaj sistemu silo $(P_4, \vec{F}_4)$ tako, da razširjeni sistem sil ~~imel~~ ima skupno prijemališče v točki $P_0(a, -a, a)$.

$$\vec{OP_0} = a(\vec{\imath} - \vec{\jmath} + \vec{k}) = \frac{\vec{R} \times \vec{N}}{|\vec{R}|^2}$$

3 enačbe

$I(\mathfrak{F}) = 0$

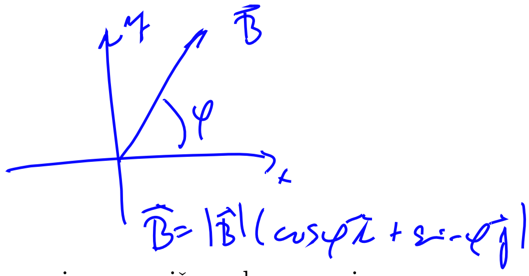
$$\vec{R} = \vec{R}(\mathfrak{F}); \qquad \mathfrak{F} = \{(P_1, \vec{F}_1), \ldots, (P_4, \vec{F}_4)\}$$

4 enačbe

$$P_4(x, y, z) \qquad \vec{F}_4 = f_1\vec{\imath} + f_2\vec{\jmath} + f_3\vec{k}$$

$$\vec{F}_4 \perp \vec{R}(\mathfrak{F}_0); \qquad \mathfrak{F}_0 = \{(P_1, \vec{F}_1), \ldots, (P_3, \vec{F}_3)\}$$

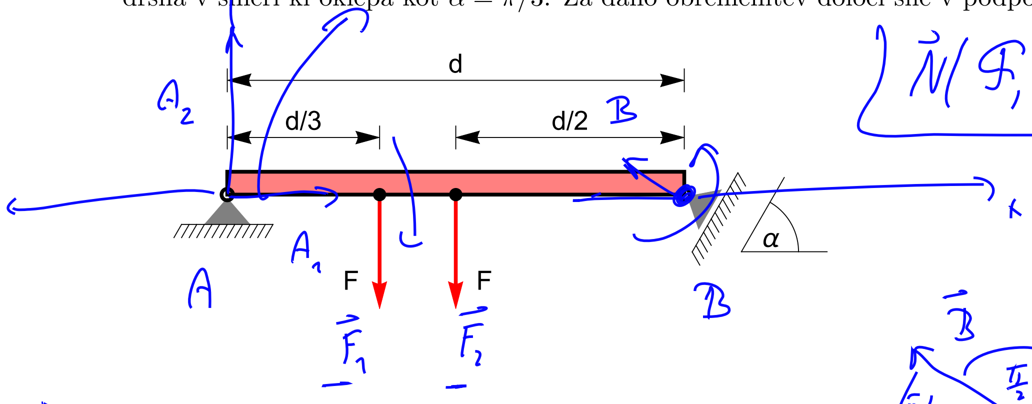
$$\vec{F}_4 \perp \vec{N}(\mathfrak{F}_0, 0)$$

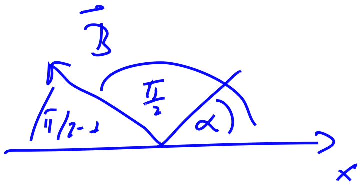


$\vec{B} = |\vec{B}|(\cos\varphi\,\vec{i} + \sin\varphi\,\vec{j})$

4. Nosilec dolžine d je podprt tako kot kaže skica. Leva podpora je nepomična, desna pa je drsna v smeri ki oklepa kot $\alpha = \pi/3$. Za dano obremenitev določi sile v podporah.

$\vec{N}(\mathfrak{F}, 0) = \vec{0}$

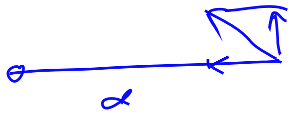




$\vec{A} = A_1\vec{i} + A_2\vec{j}$;

$\vec{B} = B\left(\cos\left(\frac{\pi}{2}+\alpha\right)\vec{i} + \sin\left(\frac{\pi}{2}+\alpha\right)\vec{j}\right) =$

$= B(-\sin\alpha\,\vec{i} + \cos\alpha\,\vec{j})$



$dB_2 = dB_{rok}$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$; $\cos\frac{\pi}{2} = 0$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$; $\sin\frac{\pi}{2} = 1$

$\alpha = \frac{\pi}{2}, \beta = \alpha$

$\vec{F}_1 = -F\vec{j}$; $\vec{F}_2 = -F\vec{j}$

$\mathfrak{F} = \{((0,0), (A_1\vec{i} + A_2\vec{j})), ((d,0), B(-\sin\alpha\,\vec{i} + \cos\alpha\,\vec{j})),$
$((\frac{d}{3},0), -F\vec{j}), ((\frac{d}{2},0), -F\vec{j})\}$

$\vec{0} = \vec{R}(\mathfrak{F}) = A_1\vec{i} + A_2\vec{j} + B(-\sin\alpha\,\vec{i} + \cos\alpha\,\vec{j}) - F\vec{j} - F\vec{j}$

$0 = A_1 - B\sin\alpha$
$0 = A_2 + B\cos\alpha - 2F$

$B = \frac{1}{\cos\alpha}(2F - A_2) = \frac{1}{\cos\alpha}\left(\frac{12}{6} - \frac{7}{6}\right)F = \frac{5F}{6\cos\alpha}$

$A_1 = B\sin\alpha = \frac{5F}{6\cos\alpha}\sin\alpha = \frac{5}{6}F\tan\alpha$

$$\vec{0} = \vec{N}(\mathcal{F}, B) = \vec{k}\left(-d A_2 + \frac{d}{2} F + \frac{2}{3} d F\right) = -d A_2 + \left(\frac{d}{2} + \frac{2d}{3}\right) F = 0$$

$$A_2 = \left(\frac{1}{2} + \frac{2}{3}\right) F = \frac{3+4}{6} F = \underline{\underline{\frac{7}{6} F}}$$

$$\vec{0} = \vec{N}(\mathcal{F}, A) = \vec{k}\left(-\frac{d}{3} F - \frac{d}{2} F + B d \cos\alpha\right)$$

$$0 = -\frac{5}{6} F + B \cos\alpha \Rightarrow \underline{\underline{B = \frac{5 F}{6 \cos\alpha}}}$$