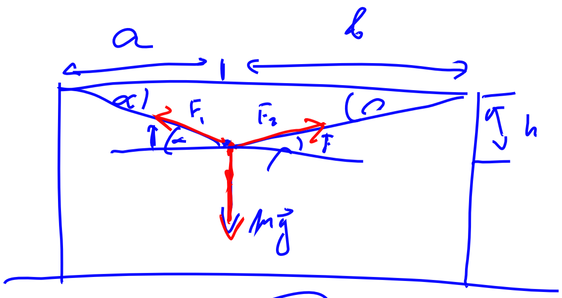


Vaje 18. marec 2021

1. Utež na dveh žicah. Določi sili žic.

$$F_1 \cos\alpha = F_2 \cos\beta \quad \Rightarrow \quad F_2 = \frac{\cos\alpha}{\cos\beta} F_1$$

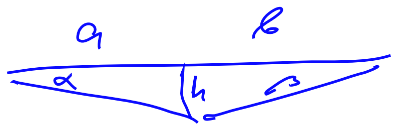
$$F_1 \sin\alpha + F_2 \sin\beta = mg$$

$$F_1 \left(\sin\alpha + \frac{\cos\alpha}{\cos\beta} \sin\beta \right) = mg$$

$$F_1 \frac{\sin\alpha \cos\beta + \cos\alpha \sin\beta}{\cos\beta} = mg$$

$$F_1 = mg \frac{\cos\beta}{\sin\alpha \cos\beta + \cos\alpha \sin\beta} = mg \frac{\cos\beta}{\sin(\alpha+\beta)}$$

$$F_2 = mg \frac{\cos\alpha}{\sin(\alpha+\beta)}$$



$$\sin\alpha = \frac{h}{\sqrt{a^2+h^2}} \qquad \cos\alpha = \frac{a}{\sqrt{a^2+h^2}}$$

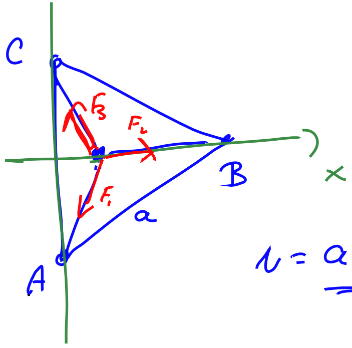
$$\sin\beta = \frac{h}{\sqrt{b^2+h^2}}, \qquad \cos\beta = \frac{b}{\sqrt{b^2+h^2}}$$

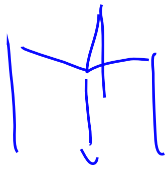
$$F_1 = mg \frac{\frac{b}{\sqrt{b^2+h^2}}}{\frac{h}{\sqrt{a^2+h^2}} \frac{b}{\sqrt{b^2+h^2}} + \frac{a}{\sqrt{a^2+h^2}} \frac{h}{\sqrt{b^2+h^2}}} =$$

$$= mg \frac{b\sqrt{a^2+h^2}}{h(a+b)}$$

$$F_2 = \frac{\cos\alpha}{\cos\beta} F_1 =$$

$$F_2 = mg \frac{a\sqrt{b^2+h^2}}{h(a+b)}$$





$r = a\frac{\sqrt{3}}{2}$

mg

$A(0, -\frac{a}{2}, 0)$ $\qquad P(a\frac{\sqrt{3}}{6}, 0, -h)$

$B(a\frac{\sqrt{3}}{2}, 0, 0)$ $\qquad \uparrow \frac{r}{3}$

$C(0, \frac{a}{2}, 0)$

$$\vec{F}_1 = F_1 \frac{\vec{PA}}{|\vec{PA}|}, \quad \vec{F}_2 = F_2 \frac{\vec{PB}}{|\vec{PB}|}, \quad \vec{F}_3 = \frac{\vec{PC}}{|\vec{PC}|}$$

$$\vec{PA} = A - P = -a\frac{\sqrt{3}}{6}\vec{i} - \frac{a}{2}\vec{j} + h\vec{k}$$

$$|\vec{PA}|^2 = a^2 \cdot \frac{3}{36} + a^2\frac{1}{4} + h^2 = a^2\left(\frac{1}{12} + \frac{1}{4}\right) + h^2 = \frac{1}{3}a^2 + h^2$$

$$|\vec{PA}| = \sqrt{\frac{1}{3}a^2 + h^2} \qquad \frac{1+3}{12}$$

$$\vec{PB} = B - P = a\sqrt{3}\left(\frac{1}{2} - \frac{1}{6}\right)\vec{i} + h\vec{k} = a\frac{1}{\sqrt{3}}\vec{i} + h\vec{k} \; ; \; |\vec{PB}| = \sqrt{\frac{1}{3}a^2 + h^2}$$

$$\vec{PC} = C - P = -a\frac{\sqrt{3}}{6}\vec{i} + \frac{a}{2}\vec{j} + h\vec{k} \; ; \; |\vec{PC}| = \sqrt{\frac{1}{3}a^2 + h^2}$$

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 - mg\vec{k} = \vec{0} \qquad / \sqrt{\frac{1}{3}a^2 + h^2}$$

$$\vec{i}: \; -a\frac{\sqrt{3}}{6}F_1 + a\frac{1}{\sqrt{3}}F_2 - a\frac{\sqrt{3}}{6}F_3 = 0 \Rightarrow -\frac{1}{2}F_1 + F_2 - \frac{1}{2}F_3 = 0$$

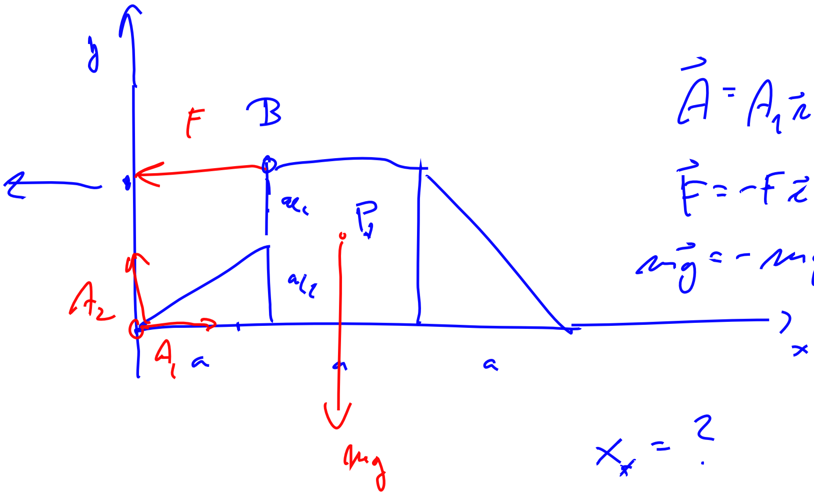
$$\vec{j}: \; -\frac{a}{2}F_1 + \frac{a}{2}F_3 = 0 \Rightarrow F_1 = F_3$$

$$\Rightarrow F_2 = F_1 = F_3$$

$$\vec{k}: \; hF_1 + hF_2 + hF_3 = mg\sqrt{\frac{1}{3}a^2 + h^2}$$

$$3hF_1 = mg\sqrt{\frac{1}{3}a^2 + h^2} \Rightarrow \underline{F_1 = F_2 = F_3 = mg\frac{\sqrt{\frac{1}{3}a^2 + h^2}}{3h}}$$

2. Homogena plošča sestavljena iz kvadrata in dveh trikotnikov je členkasto vpeta. Določi silo vrvice, in reakcijo v tečaju, ki drži ploščo v ravnovesju.



$\vec{A} = A_1\vec{i} + A_2\vec{j}$ $\quad A(0,0)$

$\vec{F} = -F\vec{i}$ $\quad B(0,a)$

$m\vec{g} = -mg\vec{j}$ $\quad P_*(\,?\,)$

$P_*(x_*, y_*)$

$x_* = ?$

	A	x
◺	$\frac{1}{4}a^2$	$\cancel{\frac{a}{3}}$
□	a^2	$a + \frac{a}{2} = \frac{3a}{2}$
◺	$\frac{1}{2}a^2$	$2a + \frac{a}{3}\quad \frac{7}{3}a$

napaka ma je $\frac{2}{3}a$

$$A = a^2\left(\frac{1}{4} + 1 + \frac{1}{2}\right) = a^2\,\frac{7}{4}$$

$$x_* = \frac{1}{A}\left(A_1 x_1^* + A_2 x_2^* + A_3 x_3^*\right) =$$

$$= \frac{4}{7a^2}\left(\frac{1}{4}a^2\cdot\cancel{\frac{a}{3}}\ \frac{2a}{3} + a^2\cdot\frac{3a}{2} + \frac{1}{2}a^2\cdot\frac{7}{3}a\right) =$$

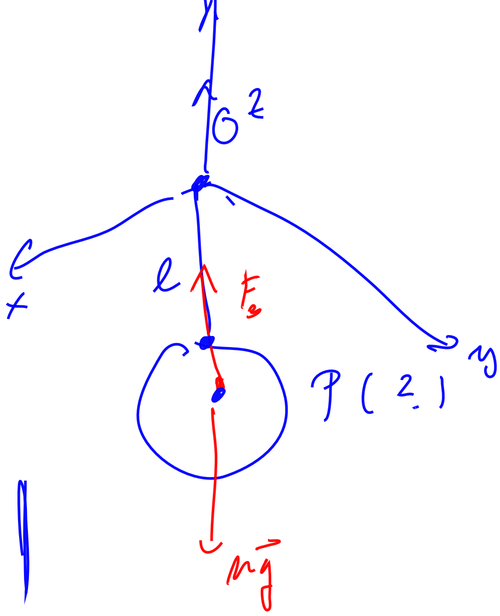
$$= \frac{4}{7}a\left(\frac{\cancel{1}\,2}{12} + \frac{3}{2} + \frac{7}{6}\right) = \frac{4}{7}a\,\frac{\cancel{1}\,2 + 18 + 14}{12} = \frac{4}{7}a\,\frac{\cancel{33}\,34}{12} = \cancel{\frac{11}{7}a}\ \frac{34}{21}$$

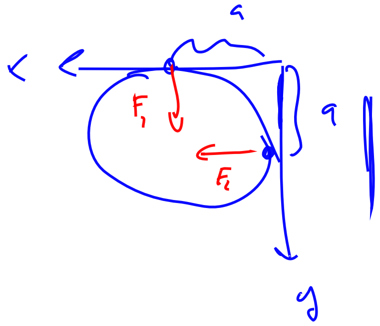
$$0 = -x^* mg + aF \Rightarrow F = mg\,\cancel{\frac{11}{7}}\,\frac{34}{21}$$

$$A_1 - F = 0 \Rightarrow A_1 = mg\,\cancel{\frac{11}{7}}\,\frac{34}{21}$$

$$A_2 - mg = 0 \Rightarrow A_2 = mg$$

3. Krogla v vogalu. Določi silo vrvice.





$$P(a, a, -h)$$

$$|\vec{OP}| = l + a = \sqrt{a^2 + a^2 + h^2}$$

$$(l+a)^2 = 2a^2 + h^2 \Rightarrow h = \sqrt{(l+a)^2 - 2a^2}$$

$$h = \sqrt{l^2 + 2al - a^2}$$

$$\vec{F}_3 = F_3 \frac{\vec{PO}}{|\vec{PO}|} = F_3 \frac{1}{l+a}(O - P) = F_3 \frac{1}{l+a}(-a\vec{i} - a\vec{j} + h\vec{k})$$

$$\vec{F}_1 = F_1 \vec{j}, \quad \vec{F}_2 = F_2 \vec{i}$$

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + m\vec{g} = 0 \qquad m\vec{g} = mg(-\vec{k})$$

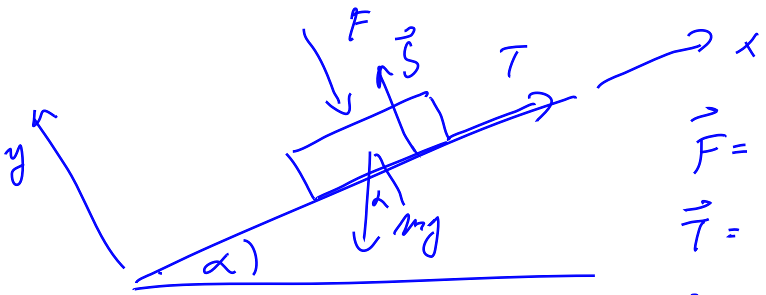
$$\left.\begin{array}{ll} \vec{i}: & F_2 - \frac{a}{l+a} F_3 = 0 \Rightarrow F_2 = \frac{a}{l+a} F_3 \\ \vec{j}: & F_1 - \frac{a}{l+a} F_3 = 0 \Rightarrow F_1 = \frac{a}{l+a} F_3 \end{array}\right\} \Rightarrow F_1 = F_2$$

$$\vec{k}: \quad \frac{h}{l+a} F_3 - mg = 0 \Rightarrow F_3 = mg \frac{l+a}{h} = mg \frac{l+a}{\sqrt{l^2 + 2al - a^2}}$$

$$h = 0 \Leftrightarrow l^2 + 2al - a^2 = 0$$

$$l_{1,2} = \frac{-2a \pm \sqrt{4a^2 + 4a^2}}{2} = -a \pm a\sqrt{2} \ ; \quad l_{min} = a(\sqrt{2} - 1)$$

4. Klada na klancu. Določi silo, da klada ne zdrsne.



$\vec F = -F\vec j \qquad \vec S = S\vec j$

$\vec T = T\vec i$

$m\vec g = mg(-\sin\alpha\,\vec i - \cos\alpha\,\vec j)$

$\vec 0 = \vec F + \vec T + m\vec g$

$\vec i: \quad 0 = T - mg\sin\alpha$

$\vec j: \quad 0 = -F - mg\cos\alpha + S$

$T \le kS$