Predavanje 24. marec 2021

Vijačna dvigalka

Dvigovanje, spuščanje klade na klancu.
Dvigovanje

$$F = mg\frac{\sin(\alpha+\alpha_0)}{\cos(\beta+\alpha_0)}. \tag{1}$$

Spuščanje

$$F = mg\frac{\sin(\alpha-\alpha_0)}{\cos(\beta-\alpha_0)}. \tag{2}$$

Navor vijačne dvigalke

$$M = Gr_0\tan(\alpha+a\alpha_0),$$

α strmina vijačnice, α_0 torni kot.

$\alpha = \rho$

$mg = G$ $\quad f = \frac{G}{L}\frac{\sin(\alpha+a\alpha_0)}{\cos(\alpha+a\alpha_0)}$

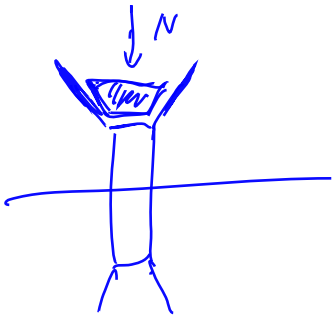
$M = r_0 \cdot L \cdot f = r_0 G \operatorname{tg}(\alpha+a\alpha_0)$

Dvigovanje: $a=1$ $\quad M = r_0 G \operatorname{tg}(\alpha+\alpha_0)$ $\quad \operatorname{tg}\alpha_0 = k$

Spuščanje $a=1$ $\quad M = r_0 G \operatorname{tg}(\alpha-\alpha_0)$

$\alpha > \alpha_0 \Rightarrow M > 0$

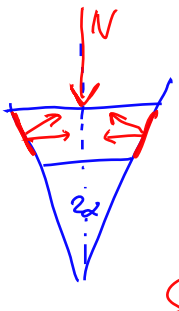
Samozapornost $M < 0$ $\quad \alpha < \alpha_0$



$f \leq kN$

Klinasti jermen

Efektivni koeficient trenja $\hat{k} = k/\sin\alpha$.



$S_1 = S_2 = S$

$S_2 \sin\alpha + S_1 \sin\alpha = N$

$2S\sin\alpha = N$

$S = \frac{N}{2\sin\alpha}$

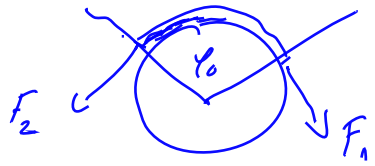
$F \leq 2kS = 2k\frac{N}{2\sin\alpha} = \frac{k}{\sin\alpha}N$

$F \leq \frac{k}{\sin\alpha}N$

$\frac{k}{\sin\alpha} > k$

Trenje vrvi na kolutu

Izpeljava formule $F_2 = F_1 \exp k\varphi_0$.

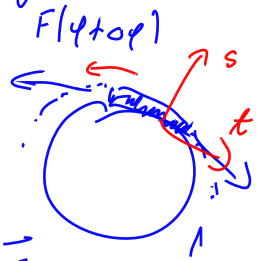


φ_0 ovojni kot

k koef trenja med vrvjo in kolutom

$$F_2 = F_1 e^{k\varphi_0}$$

$$e^{2\pi i} = 2\pi i$$

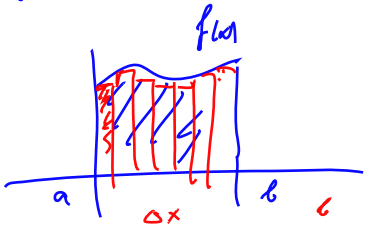


$$\vec{F}(\varphi+\Delta\varphi) - \vec{F}(\varphi) + \int_{\varphi}^{\varphi+\Delta\varphi} (\vec{s}+\vec{t})\,d\varphi = \vec{0}$$

l. $\Delta\varphi \to 0$

$\vec{F}(\varphi)$ sila vrvi $\varphi > \varphi_0$

na del vrvi $\varphi < \varphi_0$



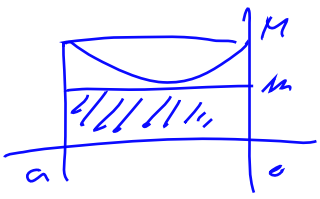
$$A = \int_a^b f(x)\,dx = \lim_{\Delta x \to 0} \sum_{i=1}^{n} \Delta x\, f(x_i) \qquad x_i = a + (i-1)\Delta x$$

$(n \to \infty)$ $\qquad \Delta x = \frac{b-a}{n}$

$$\lim \frac{1}{b-a} \int_a^b f(x)\,dx = f(a)$$

$(b \to a)$ Izrek o povprečni vrednosti

Vrednosti kvocienta S_2/S_1 pri $k = \frac{1}{2}$ za različne ovojne kote φ_0.

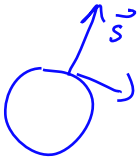


$$A = \int_a^b f(x)\,dx \qquad m \le \frac{A}{b-a} \le M$$

$$m(b-a) \le A \le M(b-a)$$

$(\vec{s}+\vec{t})(\varphi)$

$$\lim_{\Delta\varphi \to 0} \frac{1}{\Delta\varphi} \left(\vec{F}(\varphi+\Delta\varphi) - \vec{F}(\varphi) \right) + \int_{\varphi}^{\varphi+\Delta\varphi} (\vec{s}+\vec{t})\,d\varphi$$



$$\frac{d\vec{F}}{d\varphi}$$

$$\vec{0} = \frac{d\vec{F}}{d\varphi} + \vec{s} + \vec{t}$$

$$\vec{F} = F(\varphi)\,\vec{e}_\varphi \;;\; \vec{s} = s\,\vec{e}_r \;;\; \vec{t} = -t\,\vec{e}_\varphi$$

$$\frac{d\vec{F}}{d\varphi} = \frac{d}{d\varphi}(F\vec{e}_\varphi) = \frac{dF}{d\varphi}\vec{e}_\varphi + F\frac{d\vec{e}_\varphi}{d\varphi} = \frac{dF}{d\varphi}\vec{e}_\varphi - F\vec{e}_r$$

$$\vec{0} = \frac{dF}{d\varphi}\vec{e}_\varphi - F\vec{e}_r + S\vec{e}_n - t\vec{e}_\varphi$$

$$-F + S = 0$$

$$\frac{dF}{d\varphi} - t = 0$$

Primer: zdrs vrvi na kolutu škripca.

$$\frac{dF}{d\varphi} = t = kS = kF \qquad t = kS$$

$$F = S$$

$$\boxed{\frac{dF}{d\varphi} = kF}$$

$f(x)$

$$\boxed{f' = f}$$

$$\Downarrow$$

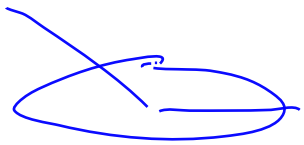
$$f = Ce^x$$

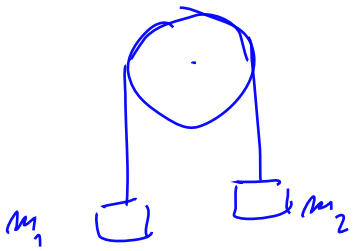
$$F = Ce^{k\varphi}$$

$$F(\varphi = 0) = Ce^0 = C$$

$$\boxed{F_2 = F(\varphi) = F_1 e^{k\varphi}}$$

$$F_2 = F(\varphi = \alpha)$$





$$m_1 > m_2$$

Dolžini k, da ne zdrsne.

$$m_2 g \le m_2 g e^{k\cdot\pi}$$

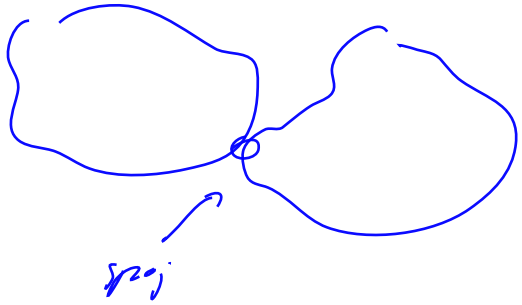
$$\frac{m_1}{m_2} \le e^{k\pi}$$

$$\log\left(\frac{m_1}{m_2}\right) \le k\pi \; ; \quad \boxed{k \ge \frac{1}{\pi}\log\frac{m_1}{m_2}}$$

$$F_2 \le F_1 e^{k\varphi}$$

Statika sistema togih teles

Spoji med telesi, sile in navori v spojih.

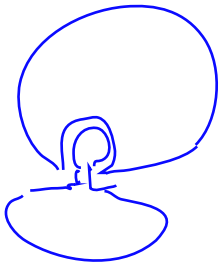


Klasifikacija spojev:

1. popolni spoj, prenos vseh sil in momentov;

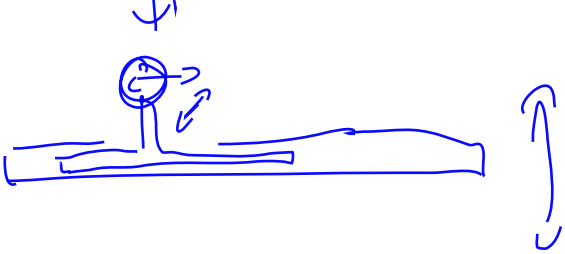
2. tečaj, prenos vseh sil in momentov pravokotnih na os tečaja;

3. križni zglob, prenos vseh sil in momenta v smeri osi zgloba;



4. krogelni zglob, prenos vseh sil brez prenosa momenta;

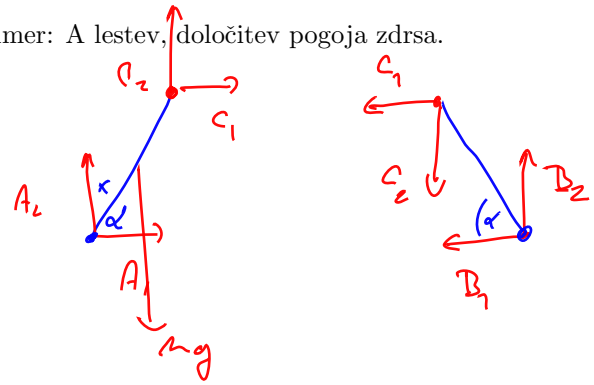
5. linijski drsnik, prenos sil pravokotnih na smer drsnika in vseh momentov;

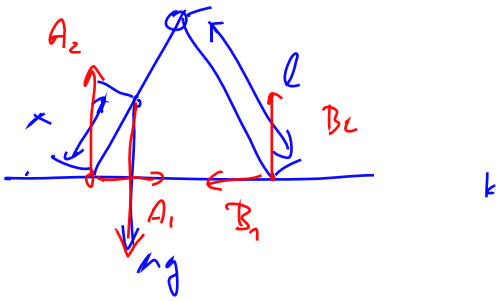


6. ploškovni drsnik, prenos sile pravokotne na ravnino drsnika in vseh momentov;

7. kombinacija drsnika in zgloba.

Primer: A lestev, določitev pogoja zdrsa.





$$\left[\begin{array}{l} A_1 + C_1 = 0 \\ A_2 + C_2 - mg = 0 \\ -mgx\cos\alpha + C_2 \ell\cos\alpha - C_1 \ell\sin\alpha = 0 \end{array}\right. \qquad \left|\begin{array}{l} -B_1 - C_1 = 0 \\ B_2 - C_2 = 0 \\ C_1 \ell\sin\alpha + C_2 \ell\cos\alpha = 0 \end{array}\right.$$

$$C_2 = -C_1 \frac{\sin\alpha}{\cos\alpha}$$

$$-mgx\cos\alpha - C_1 \frac{\sin\alpha}{\cos\alpha}\ell\cos\alpha - C_1 \ell\sin\alpha = 0$$

$$-2\ell C_1 \sin\alpha = mgx\cos\alpha$$

$$C_1 = -\frac{1}{2}mg\frac{x}{\ell}\frac{\cos\alpha}{\sin\alpha} \qquad C_2 = -C_1\frac{\sin\alpha}{\cos\alpha} = \frac{1}{2}mg\frac{x}{\ell}$$

$$A_1 = -C_1 = \frac{1}{2}mg\frac{x}{\ell}\frac{\cos\alpha}{\sin\alpha} \qquad B_1 = -C_2 = \frac{1}{2}mg\frac{x}{\ell}\frac{\cos\alpha}{\sin\alpha}$$

$$A_2 = mg - C_2 = mg\left(1 - \frac{x}{2\ell}\right) \qquad B_2 = C_2 = \frac{1}{2}mg\frac{x}{\ell}$$

$$A_1 \le kA_2 \; ; \quad B_1 \le kB_2$$

$$\frac{1}{2}mg\frac{x}{\ell}\frac{\cos\alpha}{\sin\alpha} \le kmg\left(1 - \frac{x}{2\ell}\right) \qquad \frac{1}{2}mg\frac{x}{\ell}\operatorname{ctg}\alpha \le k\frac{1}{2}mg\frac{x}{\ell}$$

$$\frac{1}{2}\frac{x}{\ell}\operatorname{ctg}\alpha \le k\left(1 - \frac{x}{2\ell}\right) \qquad \boxed{\operatorname{ctg}\alpha \le k}$$

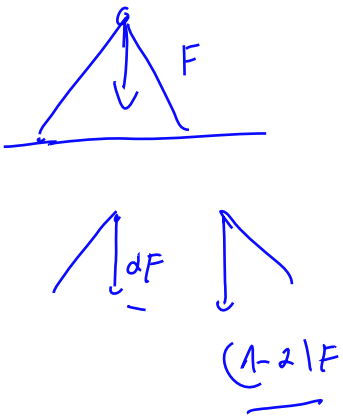
$x = 0$ $\quad 0 \le k$ ✓

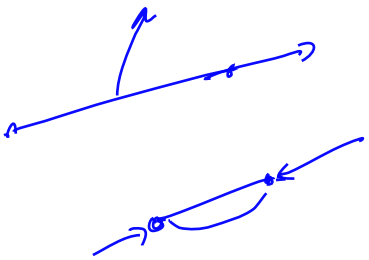
$x = \ell$ $\quad \frac{1}{2}\operatorname{ctg}\alpha \le \frac{1}{2}k = \operatorname{ctg}\alpha \le k$

Pri tem pogoju lestev ne zdrsne.

Potek reševanja nalog statike sistema togih teles:

- identifikacija zunanjih sil;
- razčelnitev sistema na toge komponente;
- identifikacija sil in momentov v spojih;
- postavitev diagramov prostih teles;
- zapis ravnotežnih enačb;
- raševanje sistema ravnotežnih enačb.

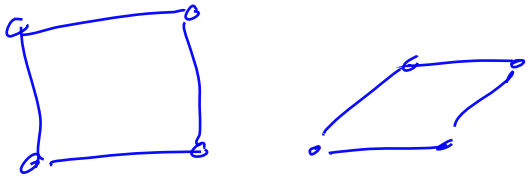




Paličje

Paličje je togi sistem sestavljen iz palic pod vplivom sil s prijemališči v spojih palic. Sile v palicah, natezne, tlačne. Enostavno paličje:

- ravninsko paličje: $2v - 3 = p$;
- prostorsko paličje: $3v - 6 = p$.

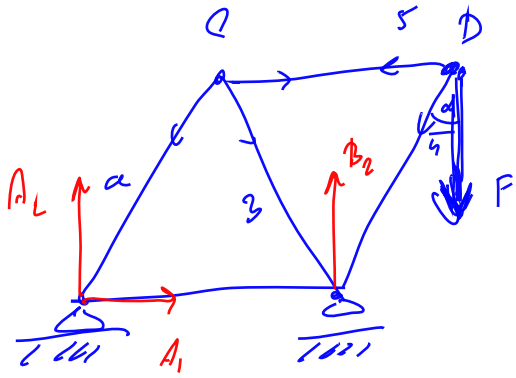


Ravnovesne enačbe paličja.

Enostavno paličje je pri statično določenih podporah statično določeno.

Vozliščna metoda

Primer: paličje treh enakostraničnih trikotnikov.



$\alpha = \frac{\pi}{6}$

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\sin\frac{\pi}{6} = \frac{1}{2}$

$a B_2 - \frac{3a}{2} F = 0$

$B_2 = \frac{3}{2} F, \quad A_1 = 0$

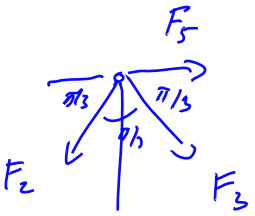
$A_L + B_2 - F = 0 \Rightarrow \boxed{A_2 = -\frac{1}{2} F}$

$-F_5 - F_4 \sin\alpha = 0$

$-F_4 \cos\alpha - F = 0$

$F_4 = -\frac{F}{\cos\alpha} = \boxed{-\frac{2F}{\sqrt{3}}}$

$F_5 = -F_4 \sin\alpha = \frac{F}{\sqrt{3}}$



$\cos\frac{\pi}{3} = \frac{1}{2}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

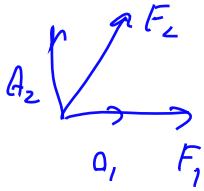
$-F_2 \cos\frac{\pi}{3} + F_3 \cos\frac{\pi}{3} + F_r = 0$

$-F_2 \cos\frac{\pi}{6} - F_3 \cos\frac{\pi}{6} = 0$

$-F_2 \frac{1}{2} + F_3 \frac{1}{2} = -\frac{F}{\sqrt{3}}$

$-F_2 - F_3 = 0 \qquad F_2 = -F_3$

$\boxed{F_3 = -\frac{F}{\sqrt{3}}}; \quad F_2 = \frac{F}{\sqrt{3}}$



$F_1 + F_2 \cos\frac{\pi}{3} = 0 \Rightarrow F_1 = -F_2 \cdot \frac{1}{2} = -\frac{F}{2\sqrt{3}}$

$A_2 + F_2 \cos\frac{\pi}{6} = 0$

$A_2 = -F_2 \frac{\sqrt{3}}{2} = -\frac{F}{2}$

Metoda prereza

Primer

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