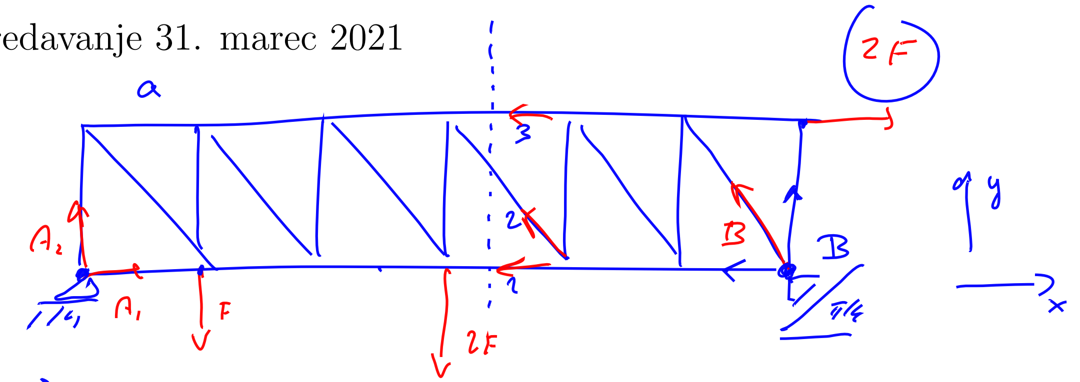
Predavanje 31. marec 2021



$$\vec{A} = A_1 \vec{i} + A_2 \vec{j} \quad ; \quad \vec{B} = B\left(-\frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}\right)$$

Metoda prereza

Primer: določitev sil v izbranih palicah.

$$-6a A_2 + 5aF + 3a \cdot 2F - a2F = 0$$

$$A_2 = \frac{1}{6}(5+6-2)F = \frac{3}{2}F$$

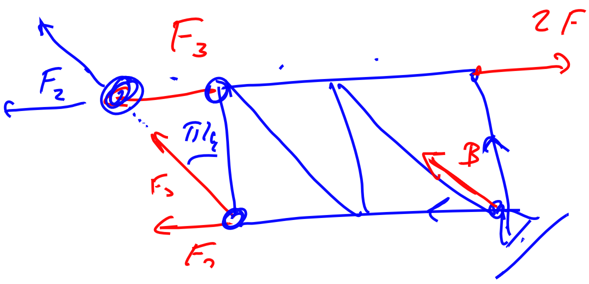
$$6a \frac{1}{\sqrt{2}} B - aF - 3a \cdot 2F - a2F = 0$$

$$B = \frac{\sqrt{2}}{6}(1+6+2)F = \frac{3}{2}\sqrt{2}F = \frac{3}{\sqrt{2}}F$$

$$A_1 - B\frac{1}{\sqrt{2}} + 2F = 0 \Rightarrow A_1 = B\frac{1}{\sqrt{2}} - 2F = \frac{3}{2}F - 2F = -\frac{1}{2}F$$

$$A_2 - F - 2F + \frac{1}{\sqrt{2}}B = \frac{3}{2}F - 3F + \frac{3}{2}F = 0 \checkmark$$

(2) Naredimo prerez:



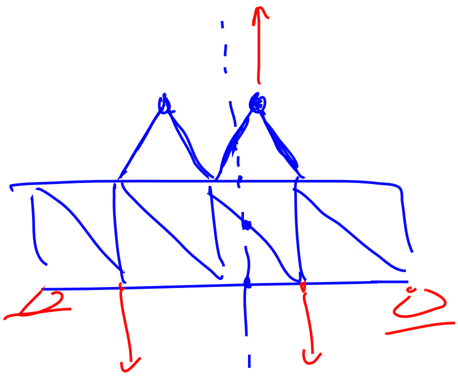
$$aF_3 - a2F + 2a\frac{1}{\sqrt{2}}B = 0 \Rightarrow F_3 = 2F - 2\frac{3}{2}F = -F \checkmark$$

$$-aF_1 - a\frac{1}{\sqrt{2}}B + 3a\frac{1}{\sqrt{2}}B = 0 \Rightarrow F_1 = -\frac{1}{\sqrt{2}}B + 3\frac{1}{\sqrt{2}}B = 2\frac{1}{\sqrt{2}}B = \cancel{\frac{3}{2}F}\ 3F$$

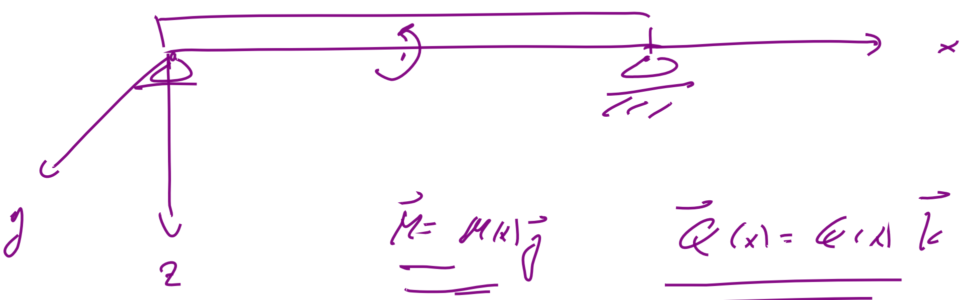
$$\frac{1}{\sqrt{2}}F_2 + \frac{1}{\sqrt{2}}B = 0 \Rightarrow F_2 = -B = -\frac{3}{\sqrt{2}}F \checkmark$$

$$-F_1 - \frac{1}{\sqrt{2}}F_2 - F_3 - \frac{1}{\sqrt{2}}B + 2F = 0$$

$-3F - \cancel{\frac{3}{2}}F + \cancel{\frac{3}{2}}F + F - \cancel{\frac{3}{2}}F + 2F = 0$ ✓

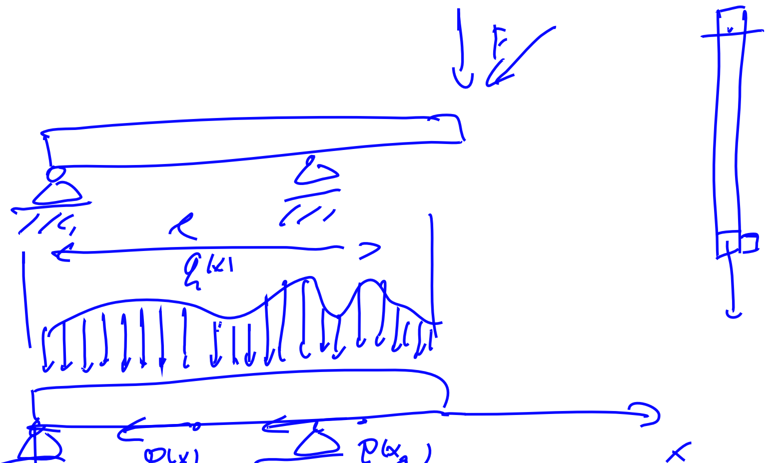


Primerjava vozliščne metode in metode prereza.



$\vec{M} = M(x)\vec{j}$ $\qquad$ $\vec{Q}(x) = Q(x)\vec{k}$

Ravninski nosilci



prečna

$q(x)$ gostota linijske obremenitve $\qquad \vec{q} = q(x)\,\cancel{\vec{j}}\ \vec{k}$

$p(x)$ gostota osne linijske obremenitve

Linijska obremenitev nosilca

Rezultanta linijske obremenitve je integral gostote linijske obremenitve

$$\vec{F} = \int_0^l \left(q(x)\,\cancel{\vec{j}}\,\vec{k} + p(x)\,\vec{i}\right) dx = \int_0^l p(x)\,dx\ \vec{i} + \int_0^l q(x)\,dx\ \cancel{\vec{j}}\,\vec{k}$$

Konstantna linijska obremenitev

$\Updownarrow$

gostota je konstantna

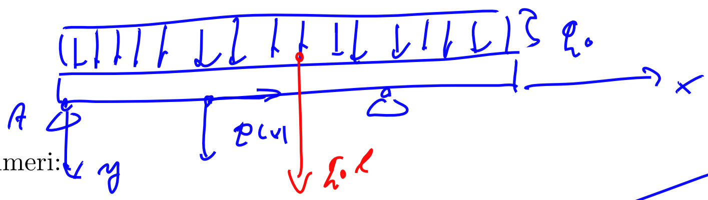
Ekvipolentna točkovna obremenitev.

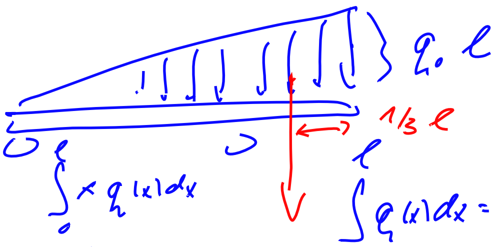
$$q(x) = q_0$$

$$\int_0^l q(x)\,dx = \int_0^l q_0(x)\,dx = q_0 l \quad ; \quad [q_0] = \frac{N}{m}$$

$q(x) = q_0 x$

linearna obremenitev





Primeri:

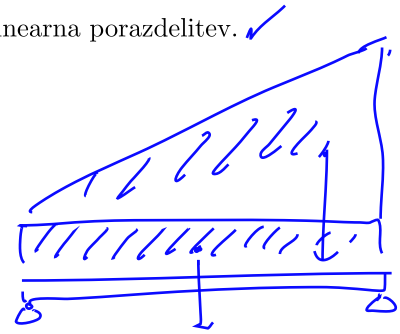
- enakomerna(konstantna) porazdelitev; ✓

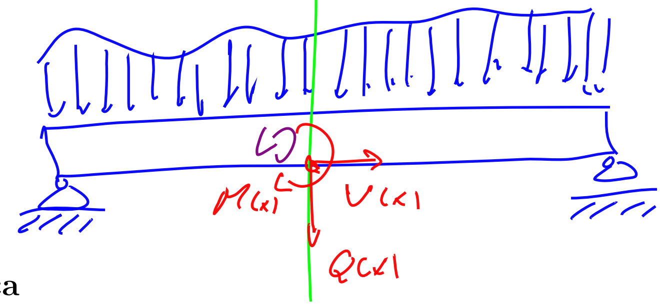
$$\vec{N} \to \vec{N}(A) = \int_0^l x\,q(x)\,dx\ \ \vec{k}\ \cancel{\vec{j}}$$

$$r \cdot \int_0^l q(x)\,dx = \int_0^l x\,q(x)\,dx \qquad \int_0^l q(x)\,dx = \frac{1}{2}\,l^2 q_0$$

$$r = \frac{\int_0^l x\,q(x)\,dx}{\int_0^l q(x)\,dx} = \frac{N}{F}$$

- linearna porazdelitev. ✓





Notranje količine nosilca

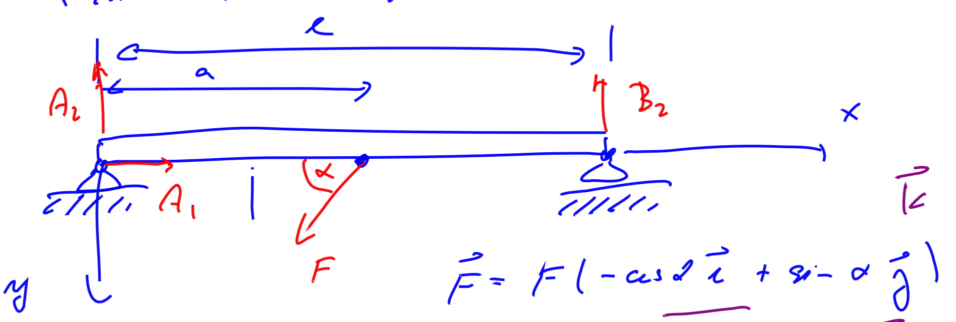
Navidezni prerez nosilca.

- osna sila $V(x)$; sila desnega dela nosilca na levi del nosilca v osni smeri pri navideznem prerezu x
- prečna sila $Q(x)$;
- upogibni moment $M(x)$.

v prečni smeri

moment desnega dela na levi del

pozitiven v obratni smeri urinega kazalca



$\vec{F} = F(-\cos\alpha\,\vec{i} + \sin\alpha\,\vec{j})$

$-lB_2 + aF\sin\alpha = 0$

$B_2 = \frac{a}{l}F\sin\alpha$

$lA_2 - (l-a)F\sin\alpha = 0$

$A_2 = \frac{l-a}{l}F\sin\alpha$

$A_1 - F\cos\alpha = 0 \qquad A_1 = F\cos\alpha$

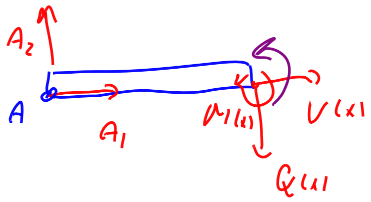
$A_2 + B_2 - F\sin\alpha = F\sin\alpha - F\sin\alpha = 0$

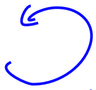
Določitev notranjih količin z metodo prereza

Primer: točkovno obremenjen enostavno podprt nosilec:

- potek osne sile;
- potek prečne sile;
- potek upogibnega momenta.

$0 < x < a$





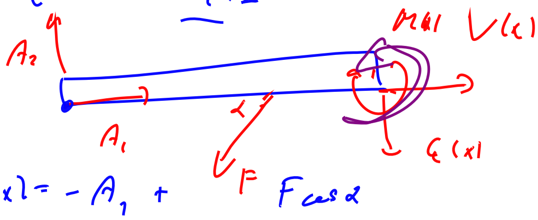
v skriptah

$A_1 + V(x) = 0 \Rightarrow V(x) = -A_1$

$-A_2 + Q(x) = 0 \Rightarrow Q(x) = A_2$

$xQ(x) + (-M(x)) = 0 \Rightarrow -M(x) = +xQ(x) = +xA_2$

$a < x < l$



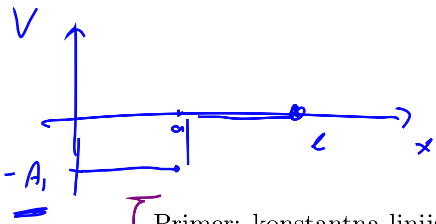
$A_1 - F\cos\alpha + V(x) = 0 \Rightarrow V(x) = -A_1 + F\cos\alpha$

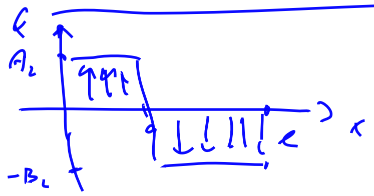
$-A_2 + F\sin\alpha + Q(x) = 0 \Rightarrow Q(x) = A_2 - F\sin\alpha = \left(\frac{l-a}{l} - 1\right)F\sin\alpha = -\frac{a}{l}F\sin\alpha = -B_2$

$aF\sin\alpha + xQ(x) + (-M(x)) = 0 \Rightarrow -M(x) = -xQ(x) - aF\sin\alpha =$

$= -x(A_2 - F\sin\alpha) - aF\sin\alpha =$

$$= -xA_2 + F\sin\alpha\,(x-a) \qquad M(x) = xA_2 - F\sin\alpha\,(x-a)$$



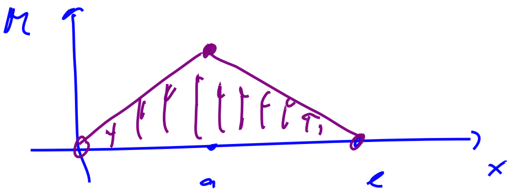


$$Q(x=0) = A_2$$

$$Q(x=l) = -B_2$$

Primer: konstantna linijska obremenitev enostavno podprtega nosilca:

- potek osne sile;
- potek prečne sile;
- potek upogibnega momenta.



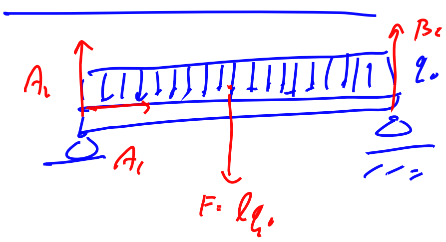
$$a < x < l \Rightarrow M(x) = xA_2 - (x-a)F\sin\alpha =$$

$$= x\frac{l-a}{l}F\sin\alpha - (x-a)F\sin\alpha =$$

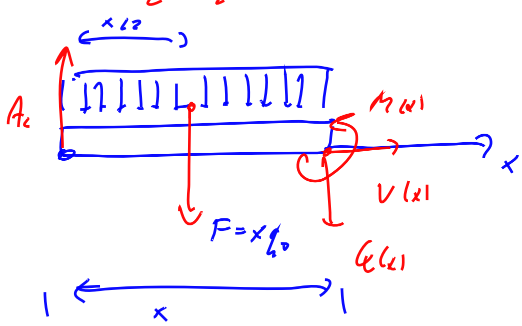
$$= F\sin\alpha\left(x\left(1-\frac{a}{l}\right) - (x-a)\right) =$$

$$= F\sin\alpha\left(a - \frac{a}{l}x\right)$$

$$M(x=l) = F\sin\alpha\left(a - \frac{a}{l}\cdot l\right) = 0$$



$$A_2 = B_2 = \frac{1}{2}lq_0 \qquad A_1 = 0$$



$$0 + V(x) = 0$$

$$-A_2 + xq_0 + Q(x) = 0$$

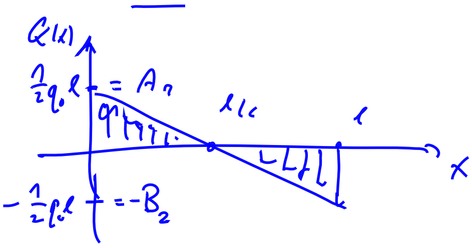
$$Q(x) = A_2 - xq_0 = \frac{1}{2}lq_0 - xq_0$$

$$Q(x) = q_0\left(\frac{1}{2}l - x\right)$$

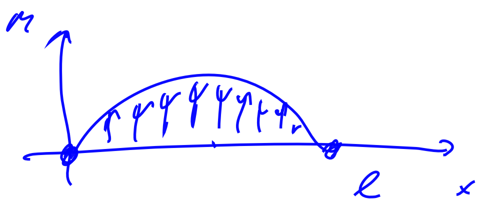
$$-\frac{x}{2}\cdot xq_0 - xQ(x) + M(x) = 0 \Rightarrow M(x) = \frac{1}{2}q_0x^2 + xQ(x) =$$

$$= \frac{1}{2}q_0x^2 + xq_0\left(\frac{1}{2}l - x\right) =$$

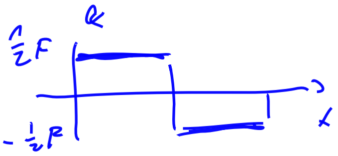
$$= q_0\left(\frac{1}{2}xl - \frac{1}{2}x^2\right) = \frac{1}{2}q_0x(l-x)$$

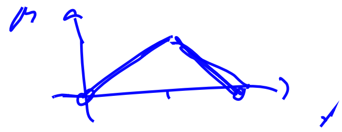


$$Q(x=l) = -B_2\,;\quad Q(x=0) = A_2$$



Točkovna obremenitev $\quad a = l/2, \; \alpha = \pi/2 \; ; \; F = q_0 l$





$\lim_{x\to 0} -A_2 + Q(x) = 0$

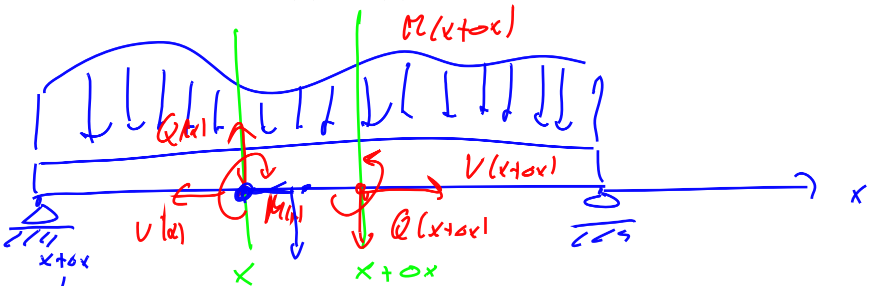
$Q(x=l) = A_2$

Ugotovitve: pri enostavno podprtem nosilcu velja:

1. prečna sila je na levem krajišču enaka sili leve podpore;
2. prečna sila je na desnem krajišču enaka negativni vrednosti sili desne podpore;
3. prečna sila ima pri točkovni obremenitvi v točkah obremenitve nezveznosti s skokom, ki je enak sili obremenitve v tej točki;
4. upogibni moment je enak nič v krajiščih;
5. upogibni moment je pri točkovno obremenjenem nosilcu odsekoma linearen.

Določitev notranjih količin z diferencialno metodo

Izpeljava formul za zvezno linijsko obremenitev $p(x)\vec{\imath} + q(x)\vec{k}$.



$x: \; V(x+\Delta x) - V(x) + \int_x^{x+\Delta x} p(s)\,ds = 0$ za vsak Δx

$\lim_{\Delta x\to 0}\left(\frac{V(x+\Delta x)-V(x)}{\Delta x} + \frac{1}{\Delta x}\int_x^{x+\Delta x} p(s)\,ds = 0\right) \Rightarrow \frac{dV}{dx} + p(x) = 0 \Rightarrow \frac{dV}{dx} = -p(x)$

p zvezna funkcija

$z: \; Q(x+\Delta x) - Q(x) + \int_x^{x+\Delta x} q(s)\,ds = 0 \Rightarrow$ q zvezna $\frac{dQ}{dx} = -q(x)$

Če je linijska obremenitev zvezna v okolici x_0, potem pri $x = x_0$ veljajo ravnovesne enačbe

$$dV/dx = -p(x), \qquad dQ/dx = -q(x), \qquad dM/dx = Q(x).$$

$M(x+\Delta x) - M(x) - \Delta x\, Q(x+\Delta x) - \int_x^{x+\Delta x} (s-x)\, q(s)\,ds = 0 \qquad \frac{1}{\Delta x};\; \lim_{\Delta x\to 0}$

$\frac{dM}{dx} - Q(x) - (s-x)\,q(s)\Big|_{s=x} = 0$

$$\lim_{\Delta x \to 0} Q(x+\Delta x) = Q(x)$$

konstanta

$$\boxed{\frac{dM}{dx} = Q(x)}$$

Primer: ~~linearna~~ linijska obremenitev enostavno podprtega nosilca.

$$\frac{dV}{dx} = -p(x) \; ; \quad \frac{dQ}{dx} = -q(x) \; ; \quad \boxed{\frac{dM}{dx} = Q(x)}$$

$p(x) = 0$

$q(x) = +q_0$

$$\frac{dV}{dx} = 0 \Rightarrow V = konst. = 0$$

$$\frac{dQ}{dx} = -q_0 \Rightarrow Q(x) = -q_0 x + C_1$$

$$Q(x=0) = A_z = C_1$$

$$\frac{dM}{dx} = Q(x) = -q_0 x + C_1 \Rightarrow M = -\frac{1}{2} q_0 x^2 + C_1 x + C_2$$

$$M(x=0) = 0 \quad \text{in} \quad M(x=l) = 0$$

$$C_2 = 0$$

$$-\frac{1}{2} q_0 l^2 + C_1 l = 0 \Rightarrow C_1 = \frac{1}{2} q_0 l$$

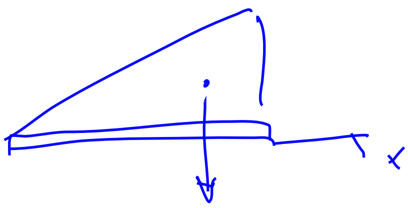
$$M(x) = -\frac{1}{2} q_0 x^2 + \frac{1}{2} q_0 l x = \frac{1}{2} q_0 x (l-x)$$

linearna linijska $q(x) = x q_0$

$$\frac{dQ}{dx} = -q(x) = -x q_0 \Rightarrow Q = -\frac{1}{2} q_0 x^2 + C_1$$

$$\frac{dM}{dx} = Q(x) = -\frac{1}{2} q_0 x^2 + C_1 \Rightarrow M = -\frac{1}{6} q_0 x^3 + C_1 x + C_2$$

$$M(x=0) = M(x=l) = 0$$



Primer: konstantna linijska obremenitev konzolnega nosilca.