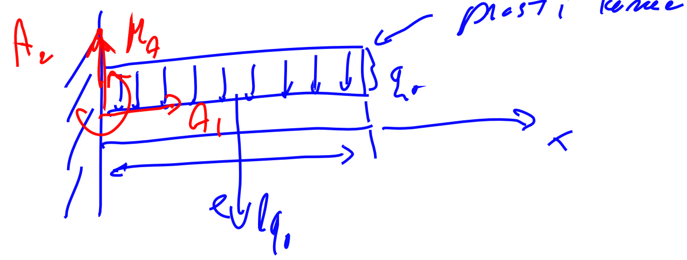


Predavanje 7. april 2021

$M(x=l)=0$; $Q(x=l)=0$ To velja na prostem koncu

Primer: konstantna linijska obremenitev konzolnega nosilca.

$0=-q_0 l+C_1$

$C_1=lq_0$

$\boxed{\frac{dM}{dx}=Q(x)}$ $\quad \frac{dQ}{dx}=-q(x)=-q_0 \Rightarrow Q=-q_0x+C_1$

$\frac{dM}{dx}=-q_0x+C_1 \Rightarrow M=-\frac{1}{2}q_0x^2+C_1x+C_2$

$\frac{d^2M}{dx^2}=\frac{dQ}{dx}=-q(x)$; $\boxed{\frac{d^2M}{dx^2}=-q_0}$

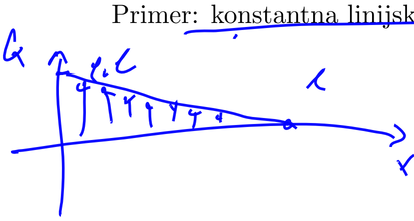
$0=M(x=l)=-\frac{1}{2}q_0l^2+q_0l^2+C_2$

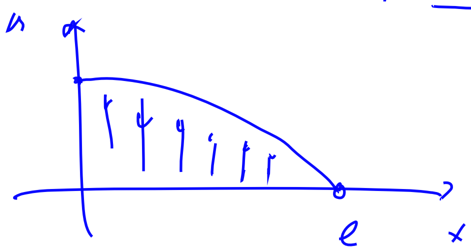
$C_2=-\frac{1}{2}q_0l^2$

Primer: konstantna linijska obremenitev prevesnega nosilca.

$M=-\frac{1}{2}q_0x^2+lq_0x-\frac{1}{2}q_0l^2=$

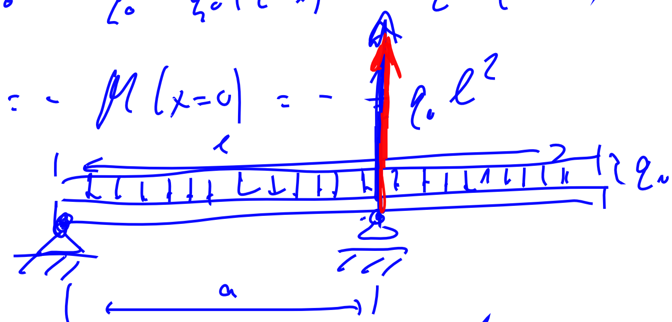
$=-\frac{1}{2}q_0(x^2-2lx+l^2)=-\frac{1}{2}q_0(x-l)^2$





$Q=-q_0x+lq_0=q_0(l-x)$ $\quad A_z=Q(x=0)=lq_0$ ✓

$M_A=-M(x=0)=-\frac{1}{2}q_0l^2$



$M_1: \ 0\le x<a$

$M_2: \ a<x\le l$

$\frac{d^2M_1}{dx^2}=-q_0$ $\quad \frac{d^2M_2}{dx^2}=-q_0$

$M_1(x)=-\frac{1}{2}q_0x^2+C_1x+C_2$ $\quad M_2(x)=-\frac{1}{2}q_0x^2+C_3x+C_4$

$\to M_1(x=0)=0 \Rightarrow C_2=0$ $\quad M_2(x=l)=0$

$Q=\frac{dM}{dx}$; $\frac{dM}{dx}(x=l)=0$

$M_1(x=a)=M_2(x=a)$

$$-\frac{1}{2}q_0 a^2 + C_1 a = -\frac{1}{2}q_0 a^2 + C_3 a + C_4 \Rightarrow C_1 a = C_3 a + C_4$$

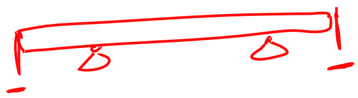
$$-\frac{1}{2}q_0 l^2 + C_3 l + C_4 = 0 \Rightarrow C_4 = \frac{1}{2}q_0 l^2 - q_0 l^2 = -\frac{1}{2}q_0 l^2$$

$$-q_0 l + C_3 = 0 \Rightarrow C_3 = q_0 l$$

$$C_1 = q_0 l - \frac{1}{2}q_0 \frac{l^2}{a}$$

Ugotovitve: pri enostavno podprtem prevesnem nosilcu velja:

1. prečna sila na prostih krajiščih je enaka nič;
2. v podporah ima prečna sila skok enak sili podpore;
3. upogibni moment je enak nič v prostih krajiščih.



$0 \le x < a$: $Q(x) = -q_0 x + C_1 =$

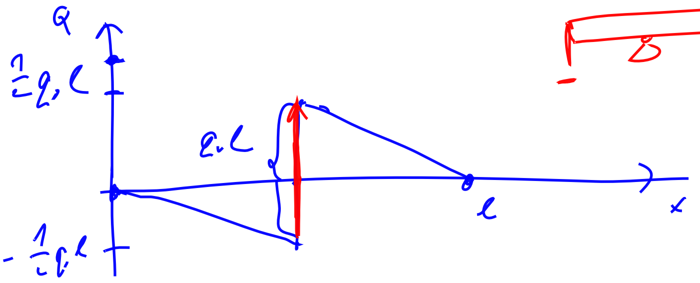
$= -q_0 x + q_0 l - \frac{1}{2} q_0 \frac{l^2}{a}$

$$Q(x=0) = q_0 l\left(1 - \frac{1}{2}\frac{l}{a}\right)$$

$$Q(x=a) = q_0\left(-a + l - \frac{1}{2}\frac{l^2}{a}\right)$$

$a = \frac{l}{2}$

$$Q\left(x = \frac{l}{2}\right) = q_0\left(-\frac{l}{2} + l - \frac{1}{2}\frac{l^2}{l}\cdot 2\right) = -q_0 \frac{l}{2}$$

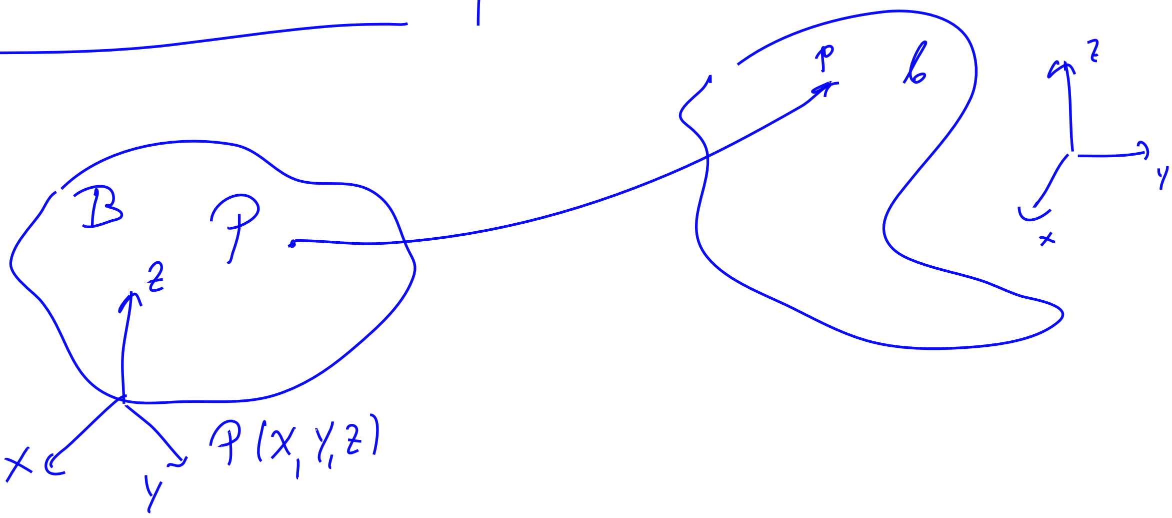


$\frac{l}{2} < x < l$ $\quad Q(x) = -q_0 x + C_3 = q_0(l - x)$

Robni pogoji diferencialne metode.

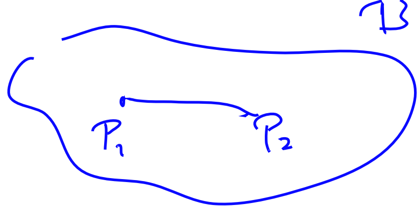
TRDNOST

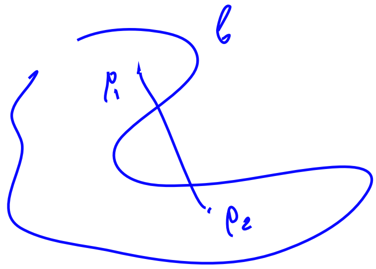
Pisava.



Referenčni(nedeformiran) položaj: B,P, P(X,Y,Z).

Prostorski(deformiran) položaj: b,p, p(x,y,z).





Mere deformacije:

- relativna sprememba dolžin

$$\epsilon_1 = \frac{|p_1p_2| - |P_1P_2|}{|P_1P_2|}; = \frac{|p_1p_2|}{|P_1P_2|} - 1$$

- Cauchyjeva mera deformacije

$$\epsilon_2 = \frac{|p_1p_2|^2 - |P_1P_2|^2}{|P_1P_2|^2} = \left(\frac{|p_1p_2|}{|P_1P_2|}\right)^2 - 1$$

;

$$\boxed{\log(1+x) \doteq x}$$

$|x| \ll 1$

- logaritemska mera

$$\epsilon = \log \frac{|p_1p_2|}{|P_1P_2|}$$

.

$\varepsilon_1 = 0$

$\varepsilon_2 = 0$

Toga deformacija $|p_1p_2| = |P_1P_2|$ $\varepsilon = \log 1 = 0$

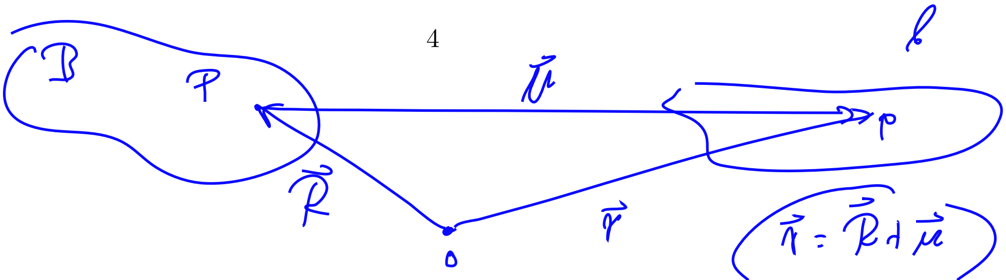
Pri togem pomiku je mera deformacije enaka nič.

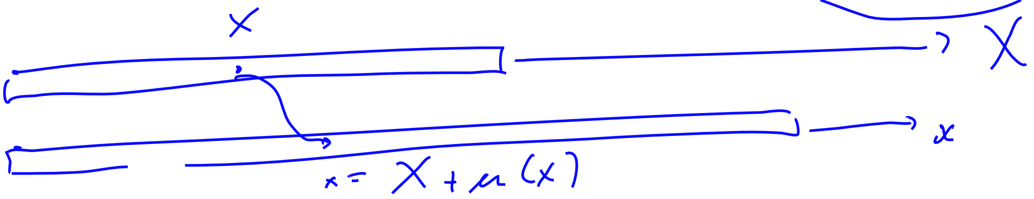
$$\left.\begin{array}{l} \varepsilon_2 = \left(\dfrac{|p_1p_2|}{|P_1P_2|}\right)^2 - 1 \\ \varepsilon_1 = \dfrac{|p_1p_2|}{|P_1P_2|} - 1 \Rightarrow \end{array}\right\} \Rightarrow \varepsilon_2 = (1+\varepsilon_1)^2 - 1 = 2\varepsilon_1 + \varepsilon_1^2$$

$$\boxed{\varepsilon_2 = 2\varepsilon_1 + \varepsilon_1^2}$$

$|\varepsilon_1| \ll 1 \Rightarrow \varepsilon_2 \approx 2\varepsilon_1$

Za majhne deformacije je $\epsilon_2 = 2\epsilon_1$, $\epsilon_1 = \epsilon$. $\varepsilon \doteq \varepsilon_1$





Osna deformacija

Pisava: referenčni položaj X, deformirani položaj x; funkcija pomika $x = X + u(X)$.

Enakomerna deformacija $|p_1 p_2| = k|P_1 P_2|$, $k > 0$.
Mere deformacije enakomerne deformacije.

$$\varepsilon_1 = \frac{|p_1 p_2|}{|P_1 P_2|} - 1 = k - 1 \qquad \varepsilon_2 = \left(\frac{|p_1 p_2|}{|P_1 P_2|}\right)^2 - 1 = k^2 - 1$$

$$\varepsilon = \log \frac{|p_1 p_2|}{|P_1 P_2|} = \log k \qquad \varepsilon_1, \varepsilon_2, \varepsilon \in \mathbb{R}$$

$\varepsilon_1 > 0 \iff k > 1$; raztег $\qquad \varepsilon_1 < 0 \iff k < 1$ skrčitev

Pomik enekomerne deformacije

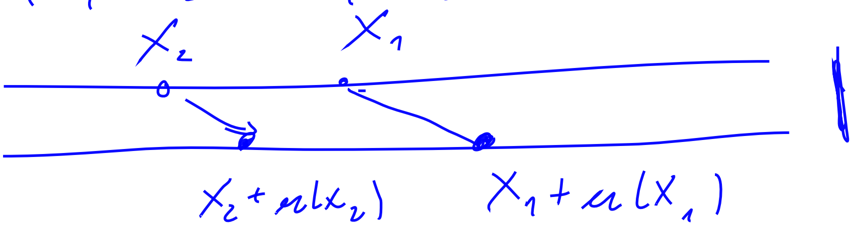
$$P_1, P_2 ; \quad X_1, X_2$$

$$P_1 \to p_1 \quad x_1 = X_1 + u(X_1) \qquad P_2 \to p_2 ; \quad x_2 = X_2 + u(X_2)$$

$$|P_1 P_2| = |X_1 - X_2|$$

$$|p_1 p_2| = |x_1 - x_2| = |(X_1 - X_2 + u(X_1) - u(X_2))| = k\,|X_1 - X_2|$$

$$X_1 > X_2 \qquad |X_1 - X_2| = X_1 - X_2$$



$$(X_1 - X_2) + (u(X_1) - u(X_2)) = k(X_1 - X_2)$$

$$u(X_1) - u(X_2) = (k-1)(X_1 - X_2)$$

$$\frac{u(X_1) - u(X_2)}{} = k - 1 \qquad X_1 = X_2 + h$$

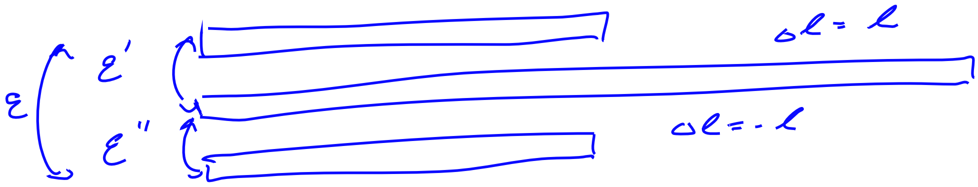
$x_1 \quad -x_2$ $\qquad\qquad\qquad\qquad$ $\lim h \to 0$

$$\boxed{\frac{du}{dx} = k-1 = \varepsilon_1} \Rightarrow u(X) = \varepsilon_1 X + C_1$$

$$u(X=0) = 0 \Rightarrow C_1 = 0; \quad u(X) = \varepsilon_1 X$$

Aditivnost mer deformacije.

$$l + \Delta l = k\, l \Rightarrow k = 1 + \frac{\Delta l}{l} \qquad \boxed{\varepsilon_1 = k-1 = \frac{\Delta l}{l}}$$



$$\varepsilon = \varepsilon' + \varepsilon''$$

Mera je aditivna

ε_1 $\qquad \varepsilon_1' = \frac{l}{l} = 1; \quad \varepsilon_1'' = \frac{-l}{2l} = -\frac{1}{2}; \quad \varepsilon_1' + \varepsilon_1'' = 1 - \frac{1}{2} = \frac{1}{2}$

$\varepsilon_1 = 0$ $\qquad$ ε_1 ni aditivna mera

ε_2 tudi ni aditivna mera

$$\varepsilon = \log\frac{|p_1 p_2|}{|P_1 P_2|}; \quad \varepsilon' = \log\frac{2l}{l} = \log 2; \quad \varepsilon'' = \log\frac{l}{2l} = \log\frac{1}{2}$$

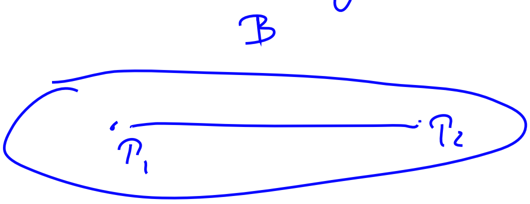
Lokalizacija mere

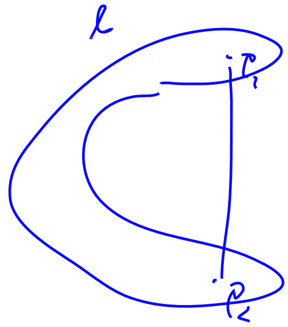
$$\varepsilon' + \varepsilon'' = \log 2 + \log\frac{1}{2} = \log 2 + \underbrace{\log 1}_{0} - \log 2 = 0$$

$$\log\frac{a}{b} = \log a - \log b$$

$$\varepsilon = \log\frac{l}{l} = \log 1 = 0$$

Logaritemska mera je aditivna.





$$\varepsilon_1 = \frac{|p_1 p_2|}{|P_1 P_2|} - 1 \qquad X_2 > X_1; \; |P_1 P_2| = X_2 - X_1$$

$$|p_1 p_2| = |X_2 - X_1 + u(X_2) - u(X_1)|$$

$$\varepsilon_1 = \frac{|X_2 - X_1 + u(X_2) - u(X_1)|}{|X_2 - X_1|} - 1 = \left| \underbrace{\frac{X_2 - X_1}{|X_2 - X_1|}}_{1} + \underbrace{\frac{u(X_2) - u(X_1)}{|X_2 - X_1|}}_{\text{majhen pri malih deformacijah}} \right| - 1$$

$$= 1 + \left| \frac{u(x_2) - u(x_1)}{x_2 - x_1} \right| - 1$$

$$\varepsilon_2 = 2\varepsilon_1; \quad \varepsilon_1 = \frac{1}{2}\varepsilon_2 = \frac{1}{2}\left(\left(\frac{|p_1 p_2|}{|P_1 P_2|}\right)^2 - 1\right) = \qquad (u(x_2) - u(x_1))^2$$

$$\frac{1}{2}\left(\frac{\left((X_2-X_1)+(u(X_2)-u(X_1))\right)^2}{(X_2-X_1)^2}-1\right)=\frac{1}{2}\left(\frac{(X_2-X_1)^2+2(X_2-X_1)(u(X_2)-u(X_1))+\downarrow}{(X_2-X_1)^2}-1\right)$$

$$=\frac{1}{2}\left(1+2\,\frac{u(X_2)-u(X_1)}{X_2-X_1}+\left(\frac{u(X_2)-u(X_1)}{X_2-X_1}\right)^2-1\right)=$$

$$\boxed{\frac{u(X_2)-u(X_1)}{X_2-X_1}}+\frac{1}{2}\left(\frac{u(X_2)-u(X_1)}{X_2-X_1}\right)^2$$

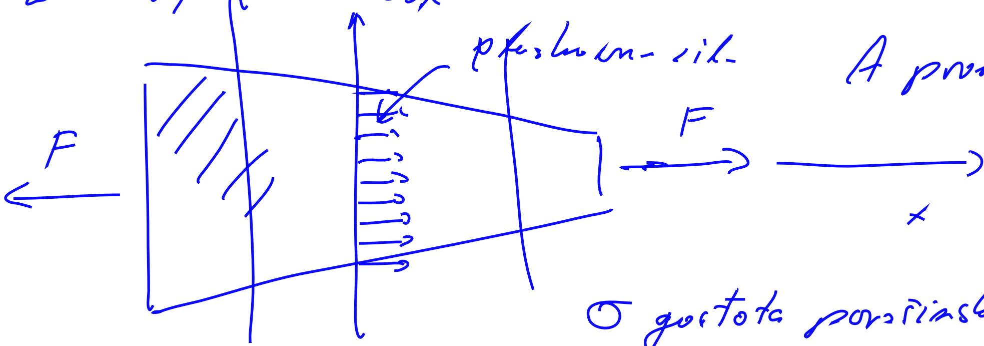
Osna napetost

Navidezni prerez.

$X_2=X_1+dX$; $\lim_{dX\to 0}$

$$\varepsilon_1=\frac{du}{dX}+\frac{1}{2}\left(\frac{du}{dX}\right)^2 \Rightarrow \boxed{\varepsilon_1 \doteq \frac{du}{dx}}$$

$\varepsilon_2=2\varepsilon_1$, $\varepsilon_1=\varepsilon=\frac{du}{dx}$



A ploščina prereza $\quad \int dx = x$

$A=A(x)$

σ gostota površinske sile $\quad \int_A \sigma\, dA \quad \int_A dA = A$

Pri konstantnem preseku je σ konstanten $\quad \sigma\cdot A$ sila drugega dela telesa na prvi del telesa

$F=\sigma A$; $\sigma=\frac{F}{A}$

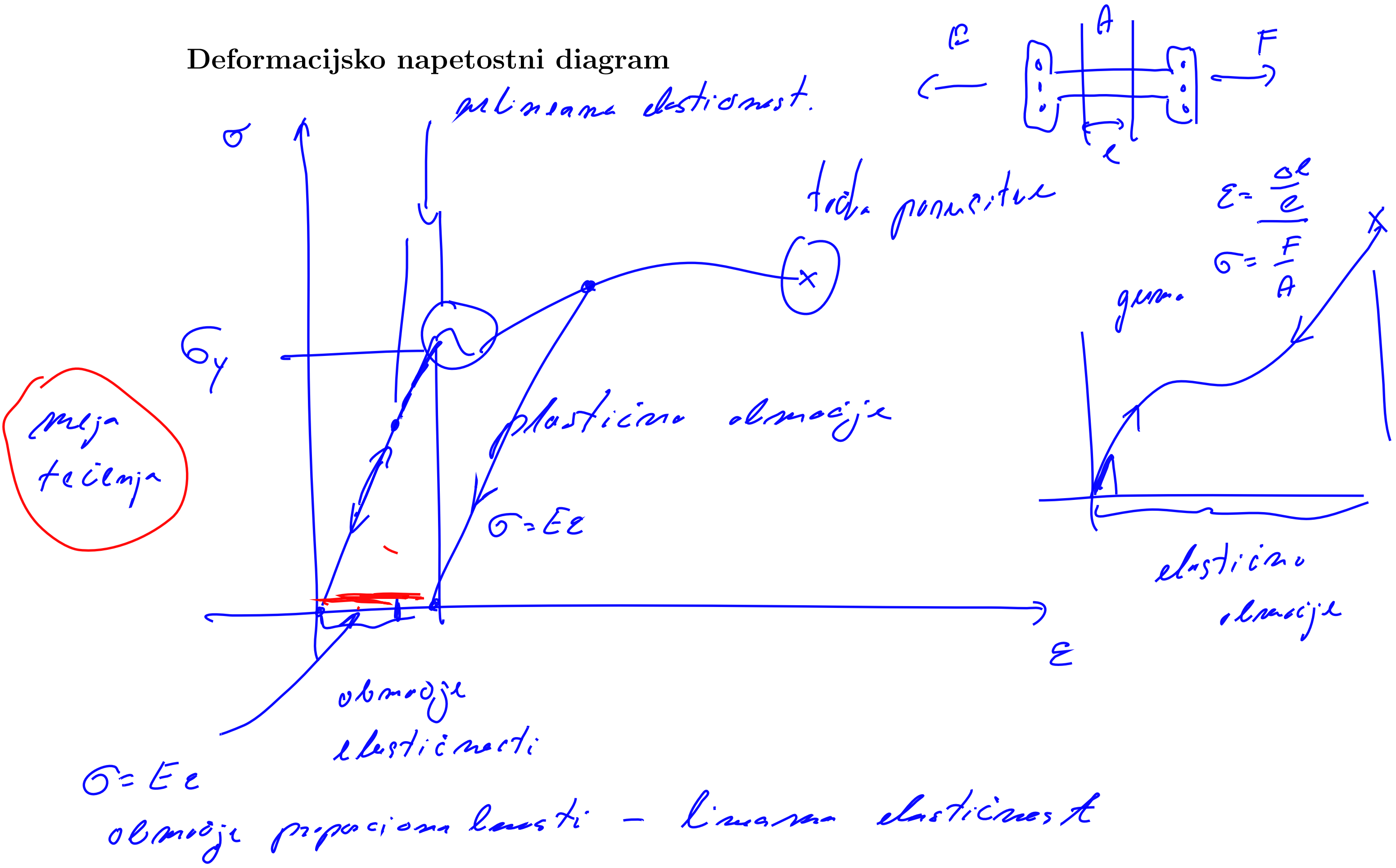
Osna napetost $\sigma = F/A$.

$$\sigma(x)=\frac{F}{A(x)}$$

$$[\sigma]=\left[\frac{F}{A}\right]=\frac{N}{m^2}=\text{Pascal (Pa)}$$

$MPa = 10^6\,Pa$; $GPa = 10^9\,Pa$

Deformacijsko napetostni diagram



Značilne točke in območja na deformacijsko napetostnem diagramu.

Tabela Youngovih modulov E, mej tečenj σ_{Y} in nateznih trdnosti σ_{S}.

Primer: določi dopustno deformacijo, da napetost ne preseže meje tečenja.

Reševanje statično nedoločenih nalog

Primer: nosilec obešen na štiri žice.

Termoelastičnost

Primer: določi napetost palice med dvema togima stenama pri spremembi temperature za ΔT.