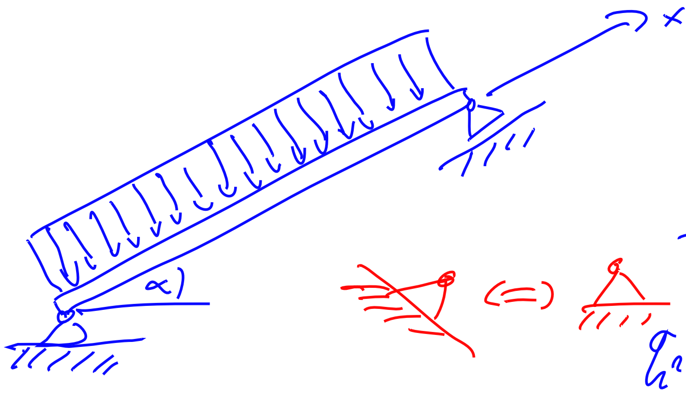
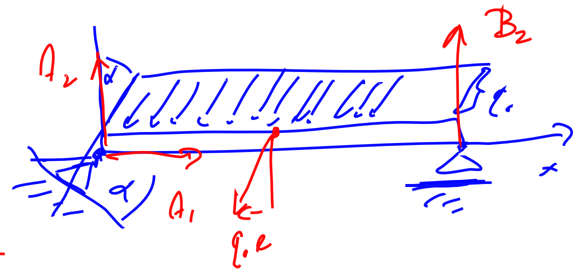
$$A_2 = B_2 = \frac{1}{2} q_0 l \cos\alpha$$

Vaje 8. april 2021

1. Enostavno podprti nagnjen nosilec je enakomerno linijsko obremenjen. Določi potek notranjih količin.

$$A_1 - q_0 l \sin\alpha = 0$$





$$q_1 = q_0 \cos\alpha$$

$$p_1 = -q_0 \sin\alpha$$

$$\vec{F} = -q_0 l \sin\alpha\, \vec{i} + q_0 l \cos\alpha\, \vec{k}$$

$$\underbrace{\frac{dM}{dx} = Q \;;\quad \frac{dQ}{dx} = -q_1}$$

$$\frac{d^2M}{dx^2} = -q_1 \Rightarrow M = -\frac{1}{2} q_1 x^2 + C_1 x + C_2 \qquad M(x=0) = 0,\quad M(x=l) = 0$$

$$0 = M(x=0) = C_2 \;;\quad 0 = -\frac{1}{2} q_1 l^2 + C_1 l \Rightarrow C_1 = \frac{1}{2} q_1 l$$

$$M = -\frac{1}{2} q_1 x^2 + \frac{1}{2} q_1 l x = \frac{1}{2} q_1 x (l - x) = \frac{1}{2} q_0 \cos\alpha\, x (l - x)$$

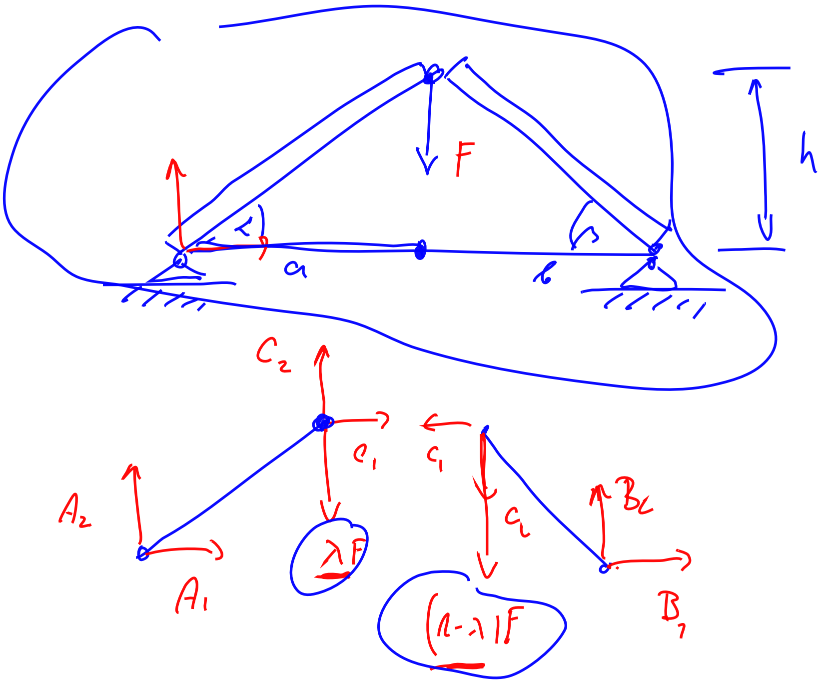
$$Q = \frac{dM}{dx} = -q_1 x + \frac{1}{2} q_1 l = q_1 \left(\frac{1}{2} l - x\right) = q_0 \cos\alpha \left(\frac{1}{2} l - x\right)$$

$$\frac{dV}{dx} = -p = -q_0 \sin\alpha \Rightarrow V = -q_0 \sin\alpha\, x + C_3$$

$$V(x=l) = 0 \Rightarrow C_3 = q_0 \sin\alpha\, l$$

$$V = q_0 \sin\alpha\, (l - x)$$

2. Tročleni lok sestavljen iz dveh ravnih nosilcev je na vrhu točkovno obremenjen. Določi potek notranjih količin.



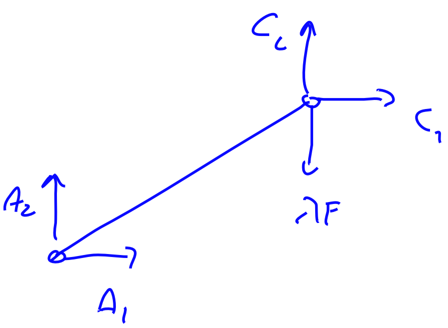
$$A_1 + C_1 = 0$$
$$A_2 + C_2 - \lambda F = 0$$
$$h A_1 - a A_2 = 0$$

$$-C_1 + B_1 = 0$$
$$B_2 - C_2 - (1-\lambda) F = 0$$
$$h B_1 + b B_2 = 0$$

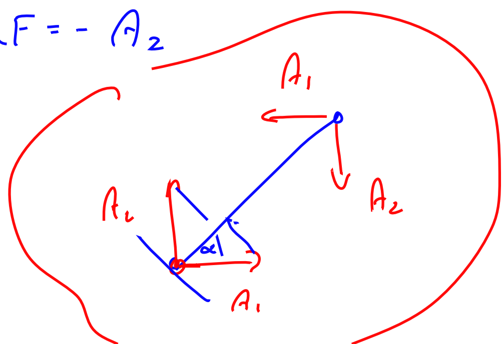
$$A_2 + B_2 - F = 0$$
$$A_1 + B_1 = 0$$
$$-a A_2 + b B_2 = 0$$

$$B_2 = \frac{a}{b} A_2$$
$$A_2 + \frac{a}{b} A_2 = F$$
$$A_2 \left(\frac{b+a}{b} \right) = F$$
$$\boxed{A_2 = \frac{b}{a+b} F \qquad B_2 = \frac{a}{a+b} F}$$

$$A_1 = \frac{a}{h} A_2 = \frac{ab}{h(a+b)} F$$
$$C_1 = -A_1 = -\frac{ab}{h(a+b)} F$$
$$B_1 = -\frac{ab}{h(a+b)} F$$
$$C_2 = \lambda F - A_2 = \left(\lambda - \frac{b}{a+b} \right) F$$

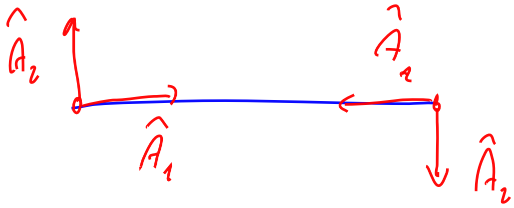


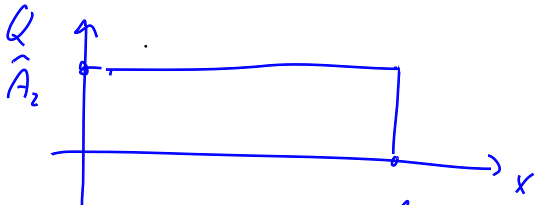
$$C_2 - \lambda F = -A_2$$



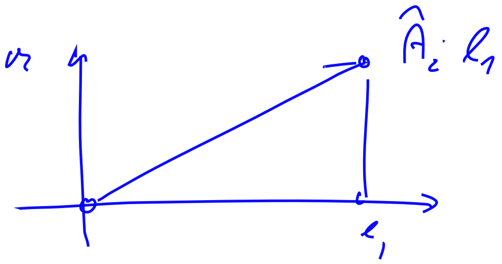
$$\hat{A}_1 = A_1 \cos\alpha + A_2 \cos(\pi/2 - \alpha) = A_1 \cos\alpha + A_2 \sin\alpha$$

$$\hat{A}_2 = - A_1 \sin\alpha + A_2 \cos\alpha$$



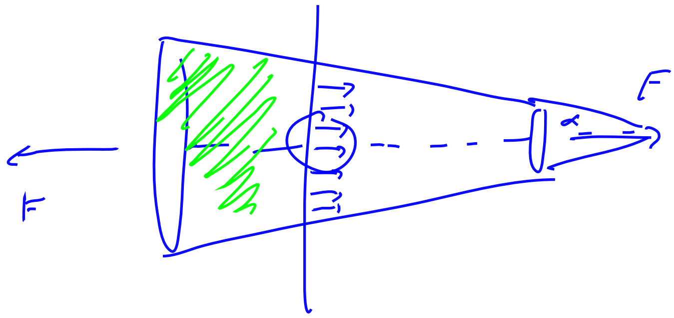


$$l_1 = \sqrt{a^2 + b^2}$$



$$\frac{dM}{dx} = Q$$

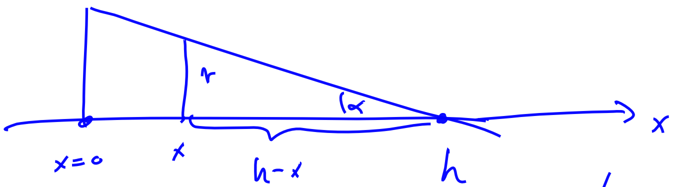
3. Osna obremenitev odsekanega stožca. Določi napetost in deformacijo.



$$\sigma(x)\cdot A(x) = F$$

$$\sigma(x) = \frac{F}{A(x)} = \frac{F}{\pi r^2(x)}$$

$$\sigma(x) = \frac{F}{\pi \, tg^2\alpha \, (h-x)^2}$$



$$tg\alpha = \frac{r}{h-x} \Rightarrow r = (h-x)\, tg\alpha$$

$$x=0: \quad \sigma(x=0) = \frac{F}{\pi h^2 tg^2\alpha} = \sigma_0$$

$$\sigma(x) = \sigma_0 \frac{h^2}{(h-x)^2} = \sigma_0 \frac{1}{(1-\frac{x}{h})^2}$$

$$\frac{F}{\pi \, tg^2\alpha} = h^2 \sigma_0$$

$$\boxed{x < h}$$

Hookov zakon

$$\boxed{\sigma = E\varepsilon} \quad ; \quad E \text{ Youngov modul}$$

$$\varepsilon = \frac{du}{dx}$$

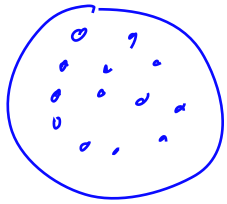
$$\sigma_0 \frac{1}{(1-\frac{x}{h})^2} = E \cdot \frac{du}{dx} \Rightarrow \boxed{\frac{du}{dx} = \frac{\sigma_0}{E} \frac{1}{(1-\frac{x}{h})^2}}$$

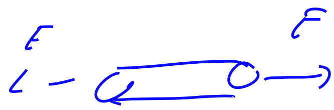
$$u = \int \frac{\sigma_0}{E} \frac{1}{(1-\frac{x}{h})^2} dx = \ldots$$

4. Razmerje površin železa in betona je na preseku železobetonskega stebra enako 1 : 9, razmerje njunih Youngovih modulov pa 6 : 1. Izračunaj kolikšen del obremenitve v stebru nosi železo in koliko beton. s

(1) beton

(2) železo





$$V = hA$$

$$\frac{V_1}{V_2} = \frac{A_1}{A_2} = 9 \qquad \frac{E_1}{E_2} = \frac{1}{6}$$

$$F = F_1 + F_2 = A_1\sigma_1 + A_2\sigma_2 \qquad F_i = \sigma_i A_i$$

$$\varepsilon_1 = \varepsilon_2 \qquad \boxed{\sigma_i = E_i \varepsilon_i} \Rightarrow \varepsilon_i = \frac{\sigma_i}{E_i}$$

$$\boxed{\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}} \qquad \sigma_1 = \frac{E_1}{E_2}\sigma_2 = \frac{1}{6}\sigma_2$$

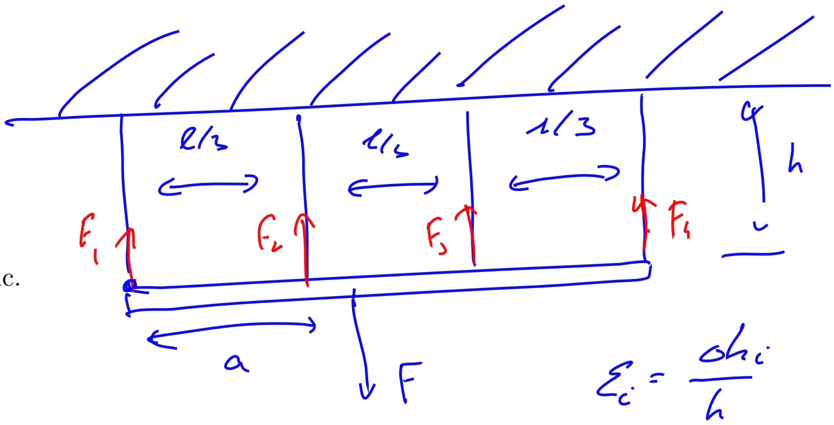
$$F = A_1 \frac{1}{6}\sigma_2 + A_2\sigma_2 = 9A_2\sigma_2\frac{1}{6} + A_2\sigma_2$$

$$= \left(\frac{3}{2} + 1\right)A_2\sigma_2$$

$$A_2\sigma_2 = \frac{2F}{5} \qquad F_2 = \frac{2}{5}F$$

Železo prevzame 40% obremenitve.

5. Nosilec na štirih žicah. Določi sile žic.



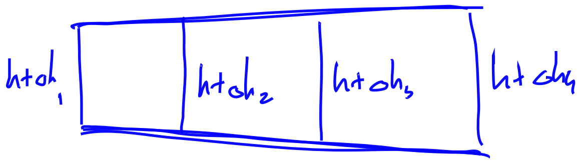
$$\varepsilon_i = \frac{\Delta h_i}{h}$$

$$F_1 + F_2 + F_3 + F_4 = F$$

$$\frac{l}{3} F_2 + \frac{2l}{3} F_3 + l F_4 = aF$$

$$F_i = A\sigma_i = AE\varepsilon_i = AE\frac{\Delta h_i}{h}$$

$$\begin{cases} \Delta h_1 + \Delta h_2 + \Delta h_3 + \Delta h_4 = \dfrac{h}{AE} F \\ \Delta h_2 + 2\Delta h_3 + 3\Delta h_4 = \dfrac{h\,3aF}{AE\,l} \end{cases}$$



$$\Delta h_2 - \Delta h_1 = \Delta h_3 - \Delta h_2 = \Delta h_4 - \Delta h_3$$

$$\frac{(h+\Delta h_2)-(h+\Delta h_1)}{\frac{l}{3}} = \frac{(h+\Delta h_3)-(h+\Delta h_2)}{\frac{l}{3}} = \frac{(h+\Delta h_4)-(h+\Delta h_3)}{\frac{l}{3}}$$

$$\begin{cases} -\Delta h_1 + 2\Delta h_2 - \Delta h_3 = 0 \\ -\Delta h_2 + 2\Delta h_3 - \Delta h_4 = 0 \end{cases}$$

Rešimo $\Delta h_1, \Delta h_2, \Delta h_3, \Delta h_4$

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