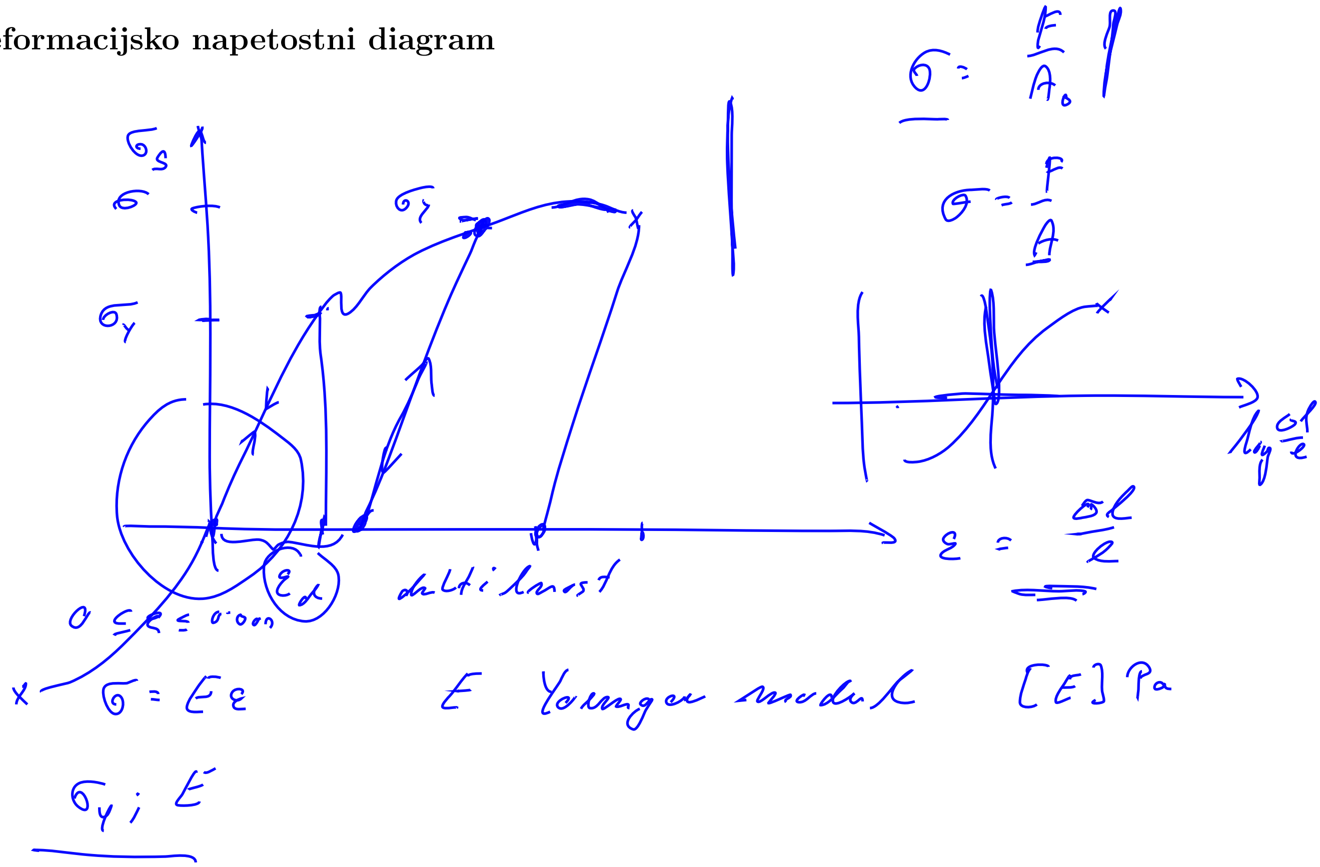
Predavanje 14. april 2021

Deformacijsko napetostni diagram



Značilne točke in območja na deformacijsko napetostnem diagramu.

Tabela Youngovih modulov E, mej tečenj σ_{Y} in nateznih trdnosti σ_{S}.

Primer: določi dopustno deformacijo, da napetost ne preseže meje tečenja.

Baker $\quad E = 120\ \mathrm{GPa}$

$$\boxed{\sigma = E\varepsilon}$$

Hookov zakon

$$\frac{F}{A} = E\left(\frac{\Delta l}{l}\right)$$

$$\sigma_Y = 60\ \mathrm{MPa} = 60 \cdot 10^6\ \mathrm{Pa}$$

$$\mathrm{GPa} = 10^9\ \mathrm{Pa}$$

$$\sigma_Y > \sigma = E\varepsilon$$

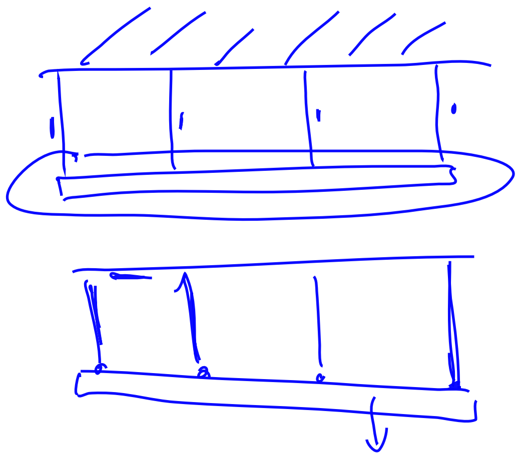
$$\Rightarrow \varepsilon < \frac{\sigma_Y}{E} = \frac{60\ \mathrm{MPa}}{120\ \mathrm{GPa}} = \frac{1}{2}\,10^{-3}$$

$$= \boxed{0.0005}$$

$F = A\sigma$ $\quad$ $\sigma = E\varepsilon$

Reševanje statično nedoločenih nalog

Primer: nosilec obešen na štiri žice.



Termoelastičnost

Primer: določi napetost palice med dvema togima stenama pri spremembi temperature za ΔT.

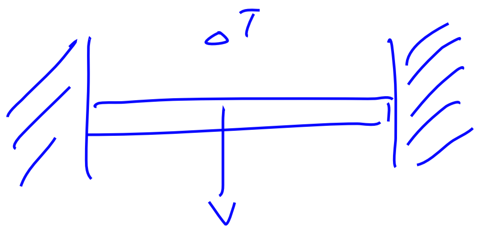
ε_T temperaturna deformacija

$$\varepsilon_T = \alpha \Delta T$$

Al $\alpha = 23 \cdot 10^{-6} \frac{1}{K}$

Fe $\alpha = 11.8 \cdot 10^{-6} K^{-1}$

beton $\alpha = 12 \cdot 10^{-6} K^{-1}$



$$\varepsilon = \varepsilon_T + \varepsilon_E = 0 \qquad \sigma = E\varepsilon_E$$

$$0 = \alpha \Delta T + \frac{\sigma}{E} \Rightarrow \sigma = -\alpha E \Delta T$$

$E = 69$ GPa ; $\alpha = 23.1 \; 10^{-6} K^{-1}$

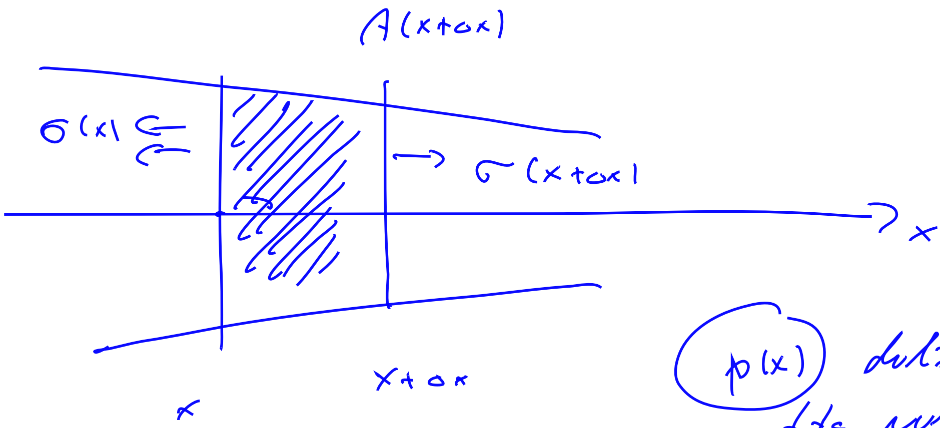
$$\sigma = -23.1 \cdot 69 \cdot 10^{9} \text{Pa} \cdot 10^{-6} K^{-1} \cdot \Delta T \doteq -1600 \cdot 10^{3} \frac{\Delta T}{K} \text{Pa}$$

$$\sigma \doteq -1.6 \cdot 10^{6} \text{Pa} \frac{\Delta T}{K} = -1.6 \text{ MPa} \frac{\Delta T}{K}$$

$\sigma_y = 40$ MPa $\qquad 1.6 \cdot \frac{\Delta T}{K} < 40$

$$\Delta T < \frac{40}{1.6} K = 25 K$$

Ravnovesna enačba osne napetosti in deformacije



$p(x)$ dolžinska gostota zunanje sile

$$\sigma(x+\Delta x)A(x+\Delta x) - \sigma(x)A(x) + \int_x^{x+\Delta x} p(s)\,ds = 0 \quad \Big/ \frac{1}{\Delta x} \quad \lim_{\Delta x \to 0}$$

Ravnovesna enačba za osno napetost

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sigma(x)A(x)\right) + p(x) = 0$$

$$\underbrace{\frac{\sigma(x+\Delta x)A(x+\Delta x) - \sigma(x)A(x)}{\Delta x}} + \underbrace{\frac{1}{\Delta x}\int_x^{x+\Delta x} p(s)\,ds} = 0$$

$$\frac{d}{dx}(\sigma A) + p(x) = 0 \qquad \sigma = E\varepsilon = E\frac{du}{dx}$$

Ravnovesna enačba za pomik

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(AE\frac{\mathrm{d}u}{\mathrm{d}x}\right) + p(x) = 0.$$

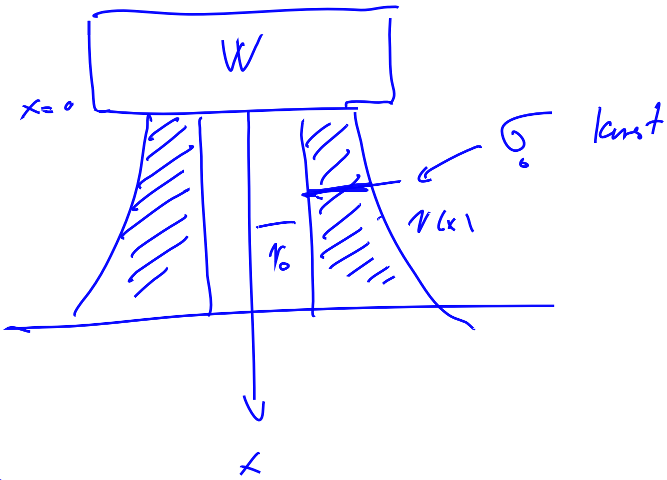
Sila teže

$$F_g = \rho g V = \rho g A l \qquad p = \rho g A$$

$$V = Al$$

Primer: sila teže: dolžinska gostota je $p(x) = \rho(x)A(x)g$.

Primer: problem vodnega stolpa



$$-\sigma_0 \cdot A_0 = W$$

$$\sigma_0 = (-)\frac{W}{A_0}$$

kompresijska napetost

$$\frac{d}{dx}(\sigma A) + p = 0 \qquad p = \rho g A(x)$$

$$\frac{d}{dx}(\sigma_0 A(x)) + \rho g A(x) = 0 \Rightarrow \frac{dA}{dx} = -\frac{\rho g}{\sigma_0} A$$

$$\frac{dA}{dx} = \overbrace{\frac{\rho g A_0}{W}}^{=k} A \; ; \qquad \boxed{\frac{dA}{dx} = kA}$$

$$\frac{de^x}{dx} = e^x$$

$$\boxed{A = Ce^{kx}}$$

$$A(x=0) = Ce^0 = C = A_0$$

$$A(x) = A_0 e^{kx} \qquad A(x) = \pi(r^2 - r_0^2)$$

$$r^2 = r_0^2 + \frac{1}{\pi} A_0 e^{kx} = r_0^2 + \frac{1}{\pi}\frac{W}{|\sigma_0|} e^{\frac{\rho g W}{W|\sigma_0|}x} = r_0^2 + \frac{1}{\pi}\frac{W}{|\sigma_0|} e^{\frac{\rho g}{|\sigma_0|}x}$$

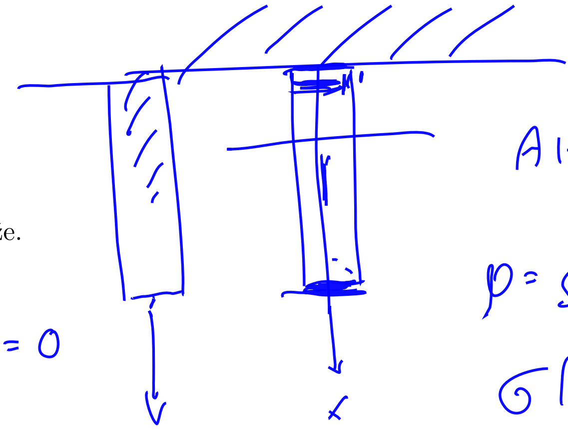
Debelina raste eksponentno.

$$e^{\frac{\rho g}{|\sigma_0|}x} \qquad \frac{\rho}{|\sigma_0|}$$

$$|\sigma_0| = \mu\,\sigma_s$$

$$\frac{\rho}{\sigma_s}$$

Primer: deformacija palice zaradi lastne teže.



$x = 0$

$A(x) = A_0$

$p = \rho g A$

$\sigma(x=l) = 0$

$$\frac{d}{dx}(A\sigma) + p(x) = 0$$

$$\sigma = E\frac{du}{dx}$$

$F = \rho g A_0 l$

$$A_0 E \frac{d^2u}{dx^2} = -\rho g A_0 \Rightarrow \frac{d^2u}{dx^2} = -\frac{\rho g}{E}$$

$$u = \frac{1}{2}\left(-\frac{\rho g}{E}\right)x^2 + C_1 x + C_2$$

$$u(x=0) = 0 \rightarrow C_2 = 0$$

$$\frac{du}{dx}(x=l) = 0$$

$$-\frac{\rho g}{E}x + C_1 \Rightarrow C_1 = \frac{\rho g}{E}l$$

$$\boxed{u = \frac{\rho g}{E}\left(l - \frac{1}{2}x\right)x}$$

$$u(x=l) = \frac{1}{2}\frac{\rho g}{E}l^2$$

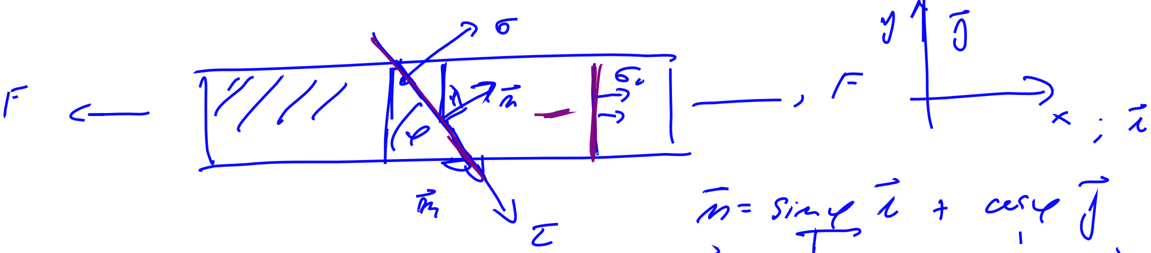
$$\Delta l = (l + u(x=l)) - l$$

$$\frac{\Delta l}{l} = \frac{u(x=l)}{l} = \frac{1}{2}\frac{\rho g}{E}l$$

$$\sigma = \frac{F}{A_0} = \rho g l = E\varepsilon \Rightarrow \boxed{\varepsilon = \frac{\Delta l}{l} = \frac{\rho g l}{E}}$$

$$\frac{du}{dx} = -\frac{\rho g}{E}x + \frac{\rho g}{E}l = \frac{\rho g}{E}(l - x)$$

deformacija ni enakomerna.



$\vec n = \sin\varphi\,\vec\imath + \cos\varphi\,\vec\jmath$

$\vec m = \cos\varphi\,\vec\imath - \sin\varphi\,\vec\jmath$

Napetostni tenzor

Poševni presek palice, strižna napetost, odvisnost od kota preseka.

$A(\sigma\vec n + \tau\vec m) = F = A_0\sigma_0\vec\imath$ $\qquad A_0 = A\cdot\cos(\frac{\pi}{2}-\varphi)$

$A_0 = A\sin\varphi$

$\sigma\sin\varphi + \tau\cos\varphi = \sin\varphi\,\sigma_0$

$\sigma\cos\varphi - \tau\sin\varphi = 0 \Rightarrow \tau = \sigma\frac{\cos\varphi}{\sin\varphi}$

$\sigma + \tau\frac{\cos\varphi}{\sin\varphi} = \sigma_0 \Rightarrow \sigma\left(1 + \frac{\cos^2\varphi}{\sin^2\varphi}\right) = \sigma_0 \Rightarrow \sigma\frac{1}{\sin^2\varphi} = \sigma_0$

Vektor napetosti $\vec t = \vec t(p, \vec n)$ je odvisen od smeri prereza. Ta odvisnost je linearna. To pomeni, da obstaja tenzor napetosti $\underline{t}$ tako, da je $\vec t = \underline{t}\,\vec n$.

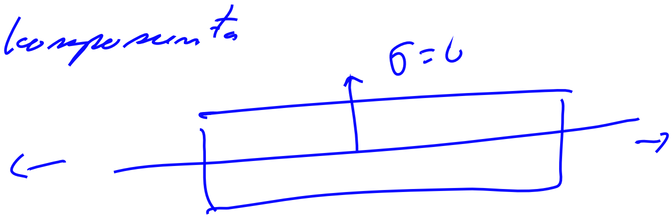
$\sigma = \sin^2\varphi\,\sigma_0 = \frac{1}{2}(1-\cos 2\varphi)\sigma_0$

$\tau = \sin\varphi\cos\varphi\,\sigma_0 = \frac{1}{2}\sin 2\varphi\,\sigma_0$

$\sin^2\varphi + \cos^2\varphi = 1$

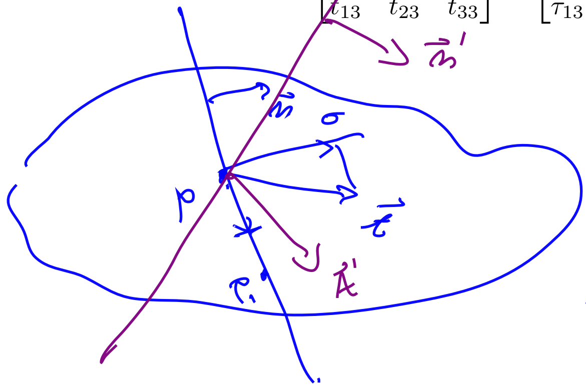
$\cos^2\varphi - \sin^2\varphi = \cos 2\varphi$

normalna komponenta $\qquad \tau$ strižna komponenta



Matrični zapis tenzorja napetosti:

$$\underline{t} = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{12} & t_{22} & t_{23} \\ t_{13} & t_{23} & t_{33} \end{bmatrix} = \begin{bmatrix} \sigma_1 & \tau_{12} & \tau_{13} \\ \tau_{12} & \sigma_2 & \tau_{23} \\ \tau_{13} & \tau_{23} & \sigma_3 \end{bmatrix}.$$



$\vec t$ gostota površinske sile

σ normalna napetost

$t_n = \sigma = \vec t\cdot\vec n \qquad \vec t_n = t_n\vec n$

τ strižna napetost

Tenzor napetosti je simetričen in ima 6 neodvisnih komponent.

$|\vec t - \vec t_n| = \tau$

$\vec t = \vec t_n + \vec t_s$

$\vec t_s = \vec t - \vec t_n$ vektor strižne napetosti

$\vec t = \vec t(p, \vec n)$

$\vec t$ je linearno odvisen od $\vec n$

$\vec t = \underline{t}(p)\,\vec n \qquad \underline{t}$ napetostni tenzor

$$\vec{t} = t_1 \vec{i} + t_2 \vec{j} + t_3 \vec{k} \qquad \vec{n} = n_1 \vec{i} + n_2 \vec{j} + n_3 \vec{k}$$

$$\begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{bmatrix} t_{11} n_1 + t_{12} n_2 + t_{13} n_3 \\ - \; - \; \cdot \\ - \; - \end{bmatrix}$$

Normalna napetost, vektor normalne napetosti; strižna napetost, vektor strižne napetosti.

$$\underline{t_{ij} = t_{ji}} \qquad \underline{\underline{t}} = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{13} & t_{23} & t_{33} \end{bmatrix}$$

$$\vec{t} = \underline{\underline{t}}\,\vec{n} \qquad t_n = \vec{t} \cdot \vec{n} = \underbrace{\vec{n} \cdot \underline{\underline{t}}\,\vec{n}} = \underbrace{\begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}}_{\vec{n}} \cdot \underbrace{\begin{bmatrix} t_{11} & t_{12} & t_{13} \\ \cdot & t_{22} & t_{23} \\ \cdot & \cdot & t_{33} \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}}_{\vec{t}}$$

$$t_s = \left| \vec{t} - \vec{t}_n \right| \quad ; \quad \vec{t}_n = t_n \cdot \vec{n} = (\vec{n} \cdot \underline{\underline{t}}\,\vec{n})\,\vec{n}$$

$$|\vec{n}| = 1 \qquad n_1^2 + n_2^2 + n_3^2 = 1$$

Pomen komponent napetostnega tenzorja v danem KS:

- diagonalni elementi so enaki normalnim napetostim v koordinatnih smereh;
- izven diagonalni elementi so enaki projekciji vektorjev strižne napetosti na koordinatne osi.

$$\vec{n} = \vec{i} \qquad \vec{t} = \underline{\underline{t}}\,\vec{n} = \underline{\underline{t}}\,\vec{i} = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{12} & t_{22} & t_{23} \\ t_{13} & & t_{33} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \underline{\begin{bmatrix} t_{11} \\ t_{12} \\ t_{13} \end{bmatrix}}$$

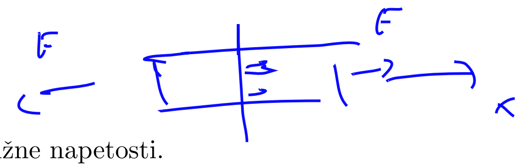
$$\vec{t}_1 = \begin{bmatrix} t_{11} \\ t_{12} \\ t_{13} \end{bmatrix} \qquad t_n = \vec{n} \cdot \vec{t} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} t_{11} \\ t_{12} \\ t_{13} \end{bmatrix} = t_{11}$$

$$\vec{t} = \vec{t}_n + \vec{t}_s \qquad \vec{t}_n = t_n \vec{n} = t_{11} \underline{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}$$

$$\begin{bmatrix} t_{11} \\ t_{12} \\ t_{13} \end{bmatrix} = \begin{bmatrix} t_{11} \\ 0 \\ 0 \end{bmatrix} + \vec{t}_s \Rightarrow \vec{t}_s = \begin{bmatrix} 0 \\ t_{12} \\ t_{13} \end{bmatrix}$$

$$\underline{\underline{t}} = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ & t_{22} & t_{23} \\ & & t_{33} \end{bmatrix} = \begin{bmatrix} \sigma_1 & \tau_{12} & \tau_{13} \\ \tau_{12} & \sigma_2 & \tau_{23} \\ \tau_{13} & \tau_{23} & \sigma_3 \end{bmatrix}$$

Osnovna napetostna stanja



- Enoosno napetostno stanje; izrčun normalne in strižne napetosti.

$$\underline{\underline{t}} = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\vec{n} = \vec{i} \qquad t_n = t_{11} = \sigma$$

$$\vec{t}_s = \vec{t} - t_n = \underline{\underline{t}}\,\vec{n} - \underbrace{(\underline{\underline{t}}\,\vec{n}) \cdot \vec{n}}\,\vec{n}$$

- Hidrostatično napetostno stanje $\underline{\underline{t}} = -p\,\underline{\underline{I}}$; v vsaki smeri je normalna napetost enaka $-p$, strižna napetost je enaka nič.

- Strižno napetostno stanje, obstaja KS v katerem je vsota diagonalnih elementov napetostnega tenzorja enaka nič. Izračun normalne in strižne napetosti.

- Ravninsko napetostno stanje.