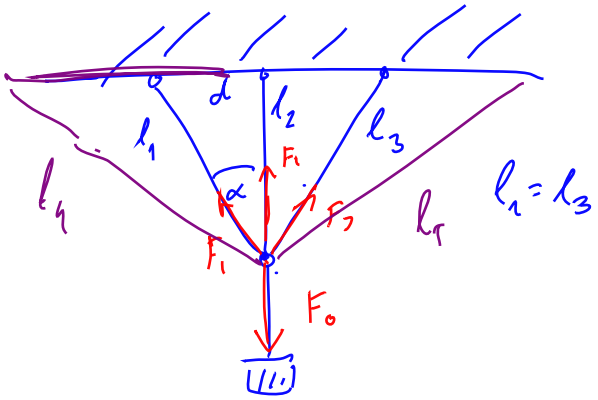
Vaje 15. april 2021

1. Utež obešena na tri palice s skupnim presečiščem.



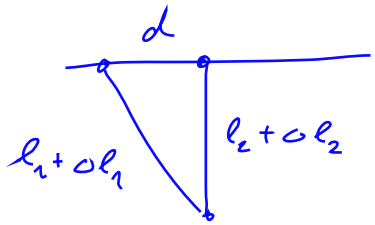
$F_1 = F_3$

$2F_1 \cos\alpha + F_2 = F_0$

$F_k = A\sigma_k = AE\varepsilon_k = AE\frac{\Delta l_k}{l_k}$

$\sigma = E\frac{\Delta l}{l}$

$\rightarrow \quad 2AE\frac{\Delta l_1}{l_1}\cos\alpha + AE\frac{\Delta l_2}{l_2} = F_0$



$\Delta l_1 = \Delta l_3$

$l_1^2 = \alpha^2 + l_2^2$

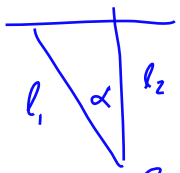
$(l_1+\Delta l_1)^2 = \alpha^2 + (l_2+\Delta l_2)^2$

$l_1^2 - l_2^2 = (l_1+\Delta l_1)^2 - (l_2+\Delta l_2)^2 = l_1^2 + 2l_1\Delta l_1 + (\Delta l_1)^2 - l_2^2 - 2l_2\Delta l_2 - (\Delta l_2)^2$

$0 = 2l_1\Delta l_1 - 2l_2\Delta l_2 + (\Delta l_1)^2 - (\Delta l_2)^2$

zanemarimo

$l_1\Delta l_1 = l_2\Delta l_2 \Rightarrow \Delta l_2 = \frac{l_1}{l_2}\Delta l_1$



$\cos\alpha = \frac{l_2}{l_1}$

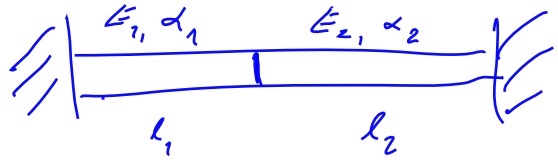
$AE\Delta l_1\left(2\frac{1}{l_1}\cos\alpha + \frac{1}{l_2}\cdot\frac{l_1}{l_2}\right) = F_0$

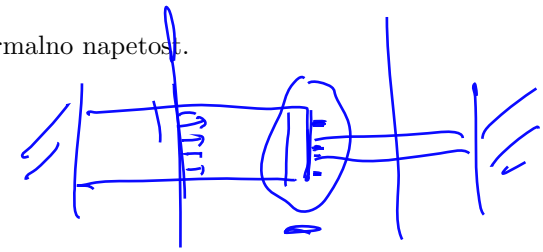
$\Delta l_1 = \frac{F_0}{AE\left(2\frac{1}{l_1}\cos\alpha + \frac{l_1}{l_2^2}\right)} = \frac{F_0}{AE\left(2\frac{l_2}{l_1^2} + \frac{l_1}{l_2^2}\right)}$

$\Delta l_1 = \frac{F_0 l_1^2 l_2^2}{AE(2l_2^3 + l_1^3)} \qquad \Delta l_2 = \frac{F_0 l_1^3 l_2}{AE(2l_2^3 + l_1^3)}$

$F_1 = A\sigma_1 = AE\frac{\Delta l_1}{l_1} = F_0\frac{l_1 l_2^2}{2l_2^3 + l_1^3}$; $F_2 = F_0\frac{l_1^3}{2l_2^2 + l_1^3}$

2. Kompozitna palica med dvema togima stenama. Določi termalno napetost.





$\varepsilon_1 \quad \varepsilon_2 \qquad A_1 = A_2 \qquad A_1\sigma_1 \quad A_2\sigma_2$

$\varepsilon_1 = \varepsilon_1^T + \varepsilon_1^E \; ; \quad \varepsilon_1^T = \alpha_1 \Delta T \quad \varepsilon_1^E = \sigma_1 / E_1$

$\varepsilon_1 = \frac{\Delta l_1}{l_1} \; ; \quad \varepsilon_2 = \frac{\Delta l_2}{l_2}$

$l_1 + \Delta l_1 + l_2 + \Delta l_2 = l_1 + l_2 \Rightarrow \boxed{\Delta l_1 + \Delta l_2 = 0}$

$E_1 \varepsilon_1^E = E_2 \varepsilon_2^E \quad \Leftarrow \quad \sigma_1 = \sigma_2$

$E_1 (\varepsilon_1 - \alpha_1 \Delta T) = E_2 (\varepsilon_2 - \alpha_2 \Delta T) \quad A_1\sigma_1 = A_2\sigma_2$

$E_1 \left(\frac{\Delta l_1}{l_1} - \alpha_1 \Delta T \right) = E_2 \left(\frac{\Delta l_2}{l_2} - \alpha_2 \Delta T \right) \qquad \Delta l_2 = -\Delta l_1$

$\Delta l_1 \left(\frac{E_1}{l_1} + \frac{E_2}{l_2} \right) = \Delta T \left(-\alpha_2 E_2 + \alpha_1 E_1 \right)$

$\Delta l_1 = \frac{\alpha_1 E_1 - \alpha_2 E_2}{\frac{E_1}{l_1} + \frac{E_2}{l_2}} \Delta T \qquad \Delta l_2 = -\Delta l_1$

$\sigma_1 = E_1 \varepsilon_1^E = E_1 \left(\varepsilon_1 - \varepsilon_1^T \right) = E_1 \left(\frac{\Delta l_1}{l_1} - \alpha_1 \Delta T \right)$

$= E_1 \Delta T \left(\frac{\alpha_1 E_1 - \alpha_2 E_2}{E_1 + l_1 E_2 / l_2} - \alpha_1 \right) = E_1 \Delta T \frac{\alpha_1 E_1 - \alpha_2 E_2 - \alpha_1 E_1 - \alpha_1 l_1 E_2 / l_2}{E_1 + l_1 E_2 / l_2}$

$\sigma_1 = - \frac{\alpha_2 E_2 + \alpha_1 E_2 l_1 / l_2}{E_1 + l_1 E_2 / l_2} E_1 \Delta T = - \frac{\alpha_2 + \alpha_1 l_1 / l_2}{E_1 + E_2 l_1 / l_2} E_1 E_2 \Delta T$

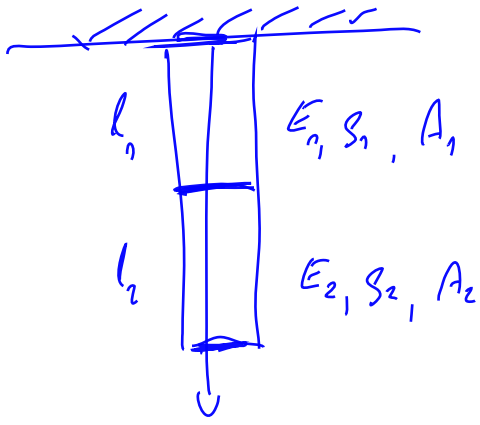
Posebni primer $l_1 = l_2$

$$\sigma_1 = \sigma_2 = -\frac{\alpha_1 + \alpha_2}{E_1 + E_2} E_1 E_2 \Delta T$$

$\alpha_1 = \alpha_2$; $E_1 = E_2$ $\Rightarrow$ $\sigma = -\alpha E \Delta T$

~~3. Osna obremenitev odsekanega stožca. Določi napetost in deformacijo.~~

4. Razteg kompozitne palice zaradi lastne teže.



$$\frac{d}{dx}\left(AE\frac{du}{dx}\right) + p(x) = 0$$

$$A_1 E_1 \frac{d^2 u_1}{dx^2} + \rho_1 g A_1 = 0$$

$$\frac{d^2 u_1}{dx^2} = -\frac{\rho_1 g}{E_1} \Rightarrow u_1 = -\frac{1}{2}\frac{\rho_1 g}{E_1}x^2 + C_1 x + C_2$$

$$u_2 = -\frac{1}{2}\frac{\rho_2 g}{E_2}x^2 + C_3 x + C_4$$

$u_1(x=0) = 0 \qquad C_2 = 0$

$u_1(x=l_1) = u_2(x=l_1)$

$\sigma_1(x=l_1) = \sigma_2(x=l_1) \qquad E_1 \frac{du_1}{dx}(x=l_1) = E_2 \frac{du_2}{dx}(x=l_1)$

$\sigma_2(x=l_2+l_1) = 0 \qquad E_2 \frac{du_2}{dx}(x=l_2+l_1) = 0$

$$-\rho_1 g l_1 + E_1 C_1 = -\rho_2 g l_1 + E_2 C_3 = +\rho_2 g l_2 \Rightarrow C_1 = \frac{g(\rho_2 l_2 - \rho_1 l_1)}{E_1}$$

$$-\frac{1}{2}\frac{\rho_1 g}{E_1} l_1^2 + C_1 l_1 = -\frac{1}{2}\frac{\rho_2 g}{E_2} l_1^2 + C_3 l_1 + C_4$$

$$-\rho_2 g(l_1+l_2) + E_2 C_3 = 0 \Rightarrow C_3 = \frac{1}{E_2}\rho_2 g(l_1+l_2)$$

$$C_4 = \frac{1}{2} g l_1^2 \left(\frac{\rho_2}{E_2} - \frac{\rho_1}{E_1}\right) + l_1(C_1 - C_3) =$$

$$= \frac{1}{2} g l_1^2 \left(\frac{\rho_2}{E_2} - \frac{\rho_1}{E_1}\right) + l_1 g \left(\frac{\rho_2 l_2 - \rho_1 l_1}{E_1} - \frac{\rho_2 l_1 - \rho_2 l_2}{E_2}\right) =$$

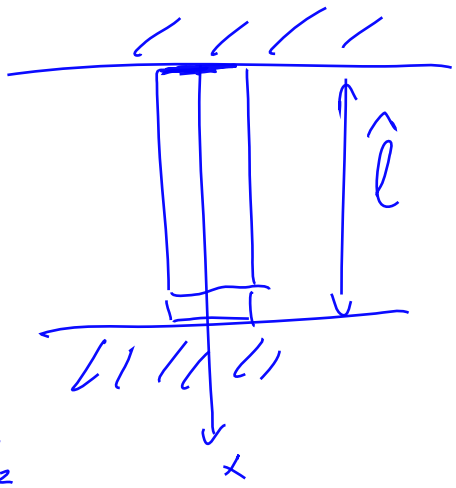
$$= \frac{1}{2} g l_1^2 \left(\frac{\rho_2}{E_2} - \frac{\rho_1}{E_1}\right) + l_1 g \left(\frac{\rho_2 l_2 - \rho_1 l_1}{E_1} - \frac{\rho_2 l_1 + \rho_2 l_2}{E_3}\right)$$

$$u_2(l_1+l_2) = -\frac{1}{2}\frac{\rho_2 g}{\varepsilon_2}(l_1+l_2)^2 + \rho_2 g(l_1+l_2)^2 +$$

$$\frac{1}{2} g l_1^2 \left(\frac{\rho_2}{\varepsilon_2} - \frac{\rho_1}{\varepsilon_1}\right) - l_1^2 g(\rho_1+\rho_2)$$

.

5. Valj, visi s stropa in se zaradi lastne teže raztegne, se dotakne togih tal. Valj segrejemo za ΔT. Določi napetostno stanje v valju. Pri kateri spremembi temperature bo v celem valju napetost kompresibilna.



$u(x=0)=0$

$u_2(x=0)=0$

$\varepsilon = \varepsilon_1 + \varepsilon_T + \varepsilon_2$

$\frac{du}{dx} = \frac{du_1}{dx} + \alpha\Delta T + \frac{du_2}{dx}$

$u = u_1 + \alpha\Delta T x + u_2 + C_1 \Rightarrow 0 = u_2(x=0) + C_1$

$C_2 = -u_2(x=0)$

$0 = u(x=\hat{l}) = u_1(x=\hat{l}) + \alpha\Delta T\,\hat{l} + u_2(\hat{l}) - u_2(0)$

Iščemo

$$\begin{cases} u_1 = -\frac{1}{2}\frac{\rho g}{E}x^2 + A_1 x + A_2 \\ u_1(x=l) = \frac{1}{2}\frac{\rho g}{E}l^2 \end{cases} ; \quad A_2 = 0$$

$\hat{l} = l + \frac{1}{2}\frac{\rho g}{E}l^2$

$u_1(x) = -\frac{1}{2}\frac{\rho g}{E}x^2 + A_1 x$

$\frac{du}{dx}(x=l) = 0 \quad A_1 = \frac{\rho g}{E}l$

$-\frac{\rho g}{E}l + A_1 = 0$

$u_1(x) = \frac{\rho g}{E}x\left(l - \frac{1}{2}x\right)$

$\hat{l} = l$

$u_1(\hat{l}) = \frac{\rho g}{E}\left(l + \frac{1}{2}\frac{\rho g}{E}l^2\right)\left(l - \frac{1}{2}\left(l + \frac{1}{2}\frac{\rho g}{E}l^2\right)\right)$

$0 = \frac{1}{2}\frac{\rho g}{E}l^2 + \alpha\Delta T l + u_2(l) - u_2(0)$

$$u_2(l) - u_2(0) = -\alpha \Delta T l - \frac{1}{2}\frac{\rho g}{E} l^2$$

$$\frac{du_2}{dx} = \frac{u_2(l) - u_2(0)}{l} = -\alpha \Delta T - \frac{1}{2}\frac{\rho g}{E} l$$

$$\varepsilon_2 = \frac{\sigma_2}{E} \qquad \sigma_2 = E\frac{du_2}{dx} = -\frac{1}{2}\rho g l - \alpha E \Delta T$$

$$\sigma = \sigma_1 + \sigma_2 = \rho g (l - x) - \frac{1}{2}\rho g l - \alpha E \Delta T$$

$$\sigma = \frac{1}{2}\rho g l - \rho g x - \alpha E \Delta T < 0$$

$$\frac{1}{2}\rho g l - \alpha E \Delta T = 0 \Rightarrow \Delta T = \frac{\rho g l}{2\alpha E}$$

6. Podan je napetostni tenzor

$$\underline{\underline{t}} = \begin{bmatrix} -24 & 16 & -8 \\ 16 & 24 & 0 \\ -8 & 0 & 0 \end{bmatrix}.$$

Izračunaj normalno in strižno napetost na ravnino, ki ima normalo v smeri vektorja $\vec{\imath} - \vec{k}$.