University of Ljubljana
Institute of Mathematics, Physics and Mechanics
Ljubljana, Slovenia

The 5th Slovenian International Conference on Graph Theory

— Program and Book of Abstracts —

Bled, Slovenia, June 22–27, 2003

Ljubljana, 2003

Foreword

It is our pleasure to welcome you at Bled, the site of the Fifth Slovenian Conference on Graph Theory.

International graph theory meetings in Slovenia are becoming regular events, organized every four years. As the first one we consider the meeting on Algebraic and Topological Graph Theory that was organized by Wilfried Imrich, Tory Parsons, and Tomo Pisanski in Dubrovnik (now in Croatia) in 1985. The second meeting was held at Lake Bled in 1991 and coincided with the declaration of Slovenian independence. This caused a slight inconvenience to the 30 participants but the meeting will be remembered as a successful and adventurous event. The third and the fourth Slovenian conferences followed in 1995 and 1999.

In the tradition of the former meetings, the conference is strongest in the areas of algebraic and topological graph theory, but we are glad that also other branches of graph theory are well represented. We express our thanks to all of you for attending this conference and wish you a mathematically productive week as well as a pleasant and relaxed stay in Slovenia.

This collection contains abstracts of the talks and the program of the conference. The proceedings of the conference will be published as a special volume of Discrete Mathematics after a thorough refereeing procedure following the standards of the journal.

The organizers are grateful to all those who helped make this meeting possible. Special thanks go to the Slovenian Ministry of Science and Technology, to the Institute of Mathematics Physics and Mechanics at the University of Ljubljana, to the Faculty of Mathematics and Physics at the University of Ljubljana, and to the Faculty of Education at the University of Maribor for their financial support.

Ljubljana, June 12, 2003

Boštjan Brešar Martin Juvan Sandi Klavžar Dragan Marušič
Aleksander Malnič Bojan Mohar Tomaž Pisanski

PROGRAM

Sunday, June 22

14:15 – 14:30	**Opening**
14:30 – 14:50	*S. Klavžar:* Square-free colorings of graphs
14:55 – 15:15	*B. Brešar:* Cubic inflation, mirror graphs, Platonic graphs, and partial cubes
15:30 – 15:50	*W. Imrich:* Cartesian products of graphs
15:55 – 16:15	*R. Häggkvist:* Bootstrapping theorems by Petersen, Baebler, Belck and Gallai
16:20 – 16:45	**Coffee break**
16:45 – 17:35	*J. Širáň:* Classification of regular maps of negative prime Euler characteristic

Monday, June 23

9:00 – 9:50	*C. Praeger:* Homogeneous factorisations of graphs and digraphs
10:00 – 10:50	*M. Muzychuk:* On isomorphism problem for Cayley graphs
11:00 – 11:30	**Coffee break**
Session 1	
11:30 – 11:50	*D. Marušič:* Semiregular elements in 2-closed groups of square-free degree
12:00 – 12:20	*R. Jajcay:* Arithmetic conditions for vertex-transitivity of graphs
12:30 – 12:50	*M. Boben:* Irreducible configurations and graphs
Session 2	
11:30 – 11:50	*J. Huang:* The structure of bi-arc graphs
12:00 – 12:20	*H.-J. Lai:* Hamiltonian claw-free graphs
12:30 – 12:50	*N. López:* Structural properties of eccentric digraphs
13:00 – 15:00	**Lunch time**
Session 1	
15:00 – 15:20	*M. Kriesell:* Contracting locally connected graphs of prescribed vertex connectivity
15:30 – 15:50	*S. Bessy:* Spanning a strong digraph by α circuits: A proof of Gallai's conjecture
16:00 – 16:20	*O. Hudry:* Extremal values for the codes identifying vertices in graphs
Session 2	
15:00 – 15:20	*A. Jurišić:* TBA
15:30 – 15:50	*Š. Miklavič:* An equitable partition for a Q-polynomial kite-free distance-regular graph
16:00 – 16:20	*E. van Dam:* Combinatorial designs with two singular values
16:30 – 17:00	**Coffee break**
Session 1	
17:00 – 17:20	*T. Biyikoglu:* The rank of a cograph
17:30 – 17:50	*H. Bielak:* Ramsey and 2-local Ramsey numbers for some graphs
18:00 – 18:20	*J. Moncel:* The pentomino exclusion problem and generalization
18:30 – 18:50	*A. Vietri:* Arc-coloring of directed hypergraphs and chromatic number of walls
Session 2	
17:00 – 17:20	*P. Šparl:* Homomorphisms of hexagonal graphs to odd cycles
17:30 – 17:50	*A. Orbanić:* Blanuša double
18:00 – 18:20	*A. Žitnik:* Series parallel extensions of plane graphs to dual-eulerian graphs
18:30 – 18:50	*T. Ryuzo:* Transferability of graphs

Tuesday, June 24

9:00 – 9:50	*P. Cameron:* Permutations and codes: polynomials, bases, and covering radius
10:00 – 10:50	*B. Servatius:* Combinatorial pointed pseudo-triangulations
11:00 – 11:30	**Coffee break**
Session 1	
11:30 – 11:50	*C. P. Bonnington:* Toroidal triangulations are geometric
12:00 – 12:20	*A. Breda d'Azevedo:* Non-orientable regular hypermaps with few faces
12:30 – 12:50	*Y. S. Kwon:* New regular embeddings of n-cubes Q_n
Session 2	
11:30 – 11:50	*B. Zmazek:* Dominating graph bundles
12:00 – 12:20	*A. Vesel:* On resonance graphs of catacondensed hexagonal graphs: structure, coding, and hamilton path algorithm
12:30 – 12:50	*I. Peterin:* Fast recognition of subclasses of almost-median graphs
13:00 – 15:00	**Lunch time**
15:00 – 15:40	*M. E. Watkins:* Growth in Bilinski diagrams
Session 1	
15:45 – 16:05	*M. Klin:* A census of spreads in small $GQ(s,t)$ graphs and corresponding antipodal distance regular graphs of diameter 3
Session 2	
15:45 – 16:05	*G. Fijavž:* Weighted Hadwiger conjecture
16:15 – 16:45	**Coffee break**
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16:45 – 17:05	*T. Pisanski:* Bokowski systems of curves
17:15 – 17:35	*H. Gropp:* Configurations — 75 years after the death of Ernst Steinitz
17:45 – 18:05	*R. Bacher:* Choosing trees in an orchard
18:15 – 18:35	*A. Malnič:* An infinite family of regular partial cubes
Session 2	
16:45 – 17:05	*M. Zaveršnik:* Short cycles connectivity
17:15 – 17:35	*D. Fronček:* Magic type labelings and tournament scheduling
17:45 – 18:05	*J. Žerovnik:* A remark on n-tuple colorings of planar graphs with large odd girth
18:15 – 18:35	*P. Shariefuddin:* On scores of oriented graphs

Wednesday, June 25

9:00 – 9:50	*B. Grünbaum:* Graphs of polyhedra; polyhedra as graphs
10:00 – 10:50	*V. Chepoi:* Algebraic characterization of weakly median graphs
10:55 – 11:15	**Coffee break**
11:15 – 11:55	*T. Tucker:* Balanced regular Cayley maps for abelian groups
12:00 – 12:30	*M. Conder:* Regular Cayley maps for finite abelian groups, II
	Excursion in the afternoon

Thursday, June 26

9:00 – 9:50	*M. N. Ellingham:* The nonorientable genus of complete tripartite graphs
10:00 – 10:50	*P. Hell:* Homomorphisms to graphs with bounded degrees
11:00 – 11:30	**Coffee break**
Session 1	
11:30 – 11:50	*K. Ando:* Vertices of degree 5 in a minimally contraction critically 5-connected graph
12:00 – 12:20	*T. Böhme:* Connectivity and minors in large graphs
12:30 – 12:50	*D. Bokal:* The minor crossing number
Session 2	
11:30 – 11:50	*M. Šajna:* Mobility of vertex-transitive graphs
12:00 – 12:20	*T. K. Lim:* Homogeneous factorisations of complete graphs with edge-transitive factors
12:30 – 12:50	*J. Šiagiová:* Infinite planar vertex-transitive and Cayley maps with a given covalence sequence
13:00 – 15:00	**Lunch time**
15:00 – 15:40	*J. Koolen:* Is there a finite number of distance-regular graphs with fixed valency?
Session 1	
15:45 – 16:05	*H. M. Mulder:* Signpost systems and their underlying graphs
Session 2	
15:45 – 16:05	*M. Hirasaka:* Construction of association schemes from difference sets
16:15 – 16:45	**Coffee break**
Session 1	
16:45 – 17:05	*A. Bonato:* Mutually embeddable vertex-transitive graphs and trees
17:15 – 17:35	*D. Delić:* Embeddings into the monoid of the random graph
17:45 – 18:05	*J. Jerebic:* The strong isometric dimension of graphs of diameter 2
Session 2	
16:45 – 17:05	*D. Catalano:* Approaching multiple-edge hypermaps
17:15 – 17:35	*M. Razpet:* The middle row of the principal cell
17:45 – 18:05	*M. Małafiejski:* On the chordless graphs
19:00 –	**Conference dinner (Kunstelj, Radovljica)**

Friday, June 27

9:00 – 9:50	*R. Nowakowski:* Well covered graphs, decompositions and associated vector spaces
10:00 – 10:50	*K. Kawarabayashi:* On disjoint paths problem
11:00 – 11:30	**Coffee break**
11:30 – 11:50	*B. Mohar:* Separation and rigidity indices of graphs
12:00 – 12:20	*D. M. Van Wieren:* On the domination number of the torus
12:30 – 12:50	*N. Bouz-Asal:* Graph processes derived from statistical correlation matrices

LIST OF ABSTRACTS

Vertices of degree 5 in a minimally contraction critically 5-connected graph

Kiyoshi Ando, Ken-ichi Kawarabayashi

A k-connected graph is said to be contraction critically (resp. minimally) k-connected if the contraction (resp. the removal) of every edge results in a graph which is not k-connected. We prove that each vertex of minimally contraction critically 5-connected graph has at least two neighbours of degree 5.

Choosing trees in an orchard

Roland Bacher

Given a finite generic (no alignment of 3 points) set of points in $\mathbb{R}^2$ (or more generally in $\mathbb{R}^d$), there exists a natural equivalence relation of the points into at most 2 classes. This yields a rule to plant two kinds of trees at generic locations in an orchard. If time permits it, I present also the "dual" version of this theorem. It yields a natural equivalence relation or a natural orientation on pseudolines in a generic configuration of projective pseudolines.

Spanning a strong digraph by α circuits: A proof of Gallai's conjecture

Stephane Bessy, Stephan Thomasse

In 1964, Tibor Gallai asked whether every strongly connected directed graph D is spanned by α directed circuits, where α is the stability of D. We give a proof of this conjecture.

Ramsey and 2-local Ramsey numbers for some graphs

Halina Bielak

Let F, G, H be simple graphs with at least two vertices. The Ramsey number $R(G,H)$ is the smallest integer n such that in arbitrary two-colouring (say red and blue) of K_n a red copy of G or a blue copy of H is contained (as subgraphs). If $G \cong H$ we write $R(G)$ instead of $R(G,G)$.

A *local k-colouring* of a graph F is a colouring of the edges of F in such a way that the edges incident to each vertex of F are coloured with at most k different colours. The *k-local Ramsey number* $R_{k\text{-}loc}(G)$ of a graph G is defined as the smallest integer n such that K_n contains a monochromatic subgraphs G for every local k-colouring of K_n. The existence of $R_{k\text{-}loc}(G)$ is proved by Gyárfás, Lehel, Schelp and Tuza (see A. Gyárfás, J. Lehel, R.H. Schelp, Z.S. Tuza, *Ramsey Numbers for Local Colorings*, Graphs and Combinatorics **3** (1987), 267–277).

We consider edge 2-colouring and edge local 2-colouring. We study the relation between the 2-local Ramsey number $R_{2\text{-}loc}(G)$ and the Ramsey number $R(G)$, where G is a disjoint union of some cycles or G is a disjoint union of some trees.

The rank of a cograph

Tuerker Biyikoglu

I will talk about the rank of the adjacency matrix of a cograph. A graph is called cograph if it has no induced subgraph P_4. The rank of a cograph is equal to the number of distinct nonzero columns of its adjacency matrix.

Irreducible configurations and graphs

Marko Boben

Cubic bipartite graphs with girth at least 6 correspond to symmetric (v_3) configurations. In 1887 V. Martinetti described a simple construction method which enables one to construct all (v_3) configurations from a set of so-called *irreducible* configurations. The result has been cited quite often since its publication, both in the sense of configurations and graphs. But after a careful examination, the list of irreducible configurations given by Martinetti has turned out to be incomplete. We will give the description of all irreducible configurations and corresponding graphs, including those which are missing in the Martinetti's list.

Connectivity and minors in large graphs

Thomas Böhme

It is proved that for all positive integers k, s and t there is an $f(k)$, depending on k only, and an $N(k, s, t)$ such that every graph on $n \geq N(k, s, t)$ vertices with at least $f(k) \cdot n$ edges either contains a k-connected subgraph of order at least s or t pairwise disjoint k-connected subgraphs. The proof is based on classical theorem by Mader. In the talk some applications and modifications of the above theorem will be discussed. The presented results are partial joint work with A. V. Kostochka.

The minor crossing number

Drago Bokal, Gašper Fijavž, Bojan Mohar

The minor crossing number $\mathrm{mcr}(G)$ is defined as the minimum crossing number over all graphs that contain G as a minor. It links the powerful theory of graph minors with the notion of crossing number of a graph.

The graphs with $\mathrm{mcr}(G) = 1$ have been characterized by Robertson and Seymour and this result provided most motivation for our research. We prove that the minimum is always attained at some cubic graph, that the crossing number of G is bounded from above by some function of $\mathrm{mcr}(G)$ and maximum degree $\Delta(G)$, and that examining $\mathrm{mcr}(G)$ it is enough to consider the blocks of G. We also establish some results about the minor crossing number of complete graphs.

Toroidal triangulations are geometric

Dan Archdeacon, C. Paul Bonnington, Jo Ellis-Monaghan

Steinitz's Theorem states that a graph is the 1-skeleton of a convex polyhedron if and only if it is 3-connected and planar. The polyhedron is called a geometric realisation of the embedded graph. Its faces are bounded by convex polygons whose points are co-planar. When does a graph embedded on the torus have a geometric realisation in 3-space? In this talk we prove that every toroidal triangulation has a geometric realisation. The proof involves an interesting detour into 4-space.

Mutually embeddable vertex-transitive graphs and trees

Anthony Bonato, Claude Tardif

Two graphs are *mutually embeddable* or *me* if they are isomorphic to induced subgraphs of each other. While any two finite me graphs are isomorphic, there are many examples of pairwise me non-isomorphic infinite graphs. Closely related to me graphs are the *universal* graphs, which embed all graphs of smaller orders. A fascinating example of a universal graph is the infinite random graph, R. The graph R is vertex-transitive, and this fact in part motivates the following problem: find large (say, of cardinality $2^{\aleph_0}$) families of pairwise me non-isomorphic vertex-transitive countable graphs. We will give examples of such families by using the weak Cartesian product and its unique factorization properties. To our knowledge, this is one of the first applications of the unique factorization property of the weak Cartesian product with infinitely many factors. Our techniques give large families of me universal graphs for all infinite cardinals.

We conjecture that if a tree T (of any infinite order) is me with another non-isomorphic one, then T is me with an *infinite* family of pairwise non-isomorphic trees. The conjecture is true in the case of *rayless* trees; that is, trees that contain no infinite path as an induced subgraph. Our proof of this result depends on Halin's fixed element theorem for rayless trees.

Graph processes derived from statistical correlation matrices

Nadia Bouz-Asal, Almasa Skaka, Douglas Van Wieren

A simple graph can be produced from n nontrivial finite vectors of equal length and a value $p \in [-1, 1]$ by a simple procedure: One uses the vectors as the set of vertices and constructs an edge between two vertices if and only if the correlation coefficient between the corresponding vectors is greater than p. Without the value p, the natural result is a monotonic graph process, an ordering of edges in K_n. A similar method uses n random variables. Importantly, the resulting graph or graph process has *meaning* in the same sense that the correlation coefficient has *meaning.* (Thus, examination of the resulting structure and its substructures — such as cliques and/or independent sets — implicitly provides inference material. With graph processes *per se,* it's also appropriate to discuss the implications of distinct hitting times for various properties.) Here, we examine the described procedure for a form of robustness. Given an ordering of the $\binom{n}{2}$ edges in K_n, we demonstrate how to construct n vectors, polynomial in length and using positive integers, such that the $\binom{n}{2}$ correlation coefficients have exactly the required order.

Non-orientable regular hypermaps with few faces

Antonio Breda d'Azevedo, Steve Wilson

A map, or a cellular division of a compact surface, is often viewed as a cellular imbedding of a connected graph in a compact surface. It generalises to a hypermap by replacing graph by a hypergraph. In this paper we classify the non-orientable regular maps and hypermaps with size a power of 2, the non-orientable regular maps and hypermaps with 1, 2, 3, 5 faces and give a sufficient and necessary condition for the existence of regular hypermaps with 4 faces on non- orientable surfaces. For maps we classify the non-orientable regular maps with a prime number of faces. These results can be useful in classifications of non-orientable regular hypermaps or in non-existence of regular hypermaps in some non-orientable surface.

Cubic inflation, mirror graphs, Platonic graphs, and partial cubes

Boštjan Brešar, Sandi Klavžar, Alenka Lipovec, Bojan Mohar

Mirror graphs are a common generalization of even cycles and hypercubes. A connected graph $G = (V, E)$ is called a mirror graph if there exists a partition $\mathcal{P} = \{E_1, E_2, \ldots, E_k\}$ of E such that for all $i \in \{1, \ldots, k\}$ the following holds: (i) $G - E_i$ consists of two connected components G_1^i, G_2^i, and (ii) there is an automorphism α_i of G that maps G_1^i isomorphically onto G_2^i and every edge of E_i is invariant under α_i.

Mirror graphs are bipartite, vertex-transitive, and closed for Cartesian products. Cubic inflation is an operation that transforms a plane graph into a cubic plane graph; its result can be described as the dual of the barycentric subdivision of a plane graph.

In this talk we present a characterization of (plane) mirror graphs that can be obtained by cubic inflation. There are only five such graphs, namely the inflated Platonic graphs. In turn, a characterization of Platonic graphs involving a partition of a graph into special straight-ahead walks is derived. As an application of both concepts, some new regular isometric subgraphs of hypercubes are constructed.

Permutations and codes: polynomials, bases, and covering radius

Peter Cameron

There are many analogies between sets (or groups) of permutations and (linear) codes. Some of these, particularly concerning minimum distance and decoding, have been studied since the 1970s. Others, such as a relationship between the cycle index and the Tutte polynomial, have been looked at recently. Questions about covering radius for permutation sets and groups appear to be quite new. In the talk, I will survey what is known about these topics, concentrating on the newer results.

Approaching multiple-edge hypermaps

Domenico Catalano

An oriented map on a compact connected oriented surface without boundary S is an embedding $\mathcal{M}\colon G \to S$ of a connected graph G (in S) such that its complement in S consists of simply connected regions (called cells). A map is (orientably) regular if its orientation-preserving automorphism group acts regularly on darts (i.e. directed edges). Škoviera and Zlatoš have shown that each regular embedding $\mathcal{M}\colon G^{(k)} \to S$ with k parallel edges between two adjacent vertices of a simple graph G give rise to invariants like the *shadow* (the unique regular embedding of G which is covered by $\mathcal{M}$) the *twisting number* (a number $e \in \mathbb{Z}_k$ such that $e^2 \equiv 1 \pmod{k}$) and the *rotation gradient* (a function defined on the set of darts with values in $\mathbb{Z}_k$). Given a shadow, a twisting number and a rotation gradient there exists a regular embedding of $G^{(k)}$ with these invariants if and only if a certain system of equations in $\mathbb{Z}_k$ is solvable.

Hypermap is a generalisation of map in the sense that an hypermap is a cellular embedding $\mathcal{H}\colon G \to S$ of an hypergraph G in a compact connected surface without boundary S. Our purpose is to generalize edge-multiplicity to regular hypermaps. A hypermap is said to have multiple hyperedges if there are two or more (distinct) hyperedges sharing the same set of hypervertices. Contrary to maps the generalisation to hypermaps does not give many invariants. In general we lose the shadow and the rotation gradient.

Algebraic characterization of weakly median graphs

H.-J. Bandelt, V. Chepoi

We bring together algebraic concepts such as equational class and various concepts from graph theory for developing a structure theory for graphs that promotes a deeper analysis of their metric properties. The basic operations are Cartesian multiplication and gated amalgamation or, alternatively, retraction. Specifically, finite weakly median graphs are known to be decomposable (relative to these operations) into smaller pieces that in turn are parts of hyperoctahedra, the pentagonal pyramid, or of certain triangulations of the plane. This decomposition scheme can be interpreted as Birkhoff's subdirect representation in purely algebraic terms. Then the weakly median graphs can be identified with the discrete members of an equational class of ternary algebras satisfying five (independent) axioms on two to four points. What is remarkable in this context is that the purely algebraic avenue leads to objects that can be regarded as particular graphs, which admit quite a number of alternative characterizations. It is perhaps no accident that the prime models all have some geometric interpretation.

Regular Cayley maps for finite abelian groups, II

Marston Conder

This talk follows on from one by Tom Tucker. I will describe the use of Ito's theorem on products of abelian subgroups to derive a 3-way classification of regular Cayley maps for finite abelian groups, computational experiments to investigate these, and some surprising theoretical consequences. Also if time permits I will discuss some open questions at the end.

Embeddings into the monoid of the random graph

Anthony Bonato, Dejan Delić, Igor Dolinka

The infinite random graph (or Rado graph), written R, has been extensively investigated by both graph theorists and logicians. In particular, the automorphism group of R and its corresponding model-theoretic properties are fairly well-understood. Less is known about the endomorphism monoid of R, written End(R), which is the set of all graph homomorphisms of R to itself equipped with the operation of composition.

After a brief introduction to some of the properties of End(R), we will sketch a proof of a new result of the authors that End(R) embeds all countable semigroups; that is, it is countably universal in the class of all semigroups. This recalls the fact, known to Fraisse in the 1950's, that R is countable universal in the class of all graphs. Our proof is combinatorial in nature, and relies on certain semigroup embedding theorems of Pultr and Trnkova. Certain algebraic and computational consequences of our embedding theorem will be presented, along with evidence that End(R) satisfies an even stronger embedding property in the class of all semigroups.

The nonorientable genus of complete tripartite graphs

Mark N. Ellingham, Chris Stephens, Xiaoya Zha

Finding the surfaces on which a particular graph can be embedded is a difficult problem, and exact answers are known only for some very special classes of graphs. Even for complete graphs, it took over 70 years to find the genus of an arbitrary complete graph K_n. The complete bipartite graphs $K_{m,n}$ were somewhat easier, with the answers for orientable and nonorientable genus being given in the 1960's by Ringel. Here we examine complete tripartite graphs $K_{l,m,n}$ (where we assume $l \geq m \geq n$). In 1976 Stahl and White conjectured that the nonorientable genus is $\left\lceil \frac{(l-2)(m+n-2)}{2} \right\rceil$. We prove that this conjecture is true, with three exceptions: $K_{4,4,1}$, $K_{4,4,3}$, and $K_{3,3,3}$. We also discuss progress on a similar conjecture for the orientable genus of $K_{l,m,n}$.

Weighted Hadwiger conjecture

Gašper Fijavž

Let $G^w = (V, E, w)$ be a weighted graph: $G = (V, E)$ is its underlying graph and $w : E \to [1, \infty)$ is the edge weight function. A (circular) p-coloring of G^w is a mapping c of its vertices into a circle of circumference p, so that every edge $e = uv$ satisfies $\text{dist}(c(u), c(v)) \geq w(uv)$. The smallest p allowing a p-coloring of G^w is its weighted chromatic number, $\chi_w(G)$.

A q-basic graph is a weighted complete graph, whose edge weights satisfy triangular inequalities, and whose optimal travelling salesman tour has length q. Weighted Hadwiger Conjecture (WHC) at p states that if for every $q > p$ the graph G contains no q-basic graph as a *weighted* minor then $\chi_w(G^w) \leq p$.

We prove that WHC is true for $p < 4$ and false for $p \geq 4$.

Magic type labelings and tournament scheduling

Dalibor Frončem

Vertex-magic vertex labeling of a graph $G(V, E)$ is a bijection λ from the set V to $\{1, 2, \ldots, |V|\}$ with the property that for every vertex x the sum of the labels of all neighbors of x is equal to a given constant k. We will present an interesting connection between vertex-magic vertex labelings and schedules of incomplete round robin tournaments. A joint result with M. Meszka on non-compact round robin tournaments will be also mentioned. If time permits, we may also present some joint results with P. Kovar and T. Kovarova on vertex magic total labelings for some new families of graphs, namely products of cycles and some regular graphs.

Configurations — 75 years after the death of Ernst Steinitz

Harald Gropp

Ernst Steinitz (1871-1928) died on September 29,1928, i.e. nearly 75 years ago. His main contributions to mathematics were in the fields of algebra and polyhedra. However, his work on configurations in geometry and combinatorics should not be forgotten. Already the dissertation of Steinitz in Breslau (1894) on "Über die Construction der Configurationen n_3" contains interesting results concerning configurations and their relation to graphs and geometry.

This talk will consist of three parts.

First a short biography on Ernst Steinitz and a discussion of his work on configurations. Here an important aspect will be the political fate of Steinitz and his family because he was Jewish, until now a nearly unknown fact.

Second, the period between 1928 and 2003 will be discussed concerning the research of configurations. Here one of the few mathematicians who contributed results at all is **Branko Grünbaum**.

The last part will discuss the future of configurations in the 21^{st} century. For those who still do not know what configurations are:

Definition. *A* configuration (v_r, b_k) *is a finite incidence structure with v points and b lines such that*

(i) there are k points on each line and r lines through each point, and

(ii) through two different points there is at most one common line.

Configurations are also closely related to graphs and hypergraphs:

Remark. *A configuration (v_r, b_k) is a linear r-regular k-uniform hypergraph with v vertices and b hyperedges.*

A configuration (v_r, b_k) is equivalent to a bipartite graph with $v + b$ vertices and $vr = bk$ edges whose girth is at least 6 and whose degrees are constant within the 2 partition classes of v and b edges.

A configuration (v_r, b_k) corresponds to a regular edge-disjoint packing of b cliques K_k into a K_v.

Graphs of polyhedra; polyhedra as graphs

BRANKO GRÜNBAUM

Graphs and 3-dimensional polyhedra interact in several ways. Two of the principal ones form our topic.

A theorem due to Steinitz can be formulated so as to characterize graphs that are isomorphic to graphs formed by vertices and edges of convex polyhedra. The criterion is very simple the graphs must be planar and 3-connected. The necessity is essentially obvious, the sufficiency is quite deep. Besides a sketch of some of the proofs and the history, we shall discuss various generalizations and modifications, and present several open problems.

When one wishes to study polyhedra in 3-space that are more general than convex ones, it is far from obvious how to delimit the objects under consideration – beyond the understanding that they should be determined by polygonal faces. Any general class should certainly include polyhedra that are homeomorphic to closed solid balls, but also polyhedra such as the Kepler-Poinsot regular star-polyhedra. Moreover, the class should be closed under duality, and it is highly desirable for the class to be closed under limits in a suitable sense. All these desiderata, and more, can be achieved if one adopts a two-step process of definition. In the first step, abstract (or combinatorial) polyhedra are defined as (abstract) graphs with a certain structure, which involves a family of circuits with suitable restrictions. The second step defines geometric polyhedra as images of abstract polyhedra in 3-space, where vertices of the graphs are mapped onto points, edges onto segments, and the chosen circuits onto polygons. The talk will present details of these definitions, and their application to a variety of situations. The generality obtained allows not only satisfactory treatments for the existing theories, but solves a variety of mysteries that were previously explained as exceptions to the general rules.

Bootstrapping theorems by Petersen, Baebler, Belck and Gallai

Roland Häggkvist

See title. A typical statement which atypically is not proved in the paper is the conjecture: Every $2m+1$-regular pseudograph with at most $2m+3-3s$ leaves has a $2s$-factor.

A typical, atypically weak, corollary of theorems proved in the paper is the bootstrapped Belck-Gallai mikrotheorem: Every $3k$-regular pseudograph with at most 2 leaves has a k-factor.

The proof techniques are elementary.

Homomorphisms to graphs with bounded degrees

Pavol Hell

I will present recent results, some joint with Huang Jing and Tomás Feder, others joint with Jarik Nešetřil, on the complexity of homomorphism problems restricted to graphs with bounded degrees. A conjecture of Dyer and Greenhill proposes that degree restrictions do not change the classification of the complexity of counting homomorphisms. On the other hand, results of Galluccio, Häggkvist, Nešetřil, and the speaker indicate that degree restrictions do yield interesting new polynomial cases for the basic homomorphism problem. The main results I will present state that for list homomorphisms, degree restrictions do not help. I will also give examples, for homomorphism extension problems, where the number of new polynomial cases depends on how much the degrees are restricted. Implications for constraint satisfaction problems, with restrictions on the number of times a variable can be used, will also be discussed.

Construction of association schemes from difference sets

Sejeong Bang, Mitsugu Hirasaka

Let (Y, N) and (Z, F) be association schemes. We say that an association scheme (X, G) is an *extension* of (Y, N) by (Z, F) if there exists a closed subset $K \subseteq G$ and $x \in X$ such that $(X, G)_{xK} \simeq (Y, N)$ and $(X/K, G//K) \simeq (Z, F)$. We focus on extensions of (Y, N) by (Z, F) where each of (Y, N) and (Z, F) is the orbitals of a regular permutation group. In this talk we aim to characterize such extensions by certain algebraic properties on F and N, and define the two-dimensional Cohomology group with respect to the properties on F and N when N is an abelian group. Furthermore, we will show a construction of such extensions from a difference set when N is an elementary abelian group of rank two.

The structure of bi-arc graphs

Tomás Feder, Pavol Hell, Jing Huang

This talk will introduce a new family of intersection graphs with loops allowed, which generalizes interval graphs, interval bigraphs, and circular arc graphs of clique covering number two. This family arose in the classification of the complexity of list homomorphism problems, as the largest family for which polynomial algorithms are possible (as long as $P \neq NP$). We give a structrual characterization of bi-arc graphs, akin to the Lekkerkerker-Boland characterization of interval graphs. In particular, we show that a tree is a bi-arc graph if and only if it does not contain any of the twelve trees as a subtree. These characterizations are useful in describing polynomial list homomorphism algorithms.

Extremal values for the codes identifying vertices in graphs

Irene Charon, Olivier Hudry, Antoine Lobstein

Consider a connected undirected graph $G = (V, E)$ and a positive integer r; for any vertex v belonging to V, $B_r(v)$ denotes the ball of radius r centered at v, i.e. the set of all vertices linked to v by a path of length (with respect to the number of edges) at most r. A subset C of V is called an *r-identifying code of G* if, for all vertices v of V, the sets $B_r(v) \cap C$ are all nonempty and different. An r-identifying code of a given G is said to be *minimum* if its cardinality is minimum over the set of the r-identifying codes of G. We study here the extremal values of the cardinality of a minimum r-identifying code of any connected undirected graph G with a given number n of vertices. More precisely, we show that, for any G with n vertices, a minimum r-identifying code has at least $\lceil \log_2 n \rceil$ vertices and at most $n-1$ vertices, and we show that these bounds are reached.

Cartesian products of graphs

Wilfried Imrich

This talk is concerned with the most remarkable results about Cartesian products of finite and infinite graphs, the focus being on the interaction between structural and algorithmic properties. It begins with well known results about prime factorizations and also outlines a new lean linear recognition algorithm that will be published jointly with I. Peterin.

The presentation also traces similarities between the Cartesian and other graph products, such as the strong and the direct one. It ends with a list of open problems as well as suggestions on how to approach them.

Arithmetic conditions for vertex-transitivity of graphs

Robert Jajcay, Aleksander Malnič, Dragan Marušič

Determining the vertex-transitivity of a graph is generally a hard task (although the precise complexity of this problem is not known). Any "fast" arithmetic tests for vertex-transitivity are therefore of great interest.

In our talk, we generalize the results of J. Širáň and the first author (Australasian J. Combin. **10** (1994)), and develop new formulas for the number of closed walks of length p^r or pq, where p and q are primes, valid for all vertex-transitive graphs. Based on these formulas, several simple tests for vertex-transitivity are presented, as well as lower bounds on the orders of the smallest vertex- and arc-transitive groups of automorphisms for vertex-transitive graphs of given valence.

We demonstrate the use of our methods by applying them to the class of generalized Petersen graphs.

The strong isometric dimension of graphs of diameter two

Janja Jerebic, Sandi Klavžar

The strong isometric dimension $\mathrm{idim}(G)$ of a graph G is the least number k such that G can be isometrically embedded into the strong product of k paths. The problem of determining $\mathrm{idim}(G)$ for graphs of diameter two is reduced to a covering problem of the complement of G with complete bipartite graphs. As an example it is shown that $\mathrm{idim}(P) = 5$, where P is the Petersen graph.

TBA

A. Jurišić

(Abstract not received.)

On disjoint paths problem

Ken-ichi Kawarabayashi

In this talk, we will discuss disjoint paths problem. The topic includes k-linked graphs, k-linked graphs in sparse graphs, k-linked graphs in dense graphs, odd-k-path problem and parity-k-path problems. Some of this work is a joint work with B. Reed.

Square-free colorings of graphs

Boštjan Brešar, Sandi Klavžar

Let G be a graph and c a coloring of its edges. If the sequence of colors along a walk of G is of the form $a_1, \ldots, a_n, a_1, \ldots, a_n$, the walk is called a square walk. We say that the coloring c is square-free if any open walk is not a square and call the minimum number of colors needed so that G has a square-free coloring a walk Thue number and denote it by $\pi_w(G)$. This concept is a variation of the Thue number introduced in: [1] N. Alon, J. Grytczuk, M. Hałuszczak, and O. Riordan, Non-repetitive colorings of graphs, Random Structures Algorithms 21 (2002) 336–346.

Using the walk Thue number several results of [1] are extended. The Thue number of some complete graphs is extended to Hamming graphs. This result (for the case of hypercubes) is used to show that if a graph G on n vertices and m edges is the subdivision graph of some graph, then $\pi_w(G) \leq n - \frac{m}{2}$. Graph products are also considered. An inequality for the Thue number of the Cartesian product of trees is extended to arbitrary graphs and upper bounds for the (walk) Thue number of the direct and the strong products are also given. Using the latter results the (walk) Thue number of complete multipartite graphs is bounded which in turn gives a bound for arbitrary graphs in general and for perfect graphs in particular.

A census of spreads in small $GQ(s,t)$ graphs and corresponding antipodal distance regular graphs of diameter 3

Mikhail Klin, Sven Reichard

A pseudo-geometric $GQ(s,t)$ strongly regular graph (briefly, $GQ(s,t)$-graph) has the parameters

$$\begin{aligned} v &= (s+1)(st+1) \\ k &= s(t+1) \\ \lambda &= s-1 \\ \mu &= t+1 \end{aligned}$$

where s, t are suitable integers. The name comes from the fact that the point graph of a generalized quadrangle of order (s,t) is strongly regular with the parameters above (see, e.g., [2], [7]).

A spread in a $GQ(s,t)$-graph Γ is a partition of the vertex set of Γ into $st+1$ cliques of size $s+1$, see, e.g., [6].

A famous theorem of Brouwer [1] claims that the deletion of a spread from a $GQ(s,t)$-graph yields an antipodal distance regular graph of valency st and diameter 3, which is in fact an $(s+1)$-fold covering of the complete graph K_{st+1}. Moreover, this way of getting a distance regular graph from a suitable strongly regular graph Γ works only if Γ is a $GQ(s,t)$-graph.

In this context, Godsil and Hensel [5] asked for an example of a non-geometric $GQ(s,t)$-graph with a spread. The first such example was given in [3]; it has 96 vertices and corresponds to $s=5$, $t=3$. Many examples for sufficiently large values of s and $t=s+2$ follow from a prolificc construction in [4].

In this paper we describe all spreads (up to isomorphism) in $GQ(s,t)$-graphs with at most 64 points.

This investigation is essentially based on the computer catalogues of Ted Spence [8] for small strongly regular graphs. The table below gives a summary of our results. Here, *GS* refers to geometrical spreads, *NGS*, to non-geometrical ones. Note that we discover that the smallest (new) examples of graphs, giving a positive answer on the question by Godsil-Hensel, exist on 40 points. For some of the graphs we provide a computer-free explanation.

s	t	v	k	λ	μ	$\#srg$	$\#GQ$	$\#GS$	$\#NGS$
2	2	15	6	1	3	1	1	1	0
2	4	27	10	1	5	1	1	2	0
3	3	40	12	2	4	28	2	1	2
4	2	45	12	3	3	78	1	0	0
3	5	64	18	2	6	167	1	5	89

References

[1] A.E. Brouwer. Distance regular graphs of diameter 3 and strongly regular graphs. *Discrete Math.*, 49:101–103, 1984.

[2] A.E. Brouwer, A.M. Cohen, and A. Neumaier. *Distance Regular Graphs.* Springer-Verlag, Berlin, 1989.

[3] A.E. Brouwer, J.H. Koolen, and M.H. Klin. A root graph that is locally the line graph of the Petersen graph. *Discrete Math.*, 264:13–24, 2003.

[4] D.G. Fon-Der-Flaass. New prolific constructions of strongly regular graphs. *Adv. Geom.*, 2:301–306, 2002.

[5] C.D. Godsil and A.D. Hensel. Distance regular covers of the complete graph. *J. Comb. Th. Ser. B*, 56:205–238, 1992.

[6] W.H. Haemers and V.D. Tonchev. Spreads in strongly regular graphs. *Designs, Codes and Cryptography*, 8:145–157, 1996.

[7] A. Jurišić. *Antipodal covers.* PhD thesis, University of Waterloo, 1995.

[8] E. Spence. `http://www.maths.gla.ac.uk/~es/srgraphs.html`. Homepage.

Is there a finite number of distance-regular graphs with fixed valency?

S. Bang, A. Hiraki, J. Koolen, V. Moulton.

The polygons are distance-regular and therefore the answer is no for valency 2. However, Bannai and Ito conjectured that there are only finitely many distance-regular graphs with fixed valency $k \geq 3$. Indeed, they proved this to be the case when k equals 3 or 4, and also when $k \geq 3$ for bipartite distance-regular graphs.

In this talk I will present recent work on this conjecture and related problems. In particular, I will discuss the valency 5 case.

Contracting locally connected graphs of prescribed vertex connectivity

Matthias Kriesell

We present recent results on the existence of edges whose contraction in a locally k-connected graph keeps these properties.

New regular embeddings of n-cubes Q_n

Young Soo Kwon

We investigate highly-symmetrical embeddings of the n-dimensional cube Q_n into orientable compact surfaces which we call regular embeddings of Q_n. We derive some general results and construct some new families of regular embeddings of Q_n. In particular, for $n = 1, 2, 3(\pmod 4)$), we classify the regular embeddings of Q_n which can be expressed as balanced Cayley maps.

Hamiltonian claw-free graphs

HONG-JIAN LAI, YEHONG SHAO, MINGWUAN ZHAN

Kuipers and Veldman conjectured (1991) that any 3-connected claw-free graph with order n and minimum degree $\delta \geq \frac{n+6}{10}$ is Hamiltonian for n sufficiently large. We prove that if H is 3-connected claw-free graph with sufficiently large order n, and if $\delta(H) \geq \frac{n+5}{10}$, then either H is hamiltonian, or $\delta(H) = \frac{n+5}{10}$ and the closure of H is the line graph of G obtained from the Petersen graph P_{10} by adding $\frac{n-15}{10}$ pendant edges at each vertex of P_{10}.

Homogeneous factorisations of complete graphs with edge-transitive factors

C. H. LI, T. K. LIM, C. E. PRAEGER

A factorisation of a complete graph K_n is a partition of its edges into disjoint classes. Each class of edges in a factorisation of K_n corresponds to some spanning subgraph called a factor. A homogeneous factorisation of a complete graph is a factorisation where there exists a group G which permutes the factors transitively and a normal subgroup M of G such that each factor is M-vertex-transitive. Here, we study homogeneous factorisations of K_n such that each factor is also M-edge-transitive.

Structural properties of eccentric digraphs

Joan Gimbert, Nacho López, Mirka Miller, Joseph Ryan

The *eccentric digraph* $ED(G)$ of a digraph G represents the binary relation, defined on the vertex set of G, of being "eccentric"; that is, there is an arc from u to v in $ED(G)$ if and only if v is at maximum distance from u in G. In this work, we study several properties of such a relation. Thus, we characterize when it is symmetric, in the case that G is a graph or a non connected digraph. We also consider the connection between the complement and the eccentric digraph operators. Besides, we determine the behaviour of the iterated sequence of eccentric digraphs of any tree and some other classes of graphs.

On the chordless graphs

Robert Janczewski, Michał Małafiejski

We are given a simple graph $G = (V, E)$. Any edge $e \in E$ is a *chord* in a path $P \subseteq G$ (cycle $C \subseteq G$) iff a graph obtained by joining e to path P (cycle C) has two vertices of degree 3. Class of graphs without any chord in paths (cycles) we call *path-chordless* (*cycle-chordless*). We will prove that all these graphs are 3-colorable and can be recognized in time $O(n^2)$.

The proposed class of graphs seems to be new and interesting. First, we observed that cycle-chordless (or simple chordless) graphs are a generalization of trees and cacti, but these graphs are different than partial k-trees (and not included). Moreover, we proved that there is no finite nonempty family of graphs $\mathcal{F}$ such that chordless graphs are contained in $\mathcal{F}$-free graphs.

Theorem. *Given a connected cycle-chordless graph G with $n > 3$, there exist two non-adjacent vertices of degree at most 2.*

The consequences of the theorem are the chromatic properties of the chordless graphs:

- the chromatic number of any cycle-chordless graph is at most 3,
- the chromatic number of any path-chordless graph is at most 3.

An infinite family of regular partial cubes

B. Brešar, S. Klavžar, A. Malnič, D. Marušič

A partial cube is an isometric subgraph of a hypercube. Using the well known Winkler's necessary and sufficient condition for a graph to be a partial cube, we give a construction of an infinite family of regular partial cubes not known before.

Semiregular elements in 2-closed groups of square-free degree

T. Dobson, A. Malnič, D. Marušič, L. Nowitz

The problem of existence of semiregular elements (that is, nontrivial elements with all orbits of equal size) arose first in a graph-theoretic context. In *Discrete Math.*, **36** (1981) I asked if the automorphism group of an arbitrary vertex-transitive digraph contains such an element. The now commonly accepted version of this question involves the class of 2-closed transitive groups and is due to Klin, see *Discrete Math.* **167/168** (1997). The 2-*closure* $G^{(2)}$ of a permutation group G on a finite set V is the largest subgroup of the symmetric group S_V having the same orbits on V^2 as G. The group G is said to be 2-*closed* if it coincides with $G^{(2)}$. Further, the group G is said to be *elusive* if it is transitive and contains no semiregular element, or equivalently, it contains no fixed-point-free element of prime order. (The name is due to Cameron and is intended to suggest that such groups appear to be quite rare.)

In one of the major recently published work on the subject, Giudici determined all quasiprimitive elusive permutation groups, and showed that their respective 2-closures are not elusive, see *J. London Math. Soc.* **67** (2003). (A permutation group is *quasiprimitive* if every non-trivial normal subgroup is transitive.)

In this talk I will discuss a recent result which shows that every 2-closed transitive permutation group of square-free degree is non-elusive.

An equitable partition for a Q-polynomial kite-free distance-regular graph

Štefko Miklavič

Let Γ denote a distance-regular graph with diameter $d \geq 3$. Pick an integer i ($2 \leq i \leq d$). By a *kite of length* i (or i*-kite*) we mean a 4-tuple $xyzu$ of vertices of Γ, such that x, y and z are mutually adjacent, and $\partial(u, x) = i, \partial(u, y) = \partial(u, z) = i - 1$, where ∂ denote a path-length distance function. We say Γ is *kite-free* whenever Γ has no kites of any length. Examples of kite-free distance-regular graphs include triangle-free distance-regular graphs and classical distance-regular graphs with base $b < -1$.

Assume Γ is Q-polynomial. We investigate the extent to which Γ is 1-homogeneous in the sense of Nomura [1]. We show that either Γ is 1-homogeneous, or else Γ has a certain equitable partition of its vertex set which involves $4d - 1$ cells. We will prove this using Terwilliger's "balanced set" characterization of the Q-polynomial property [2].

References

[1] K. Nomura, Homogeneous graphs and regular near polygons, *J. Combin. Theory Ser. B* **60** (1994), 63–71.

[2] P. Terwilliger, A new inequality for distance-regular graphs, *Discrete Math.* **137** (1995), 319–331.

Separation and rigidity indices of graphs

Gašper Fijavž, Bojan Mohar

For a graph G, the *separation index* is the minimum number k of vertices $a_1, \ldots, a_k$ of G such that every two distinct vertices of G are in distinct orbits with respect to the stabilizer $\mathrm{Aut}(G)_{a_i}$ for some $i \in \{1, \ldots, k\}$. This index is compared with the *rigidity index* of G which is defined as the minimum number of vertices $a_1, \ldots, a_k$ of G such that the only automorphism of G that fixes every a_i $(1 \leq i \leq k)$ is the identity.

Vince proved that the separation index of every 3-connected planar graph is at most three. Answering several questions of Vince, we show that this result can be generalized to arbitrary surfaces but only after strengthening the connectivity assumption to include only 5-connected graphs. The only surface distinct from the sphere, where 4-connectivity is sufficient is shown to be the projective plane.

The pentomino exclusion problem and generalization

Sylvain Gravier, Julien Moncel, Charles Payan

(Abstract not received.)

Signpost systems and their underlying graphs

Henry Martyn Mulder, Ladislav Nebeský

Usually, when we travel and want to find our way from u to w, we look for signposts towards w and follow these until we arrive at our destination w. That is, at u we follow the signpost to w, by which we first arrive at v. Then at v we look for the next signpost to w. We can denote this step in our journey by the triple (u, v, w): here u is the point where we currently are, w is the final destination, and v is the point where we first arrive after following the direction of the signpost at u. In this talk we present the concept of a signpost system on a set V as a ternary relation on V satisfying three basic and natural axioms. Moreover, the underlying graph of a signpost is introduced. First results and examples are discussed. Signpost systems satisfying additional signpost axioms are studied, e.g. modular signpost systems and median signpost systems. We conclude with some open problems.

On isomorphism problem for Cayley graphs

Misha Muzychuk

Let H be a finite group. A Cayley digraph $\mathsf{Cay}(H, S), S \subseteq H$ is a graph the node set of which is H and two nodes $x, y \in H$ are connected by an arc iff $xy^{-1} \in S$. An isomorphism problem for Cayley digraphs is formulated as follows.

Given two Cayley digraphs $\mathsf{Cay}(H, S)$ and $\mathsf{Cay}(H, T)$, determine whether or not $\mathsf{Cay}(H, S) \cong \mathsf{Cay}(H, T)$.

In my talk I'll review recent results related to this problem. Among them a complete solution of the isomorphism problem for circulant graphs will be presented.

Well covered graphs, decompositions and associated vector spaces

Richard Nowakowski

This talk is survey of recent results of well covered graphs and new approaches. One of the most intriguing new ideas has been extending the well covered concept to weighted graphs and considering the associated vector space. This was introduced by Caro et al. The notion has given new insights when the underlying graph is itself well covered graph.

Blanuša double

A. Orbanić, T. Pisanski, M. Randić, B. Servatius

A snark is a non-trivial cubic graph admitting no Tait coloring. We examine the structure of the two known snarks on 18 vertices, the Blanuša graph and the Blanuša double. By showing that one is of genus 1, the other of genus 2, we obtain maps on the torus and double torus which are not 4-colorable. We also prove that the 6 known snarks of order 20 are all of genus 2.

Fast recognition of subclasses of almost-median graphs

Wilfried Imrich, Alenka Lipovec, Iztok Peterin, Petra Žigert

In this talk a class of almost-median graphs where all U_{ab} are outerplanar graphs is presented. This class includes all planar median graphs and can be recognized in almost linear time, i.e. $O(m \log n)$ time. Moreover, it is shown that all bipartite outerplanar graphs are isometric subgraphs of hypercubes and that the embeddings can be obtained in linear time.

Bokowski systems of curves

Tomaž Pisanski

The notion of a pseudoline admits a natural generalization from the projecitve plane to other closed surfaces. Simple closed curves intersecting pairwise transversely are considered. The talk is derived from a joint work with Jürgen Bokowski.

Homogeneous factorisations of graphs and digraphs

Cai Heng Li, Cheryl E. Praeger

A homogeneous factorisation of a graph is a partition of the edge set that is invariant under the action of a vertex transitive group of automorphisms such that the kernel of this action is still vertex-transitive. In particular, every self-complementary vertex-transitive graph, together with its complementary graph provides an example of a homogeneous factorisation of a complete graph. It was this observation that motivated a general study of such factorisations by Li and Praeger. Much about their structure is understood. For example we have a complete classification of the parameters of the subclass of cyclic homogeneous factorisations of complete graphs. However several fundamental questions remain unanswered.

The middle row of the principal cell

Marko Razpet

The zeros of the array $\bar{D}(i,j)$ obtained from the Delannoy numbers $D(i,j)$ modulo an odd prime p give an interesting pattern. Since the numbers $D(i,j)$ satisfy the Lucas property only the zeros of the so called principal cell are relevant. The standard combinatorial expression for $D(i,j)$ in the language of lattice paths will be derived and the entries on the middle row of the principal cell will be discussed.

Transferability of graphs

Torii Ryuzo

The transferability of a graph is an invariant arisen from the movement of a path on the graph, the behavior of the path seems as the train on the railroad. I will show the relation between the transferability and several other graph invariants.

Mobility of vertex-transitive graphs

Mateja Šajna, Gabriel Verret

We define the mobility of a graph automorphism as the minimum distance between a vertex and its image under the automorphism, and the mobility of a graph as the maximum of the mobilities of its automorphisms. In this talk,we discuss the mobility of vertex-transitive graphs and, in particular, Cayley graphs.

Combinatorial pointed pseudo-triangulations

Brigitte Servatius

A combinatorial pointed pseudo-triangulation is an assignment of large and small angles to a plane graph such that every interior face has exactly 3 small angles, the outside face has only large angles, and at every vertex there is exactly one big angle. We show that not all combinatorial pointed pseudo-triangulations have geometric realizations.

These geometric realizations, called pseudo-triangulations, are straight line plane embeddings of the given graph such that every face is a pseudo-triangle (has exactly three angles smaller than π) and the outside face is convex. Pseudo-triangulations were recently introduced and used in computational geometry for visibility problems and got much attention because of the successful implementation of an efficient algorithm to straighten a polygonal arc in a non-colliding motion in the plane.

Relaxing the condition of pointedness, we prove that all planar minimally rigid graphs may combinatorially be pseudo-triangulated and the resulting pseudo-triangulations are geometrically realizable.

On scores of oriented graphs

Pirzada Shariefuddin

In this talk, the score structure in simple digraphs, extending the concept of score structure in oriented graphs, will be presented.

Infinite planar vertex-transitive and Cayley maps with a given covalence sequence

JANA ŠIAGIOVÁ, MARK E. WATKINS

Let $\mathbf{v} = (k_1, k_2, \ldots, k_d)$ be a cyclic sequence of d integers, $d \geq 3$, such that $k_i \geq 3$, $1 \leq i \leq d$. Let $\mathcal{M}(\mathbf{v})$ denote the family of all d-valent planar maps M such that for any vertex u of M the (clockwise or counterclockwise) cyclic sequence of covalences of the faces incident with u is equal to $\mathbf{v}$.

In the talk we will consider the problem of characterizing the cyclic sequences $\mathbf{v}$ for which the family $\mathcal{M}(\mathbf{v})$ contains (1) a Cayley map, or (2) a non-Cayley map whose underlying graph is a Cayley graph, or (3) a vertex-transitive map whose underlying graph is not a Cayley graph, or (4) a homogeneous map that is not vertex-transitive, or (5) no map at all. We include an algorithmic solution of the problem (based on the theory of regular covering spaces).

Classification of regular maps of negative prime Euler characteristic

ANTONIO BREDA D'AZEVEDO, ROMAN NEDELA, JOZEF ŠIRÁŇ

In the talk we give a survey on the classification of all regular maps on nonorientable surfaces with a negative odd prime Euler characteristic (equivalently, on nonorientable surfaces of genus $p + 2$ where p is an odd prime). An interesting consequence of the classification is that there are no regular maps on nonorientable surfaces of genus $p + 2$ where p is a prime such that $p \equiv 1 \pmod{12}$ and $p \neq 13$. A number of related results will be discussed.

Homomorphisms of hexagonal graphs to odd cycles

Petra Šparl, Janez Žerovnik

The problem of deciding whether an arbitrary graph G has a homomorphism into a given graph H has been widely studied and has turned out to be very difficult.

Since an n-coloring of a graph G is a homomorphism of G to K_n, the term *H-coloring* of G has been employed to describe the existence of a homomorphism of a graph G into graph H. In such a case the graph G is said to be H-colorable. Many authors have studied the complexity of the H-coloring problem. Hell and Nešetril [1] proved that H-coloring problem is NP-complete, if H is non-bipartite graph and polynomial otherwise, assuming $P \neq NP$.

We investigate a restricted H-coloring problem, where H is an odd cycle and G an arbitrary, so called, *hexagonal graph*, which is an induced subgraph of a triangular lattice. We will consider only triangle-free hexagonal graphs, because a hexagonal graph which contains a triangle is obviously C_3-colorable but it is not C_5-colorable. It is not difficult to see that the odd girth of triangle-free hexagonal graphs is at least nine, therefore it is interesting to ask whether there is a C_5, C_7, or C_9-coloring of such a graph.

We proved the following:

Theorem. *Any triangle-free hexagonal graph is C_5-colorable.*

On the other hand, there exists a triangle-free hexagonal graph which is not C_9-colorable.

Knowing that triangle-free hexagonal graphs are C_5-colorable and are not C_9-colorable, it is interesting to ask if triangle-free hexagonal graphs are C_7-colorable.

References

[1] P. Hell and J. Nešetril, On the complexity of H-colourings, *Journal of Combinatorial Theory B* 48 (1990) 92–110.

[2] P. Šparl and J. Žerovnik, Homomorphisms of hexagonal graphs to odd cycles, *Preprint Series IMFM Ljubljana* Vol. 41 (2003) No. 879.

Balanced regular Cayley maps for abelian groups

Marston Conder, Robert Jajcay, Thomas Tucker

An orientable map M is regular if its orientation-preserving automorphism group G acts regularly (transitively, without fixed points) on the set of directed edges; if G also contains a subgroup A acting regularly on the vertex set, then M is a regular Cayley map for A. If in addition, A is normal in G, then M is a balanced regular Cayley map for A. We are interested in determining which finite groups have regular Cayley maps, focussing first on abelian groups. This talk considers the balanced case, while other talks by Conder and Jajcay consider the nonbalanced case. The problem is entirely algebraic and surprisingly subtle. It entails elementary number theory (e.g. Fermat primes), a deep understanding of the dynamics of affine automorphisms of abelian groups, relationships between the canonical forms of the p-Sylow subgroups A_p of A and the exponent of the general linear group $GL(n, p)$, significant generalizations of Ito's Theorem on the commutator of factorizations AY where Y is cyclic, and even Pontrjagin duality.

We can construct large classes of abelian groups having no regular Cayley maps, for example all those such that $A_2 \cong Z_2 \times Z_4 \times Z_8$, and large classes having balanced regular Cayley maps, for example all abelian groups of odd order. We do not have a full classification of abelian groups having a regular Cayley map, or even those having a balanced regular Cayley map.

Combinatorial designs with two singular values

Edwin van Dam, Ted Spence

We present our results on combinatorial designs whose incidence matrix has two distinct singular values. These generalize 2-designs, and include uniform multiplicative designs and partial geometric designs. The bipartite incidence graphs of designs with two singular values are precisely the bipartite graphs with four and five distinct eigenvalues.

Uniform multiplicative designs are precisely the nonsingular designs. We classify all such designs with small second singular value, generalize the Bruck-Ryser-Chowla conditions, and enumerate, partly by computer, all uniform multiplicative designs on at most 30 points.

Partial geometric designs are precisely the designs with constant replication and constant block size. We give new characterization results, and we enumerate, partly by computer, all small ones.

On the domination number of the torus

Douglas M. Van Wieren

The concern here is the domination number of the torus, $\gamma(T_{m,n})$. Directly, this paper closes out a significant subset of cases, not only calculating periodic values of $\gamma(T_{m,n})$, but also providing dominating *sets* with minimal cardinality. The work here builds from a 1994 Livingston and Stout result: For any fixed value of m, the existence of a closed-form formula in n, cyclic in nature, is assured. With that expression the value of $\gamma(T_{m,n})$ can be calculated in constant time relative to n. Unfortunately, given m as a parameter, algorithms known to produce the closed-form expression in n run in *exponential* time relative to m. In brief, the related problem has an unknown complexity. The material here pursues the critical cyclic element predicted by the Livingston and Stout material (for any general m) through the evaluation of

$$F(m) = \lim_{n \longrightarrow \infty} \frac{\gamma(T_{m,n})}{n}$$

(this limit is an overall minimum on the average-cost-per-row of domination sets for the torus). Knowledge of this quantity provides explicit, periodic values of n such that $\gamma(T_{m,n}) = nF(m)$. Here, $F(m)$ is precisely calculated whenever $m \bmod 5 \neq 3$ or $m < 13$. The nature of a closed-form formula for the two-parameter case has been a matter of some conjecture. With respect to the case when $m \bmod 5 = 3$, the best bounds here suggest that such a closed-form expression for $\gamma(T_{m,n})$ would *not* be cyclic in the usual, simple sense.

On resonance graphs of catacondensed hexagonal graphs: structure, coding, and hamilton path algorithm

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The vertex set of the resonance graph of a hexagonal graph G consists of 1-factors of G, two 1-factors being adjacent whenever their symmetric difference forms the edge set of a hexagon of G. A decomposition theorem for the resonance graphs of catacondensed hexagonal graph is proved. The theorem intrinsically uses the Cartesian product of graphs. A canonical binary coding of 1-factors of catacondensed hexagonal graphs is also described. This coding together with the decomposition theorem leads to an algorithm that returns a Hamilton path of a catacondensed hexagonal graph.

Arc-coloring of directed hypergraphs and chromatic number of walls

Andrea Vietri

We define an arc-coloring for directed hypergraphs, such that any two arcs having either intersecting tails or the same head must be colored differently. Such coloring generalizes the existing coloring for digraphs. We investigate the arc-colorings of those hypergraphs which can be represented by a suitable adjacency matrix, whereas a reduction algorithm for all the other hypergraphs is constructed, so as to avail of adjacency matrices in any case. An upper bound for the least number of required colors is established for a subclass of hypergraphs. Particular types of adjacency matrices are subsequently analyzed. We also deal with a subclass of extremely regular walls. We achieve a complete classification up to the least number of required colors, among all dimensions of such walls, and all admitted incidences of the bricks. Finally, we show that many coloring problems defined above are NP-complete, and that few others are solvable in polynomial time.

Growth in Bilinski diagrams

TOMAŽ PISANSKI, MARK E. WATKINS

A *Bilinski diagram*, named for S. Bilinski [1] and used extensively in [2], of a one-ended planar map X is a rooted map in which sets of vertices and faces are labeled with respect to their regional distance from a central "root", which may be either a vertex or face. Exact computations of the rate of growth of Bilinski diagrams are made in the case where the underlying graph is edge-homogeneous, thus extending the work of J. F. Moran [3].

In this edge-homogeneous situation, the map may be identified uniquely with a 4-tuple $\langle p, q; k, \ell \rangle$, where for each edge, p and q are the valences of its two incident vertices and k and ℓ are the covalences of its two incident faces. The growth rate of any Bilinski diagram is a function only of $p + q$ and $k + \ell$, and the individual parameters affect only the initial conditions of the system of linear recurrences from which the growth rate is calculated.

Specifically, if c_n denotes the number of vertices of X at regional distance n from the root, then

$$\sum_{n=0}^{\infty} c_n x^n = s^T (I - xM)^{-1} v,$$

where M is the coefficient matrix of the system of recurrences, v is the column vector giving the initial distribution of various map objects, and s is the column vector that determines (with multiplicities) the kinds of objects to be counted at the nth level. The growth rate is the largest modulus of the eigenvalues of M.

References

[1] S. Bilinski, *Homogene Netze der Ebene*, Bull. Internat. Acad. Yougoslave Cl. Sci. Math. Phys. Tech. **2** (1949), 63–111.

[2] B. Grünbaum and G. C. Shephard, "Tilings and Patterns," W. H. Freeman, New York, 1987.

[3] J. F. Moran, *The growth rate and balance of homogeneous tilings in the hyperbolic plane*, Discrete Math. **173** (1997), 151–186.

Short cycles connectivity

Vladimir Batagelj, Matjaž Zaveršnik

Short cycle connectivity is a generalization of ordinary connectivity. Instead by path (sequence of edges), two vertices have to be connected by sequence of short cycles, where two adjacent cycles have at least one common vertex. If two adjacent cycles share at least one edge, we talk about edge short cycles connectivity. It will be shown that the short cycles connectivity is an equivalence relation on the set of vertices V, while the edge short cycles connectivity components determine an equivalence relation on the set of edges E. Efficient algorithms for determining equivalence classes will be discussed. Short cycles connectivity can be extended to directed graphs (cyclic and transitive connectivity). Instead of short cycles we can also observe connectivity by small cliques or other families of graphs. Some applications of short cycles connectivity in analysis of large networks will be presented.

A remark on n-tuple colorings of planar graphs with large odd girth

Janez Žerovnik

In [1] the existence of planar graphs with odd girth $2k+1$ and high girth that cannot be $(2k+1,k)$-colored is left as an open question. We show that such graphs exist for arbitrary large k.

References

[1] W. Klostermeyer and C. Q. Zhang, n-tuple coloring of planar graphs with large odd girth, *Graphs Combin.* 18 (2002) 119–132.

Series parallel extensions of plane graphs to dual-eulerian graphs

Arjana Žitnik

A plane graph is dual-eulerian if it has an eulerian tour with the property that the same sequence of edges also forms an eulerian tour in the dual graph. Dual-eulerian graphs are of interest in the design of CMOS VLSI circuits. Every dual-eulerian plane graph also has an eulerian Petrie (left-right) tour.

We consider series-parallel extensions of plane graphs to graphs, which have eulerian Petrie walks. We reduce several special cases of extensions to the problem of finding hamiltonian cycles.

Dominating graph bundles

Blaž Zmazek

Let $\gamma(G)$ be the domination number of a graph G. We disprove the Vizing-like conjecture on Cartesian graph bundles. Moreover, it is shown that for any $k \geq 0$ there exists a Cartesian graph bundle $B \Box_\varphi F$ such that $\gamma(B \Box_\varphi F) \leq \gamma(B)\gamma(F) - 2k$. The domination numbers of Cartesian bundles of two cycles are determined exactly if the base graph is equal C_3, C_4 or C_5, respectively. We prove that for strong graph bundles inequality $\gamma(B \boxtimes_\varphi F) \leq \gamma(B)\gamma(F)$ holds and for any $k \geq 0$ there exist graphs B and F with $\gamma(B \boxtimes_\varphi F) \leq \gamma(B)\gamma(F) - k$.