HEURISTIC SEARCH FOR HAMILTON CYCLES IN CUBIC GRAPHS

Janez ALEŠ, Bojan MOHAR and Tomaž PISANSKI
University of Ljubljana
Department of Mathematics
61111 Ljubljana
Slovenija

August 24, 1995

Abstract. A successful heuristic algorithm for finding Hamilton cycles in cubic graphs is described. Several graphs from *The Foster census of connected symmetric trivalent graphs* and all cubic Cayley graphs of the group $PSL_2(7)$ are shown to be hamiltonian.

Proposed running head. HAMILTON CYCLES IN CUBIC GRAPHS

Key words. Cayley graph, cubic graph, Hamilton cycle, projective special linear group.

Math. Subj. Class. (1985) 05C45, 05C25

1 INTRODUCTION

The following is a well known conjecture:

Conjecture 1 *Every (cubic) Cayley graph contains a Hamilton cycle.*

In this note we report a search of Hamilton cycles in several cubic Cayley graphs. In particular, we found Hamilton cycles in 27 graphs (numbered as 108, 120 B, 128 B, 162 B, 162 C, 168 C, 168 E, 192 A, 192 C, 216 B, 216 C, 240 B, 240 C, 250, 256 A, 256 B, 256 D, 288 B, 312 B, 336 A, 336 B, 360 B, 384 B, 384 C, 480 B, 112 A, 204) from [1], for which the existence of the cycles was not known before (see Section 3). Moreover, several reasons led us to examine the group $PSL_2(7)$ for a possible counter-example to Conjencture 1. The outcome speaks in favor of the conjecture: every cubic *Cayley* graph of $PSL_2(7)$ contains a Hamilton cycle (see Section 4).

2 A HEURISTIC ALGORITHM

There is a fast exact algorithm by Brendan McKay available for searching for Hamilton cycles in cubic graphs which would probably find the same results just as fast as ours. However, we used our own heuristic algorithm that is suitable for cubic graphs with several thousands of vertices.

In this section we briefly describe a successful polynomial-time heuristic algorithm which is searching for Hamilton cycles in (bridgeless) cubic graphs. The algorithm can be generalized for several other classes of graphs. It is particularly suitable for general graphs having a 2-factor. Similar approach for Hamilton cycles in a class of random multigraphs was used in [2]. The idea of the algorithm is to start with a path which is then repeatedly augmented by combining it with the remaining cycles. The following is the description of the main part of the algorithm. Let G be a bridgeless cubic graph.

1. $F_1 := a$ 1-factor of G (It exists by Petersen's theorem.)
2. $F_2 := G \backslash E(F_1)$ is a union of disjoint cycles, $F_2 = C_1 \cup \ldots \cup C_s$.
3. $C_1 :=$ a longest cycle in F_2
4. $P := C_1 \backslash \{e\}$ for some $e \in E(C_1)$
5. Repeat $s - 1$ times:

- Π := family of paths which are generated from the path P by applying the Posa transformation [4].
- $P := \widetilde{P} \cup C_j \cup \{e\} \setminus \{f\}$ for some $\widetilde{P} \in \Pi$, $C_j \in F_2$, where $C_j \cap \widetilde{P} = \emptyset$, $f \in E(C_j)$, and $e \in F_1$ is an edge joining an endpoint of $\widetilde{P}$ with an end endpoint of the edge f.

6. Π := family of paths which are generated from path P by applying the Posa transformation.

7. If for some $P \in \Pi$, the endpoints of P are adjacent in G then $C := P \cup \{e\}$ where e is the edge connecting the endpoints of P.

The *Posa transformation* [4, p. 597], [2, p. 2-3] used in steps 5 and 6 of the algorithm is the operation which takes a path $P = w_0 w_1 \dots w_k$ in a graph G and produces a family Π of paths in G and a set of vertices $W \subseteq V(P)$ such that for every vertex $w \in W$ the family Π contains a unique path with endpoints w_0 and w which uses all the vertices in $V(P)$.

The upper limit for time complexity of this algorithm is $O(n^3 \log n)$ but we can also say $O(sn^2 \log n)$ where s is the number of cycles in F_2. This is a heuristic algorithm and it may happen that it does not find a Hamilton cycle even if there is one in the graph. In practice, there was no time problem with searching for Hamilton cycles in graphs with a PC computer because it usually takes only a few seconds to get a result.

3 GRAPHS FROM THE FOSTER CENSUS

Most of the graphs from *The Foster census* [1] are Cayley graphs of a group with a given presentation with 3 involutory generators. The following are conventions for presentations from [1, pp.5-6]:

- $a^2 = b^2 = c^2 = e$, where e is the identity of the group.
- For every given relation $f(a,b,c) = e$, the relations $f(b,c,a) = f(c,a,b) = e$ hold as well.

For 76 graphs from [1] it was not known whether they are Hamiltonian or not. In 25 Cayley graphs for the given presentation, that is in all of those that we have tested, we found Hamilton cycles. In the table below the cycles are presented with sequences of generators. Starting with an identity e of the group and using the sequence of generators we get every element of the group exactly once. We also found a Hamilton cycle in graphs numbered 112 A and 204 which are 2-cover graphs.

The graphs are given by the following presentations where the conventions mentioned above are used (labels of graphs are from [1]):

1. [108] $(abc)^3 = (ab)^2c(ba)^2c = e,$
$((ab)^2ac)^2(ba)^2bc(ba)^5bc(ac)^2(ba)^5bc(ba)^2bc(ab)^2ac(ab)^4ac(ab)^5ac(ba)^2bc$
$(ba)^2bc(ab)^5ac(ab)^2$

2. $[120B]$ $(ab)^5 = abcba.cbabc = e,$
$ca(ba)^2cbc(ab)^3c(ab)^4(ac)^4(abc)^2bac(ab)^3c(ab)^4(ac)^2abac(ab)^3(acab)^2$
$(ab)^2ac(ba)^3(cb)^2abac(ab)^2ac(ba)^2cabc(ab)^2acba$

3. $[128B]$ $ababc.babac = (abc.acb)^2 = e,$
$((ab)^7cb(ab)^7ca(ba)^7ca(ba)^7cb)^2$

4. $[162B]$ $(abc)^6 = (abac)^2babc = e,$
$((ab)^8ac)^2((ba)^2ca)^2bc(ab)^3cabacbac(ab)^5cb(ab)^7cbabacabca(ba)^2cb(ab)^2$
$(ac)^2bcac(ab)^3ca(ba)^2cabacabc(ba)^8bcbacabc$

5. $[162C]$ $(abc)^2(acb)^2 = (abcb)^3 = e,$
$bcabc(ab)^8ac(ba)^7cbacabac(ba)^7cb(ab)^8cabca(ca(ba)^2)^3cacb(ab)^6ac(ba)^8$
$b(cab)^2aca((ba)^2ca)^3$

6. $[168C]$ $(ab)^4 = (abc)^7 = (abac)^3 = e,$
$ba(bc)^3babc(ba(bc)^2)^2abca(cb)^3cacba(bc)^3bacbcabca(bc)^2bab(a(cb)^2)^2$
$abcba(bc)^3bacbc(ac)^2b(ab)^3(cb)^3ac(abac(bc)^2)^2bc(abac)^2bab(cb)^3$
$abacba(cb)^2cacba(bc)^3(bac)^3$

7. $[168E]$ $abab.acab.cbac = e,$
$(ab)^7c((ab)^8cb)^2(ab)^2ac(ab)^5ac(ba)^6c(ab)^2c$
$(ab)^3ac(ba)^2c(ab)^6ca(ba)^5c(ab)^3c(ba)^8bc(ba)^8cb$

8. $[192A]$ $(abac)^2 = e,$
$((ab)^{11}ac)^2(ab)^8ca(ba)^{11}ca(ba)^9cbca((ba)^2bc)^3(ba)^3cbac(ba)^7bc(ba)^7cbac$
$abac(ba)^6bc$

9. $[192C]$ $(abc)^4 = (ab)^bca(cb)^2ac = e,$
$((ab)^{11}ac)^2(ab)^9ac(ba)^{11}b(ca)^2bac(ba)^4cbab(cbac(ba)^4)^2ca(ba)^4ca(ba)^3bc$
$ab(ac)^2(ba)^{10}cb(ab)^4ac$

10. $[216B]$ $(ab)^6 = (ab)^2c.(ba)^3c = e$,
$(ba)^5bcac((ab)^5ac)^2((ba)^5bc)^2bc(ab)^5acbc(ab)^5ac(ba)^5(bc(ba)^4)^5babc(ab)^5$
$acbcac(ba)^5bc(ab)^5(ac(ab)^4)^2acbc$

11. $[216C]$ $(abc)^4 = (ab)^3(ac)^3 = e$,
$((ab)^8ca(ba)^8cb)^6$

12. $[240B]$ $(abc)^5 = abcbcacab = e$,
$(bc)^6ba(bc)^5(ba(bc)^6)^2ac(bc)^7acbabc((bc)^5ba)^2(bc)^3bac(bca)^2bcbac(bc)^8$
$abcba(cb)^2(ab)^2c(ba(bc)^9)^3babc(ba)^3cbca(bc)^3(abcbac)^2$

13. $[240C]$ $abcba.cbabc = e$,
$(ab)^5(cb(ab)^4)^2ca(ba)^4(bc)^2(ab)^9ac(ab)^7ca(ba)^2(bc)^2(ba)^3cb(ab)^9cbac$
$(ab)^9(ac)^2(ab)^3ac(ba)^3(bc)^2(ba)^9bc(ba)^6ca(ba)^3bc(ba)^8bc(ab)^5ac(ba)^3bc$

14. $[250]$ $(ab)^5 = (abc)^2(acb)^2 = e$,
$ac(bc)^4(ab)^2(cb)^4acbab(cb)^2abacbab(cb)^4ac(bc)^2ab(cb)^4a(bc)^2abacbaca$
$(cb)^2a(bc)^2acbca(bc)^2ac(bc)^4acba(bc)^2ab(cb)^4(ac)^2a(cb)^2abac(bc)^2$
$abaca(bc)^2abcab(cb)^3a(cb)^3acabcabac(bc)^3a(bc)^3$
$(ab)^2a(cb)^2abcbabacab(cb)^2acbabacacbaca(bc)^2abcbac(bc)^4abcba(cbcb)^2$

15. $[256A]$ $(ab)^4 = (aba.cbc)^2 = e$,
$ca(ba)^2cbca(ba)^3cababcabac(ab)^3acbc(ab)^2(abac)^2bab(cba)^2bc(ba)^2ca$
$(ba)^3(ca)^2bc(ab)^3ac(ab)^3acbabc(ab)^3acba(baca)^2(ba)^3cb(ca)^2cbacabc$
$(ab)^3ac((ba)^3bc)^2(ab)^2ac(ab)^2cb(ab)^3ca(ba)^3cb(ab)^3cbabcabaca(ba)^2cbc$
$abc(ab)^3ac(ab)^3ac(ba)^3bc(ab)^3acbc(ba)^2bc(ba)^2$

16. $[256B]$ $(abc)^4 = ababc.babac = e$,
$abcb(ab)^7ca(ba)^6cb(ab)^4acbc(ab)^6ac(ba)^7bcab(cb)^2a(bc)^2baca(ba)^7ca(ba)^7$
$cb(ab)^2(ac)^2bacb(abcb)^2abac(ba)^3ca(ba)^4cabc(ba)^3bcab((ab)^3cb)^3c(ab)^7ac$
$(ba)^7bc(ba)^6bc(ba)^2bc$

17. $[256D]$ $(ac)^2(abcb)^2 = e$,
$c(ab)^3cbcac((ba)^2bc)^2(ab)^4ac(ab)^3cabc(ab)^7ac(ab)^6aca(bca)^3cb(ab)^2c$
$(ab)^4(cba)^2ca(ba)^5cbacbc(ba)^5(ca)^2(ba)^7cbac(ba)^3ca(ba)^4cbabacba(bc)^2$
$(ab)^4cbabac((ab)^2cb)^2(ab)^5c(ba)^6cbacabc(ab)^6c(ba)^2$

18. $[288B]$ $(abc)^4 = (ab)^6 = (ab)^2(ca)^2(bc)^2 = e$,
$ab(cb)^5abcaca(cb)^2ab(cb)^2abcbabc(a(bc)^5b)^2ac(bc)^5acab(cb)^5a(bc)^4ac(bc)^5$
$a(bc)^5ba(cb)^2a(cb)^3acab(cb)^2(acb)^2a(bc)^5bacba(cb)^4(a(cb)^2)^2a(bc)^5$
$ba(bc)^4a(cb)^4a(bc)^2a(bc)^3aba(bc)^2bac(bc)^2a(bc)^3ac(bc)^2ab(cb)^2$
$a(cb)^2acbabca(bc)^5bac(bc)^5$

19. $[312B]$ $(abc)^4 = (ab)^4cbabcb = e$,
$(ab)^8cb(ab)^8c(ab)^9c(ba)^8bc(ba)^{20}b(cb(ab)^8)^2ac(ba)^8(bc(ba)^8))^2(ba)^3ca$
$(ba)^{38}ca(ba)^{10}bcb$

20. $[336A]$ $(ab)^4 = (abac)^2bcacbc = e$,
$(abc)^2abacabc(acb)^3cacb(ac)^2b(ac)^3a(bc)^2a(bc)^3b(ab)^3caba(cb)^3c(ac)^2bcabc$
$(ba)^2(bc)^2abcacbac(ab)^2(cb)^2abcacabcbabca(cb)^2c(abc)^3(bca)^2cbacabc$
$acbabcba(cb)^2cabcaca(bc)^2a(bc)^2baca(bc)^2b(abc)^3(bc)^2ba(bc)^2acabcacbaca$
$(cb)^2ca(cb)^3c(abcb)^2acb(ca)^2bacab(ac)^2(abc)^2(bc)^2bacbca(bc)^2(abac)^2(bc)^2$
$ab(acb)^3cbca(cb)^2a(cba)^2cbca(bc)^2acbcabcacacbcac(acb)^2cbca(bc)^2$

21. $[336B]$ $(ab)^4 = (abac)^3 = e$,
$(ab)^2cbaca(bc)^3ba(bc)^2bac(bc)^2(ab)^2ac(bc)^3(ac)^3babcbac(bc)^2acbacaca(bc)^3$
$babac(bc)^2abacbcabacabcbacbcacb(ac)^2abac(bc)^2abc(ba)^2bcacbc(ac)^2ba(bc)^3$
$b(ac)^2a(bc)^2acacbcacacbcac(bc)^2ac(abcbacbc)^2acbacab(ac(bc)^2)^2(ab)^2(ac)^2$
$bcabacab(cb)^2acba(bc)^2b(acb)^2cbcac(bc)^2acba(bc)^3babaca(bc)^3babcb$
$ababcbabacbcac(bc)^2a(bc)^2babc(ba)^2(bc)^2bacabacb(ac)^2abacbac(bc)^3$

22. $[360B]$ $(abac)^3 = (ab)^2ac(ba)^2bc = e$,
$((bc)^{14}ba(cb)^{11}a(bc)^{14}ba(cb)^{14}ca(cb)^{11}a(cb)^{14}ca((cb)^3a)^2)^2$

23. $[384B]$ $(ab)^6 = (abac)^4 = (ab)^2(ca)^2(bc)^2 = e$,
$(ac)^3b(a(cb)^2c)^2(ab)^2cacba(bc)^2ba(cb)^3(a(bc)^2b)^2(ab)^2c(ac)^2babcbacb(abc)^2$
$acbacabcba(cb)^2abca(cb)^4a(bc)^4ba(bc)^4abcba(cb)^2ca(bc)^3acb(ab)^2(acb)^2$
$(cb)^2ab(acbc)^2(bc)^4ab(ac)^3(bc)^3ac(bc)^3ab(acb)^3(cb)^3ca(bc)^4(ba)^3(cb)^4c$
$(a(cb)^5c)^2aba(cb)^5cacb(ca)^2(cb)^4a(bc)^2abcbabc(a(bc)^2b)^3acbc(ac)^2b$
$a(bc)^4(ab)^2cacbabacbabcabacb(ab)^2(cb)^3ca(bc)^3babcbabc$

24. $[384C]$ $(abc)^8 = (ab)^4(ac)^4 = (abc)^2(acb)^2 = e$,
$(((bc)^{23}ba)^2((cb)^{23}ca)^2)^2$

25. $[480B]$ $(ababac)^2 = (abacac)^2 = (ab)^{10} = e$,
$(ba)^9bc(ab)^5ac(ba)^9b(ca)^3bc(ab)^2c(ab)^7a(c(ab)^3)^2acabc(ab)^4c(ab)^3acbc$
$(ba)^5c(ab)^4(cbac(ab)^5)^2(ab)^2cab(abc)^2(ab)^4ac(ab)^3c(ab)^2c(ba)^2c(ba)^3$
$bcac(ab)^5ac(ba)^2bc(ba)^2cabc(ab)^4c(ab)^8(ac)^2(ab)^4c(ab)^8a(cb)^2(ab)^2c$
$(ab)^7cb(ac)^2abc(ba)^2c(ab)^2ac(ba)^2bc(ba)^2c(ab)^2ac(ba)^3(ca)^2(ba)^3bc(ac)^2$
$(ba)^9bc(ab)^5ac((ba)^6bc)^2(ab)^3c(ba)^3c(ba)^2(cb)^2c(ab)^3ac(ab)^5ac$

4 CUBIC CAYLEY GRAPHS OF $PSL_2(7)$

For q a *prime* number, $SL_2(q)$ is the *special linear group* of all 2×2 matrices with entries in Z_q and having the determinant equal to 1_q. The *projective linear group* $PSL_2(q) := SL_2(q)/\{\pm I\}$ is the $SL_2(q)$ modulo its center. The number of elements in $PSL_2(q)$ is equal to $q(q^2-1)/2$.

The group $PSL_2(7)$ is one of the smallest "strange" groups. Due to some recent discoveries [5, 6] we expected that this group might give rise to a possible counterexample to Conjencture 1. We generated all possible cubic Cayley graphs of this group, and surprisingly enough (for us) we found the following result after testing the obtained graphs for hamiltonicity:

Theorem 1 *Every cubic Cayley graph of $PSL_2(7)$ contains a Hamilton cycle.*

To generate all possible cubic Cayley graphs of $PSL_2(7)$ we need the following result, which can be easily proved:

Theorem 2 *All elements of order 2 in the group $PSL_2(7)$ are pairwise conjugate.*

We first checked all pairs $\{a, k\} \subseteq PSL_2(7)$ where $a^2 = 1$ and k is of order ≥ 3. Up to conjugacy there are exactly 21 such pairs to be considered and only 10 of them generate the whole group. Among these there are three pairs $\{a, k\}$ and $\{a, k^{-1}\}$, each pair giving the same Cayley graph. The representatives a and k for the remaining 7 graphs are:

1. $a = \begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix} \quad k = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

2. $a = \begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix} \quad k = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

3. $a = \begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix} \quad k = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

4. $a = \begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix} \quad k = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

5. $a = \begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix} \quad k = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$

$$6. \quad a = \begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix} \quad k = \begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix}$$

$$7. \quad a = \begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix} \quad k = \begin{bmatrix} 1 & 1 \\ 6 & 0 \end{bmatrix}$$

Hamilton cycles for cubic Cayley graphs of the group $PSL_2(7)$ are presented with the following sequences of generators where $h = k^{-1}$:

1. $ak(ah^4)^2(ah)^2hah^4ak^2(ah)^2ak^3(ah^2)^2h^2(ah^2)^2ak^2ah^2(ah)^2(haka)^2k$ $(ah^4ah)^2(ah^2)^2((ak)^2ah)^2akah^2akah^2ak(kaha)^2hak^2(ah^2)^2akah^2$ $(akah)^3ak^2ah^2akah(ahak)^2$

2. $ah(ak)^2(ka)^2h^2ah(ak)^3ah^3(ah)^3h^2ahak^2akah^2ak^6(ah^2)^2ak(ah^2ak)^2ah$ $(ha)^2kak^2ah^2ak^2(ah)^4ha(ka)^4(ha)^2h^2(ak)^2k^3ah^2ak^2(ak)^3(ka)^2k^2ak$ $(ah^2)^3h^4ah^3ah^6akahak^4$

3. $(ak)^2ka(ha)^2kak^4ak^2(ah)^2h^3(ak)^2kak^2(ahak)^2kah^2(ah)^3(ha)^2kah^4$ $(ahak)^2akah^3ah^2akah^2ahah^3(ah)^3hah^2ak^4ak^2(ah)^2(akah)^2hak(ah^2)^2$ $ahak^2(akah)^2ah(ak)^3(ah^2)^2(ha)^2h^2(ak^2)^2$

4. $(ak)^3ah(ak)^2ah(ak)^3(ah)^2(ak)^2(ah)^4(ak)^2ahak(ah)^4(ak)^6(ah)^3(akah)^3$ $(ah)^2(ak)^2(ah)^2akah(ak)^2ah(ak)^4ah(ak)^4(ah)^2(ak)^5(ahak)^2(ak)^2(ah)^3$ $(akah)^3ah$

5. $(akah)^2(ak)^2(ah)^3(ak)^2ah(ak)^3(ah)^3ak(ah)^4(ak)^2ah(ak)^2(ah)^2(ak)^4$ $(ah)^2ak(ah)^2akah(ak)^3(ahak)^2(ak(ah)^3)^2ahak(ah)^2ak(ah)^5(ak)^4ah$ $(ak)^4(ah)^3(ak)^4ah$

6. $h^2(ak^2)^2(ah^2ak^2)^2(ah^2)^2ak^2(ah^2)^5ak^2ah^2(ak^2)^2ah^2ak^2(ah^2)^5ak^2(ah^2)^2$ $(ak^2)^3ah^2ak^2(ah^2)^3(ak^2)^2ah^2(ak^2)^5ah^2ak^2(ah^2)^2ak^2ah^2(ak^2)^5a$

7. $ah^2ak^2(ah^2)^2(ak^2)^2(ah^2)^5(ak^2)^3(ah^2)^3(ak^2ah^2)^2(ak^2)^2ah^2(ak^2)^2ah^2$ $(ak^2)^3ah^2(ah^2ak^2)^3(ak^2)^2ak^2(ah^2)^2(ah^2ak^2)^2((ak^2)^2ah^2)^2(ak^2)^5$

The other possibility to get a cubic Cayley graph is to use three involutions. Up to conjugacy there are only 17 (unordered) triples of distinct non-trivial involutions and only 3 of them generate the whole group. The representatives a, b and c for them are:

$$1. \quad a = \begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix} \quad b = \begin{bmatrix} 1 & 2 \\ 6 & 6 \end{bmatrix} \quad c = \begin{bmatrix} 1 & 4 \\ 3 & 6 \end{bmatrix}$$

2. $a = \begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix}$ $b = \begin{bmatrix} 1 & 2 \\ 6 & 6 \end{bmatrix}$ $c = \begin{bmatrix} 2 & 2 \\ 1 & 5 \end{bmatrix}$

3. $a = \begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix}$ $b = \begin{bmatrix} 1 & 2 \\ 6 & 6 \end{bmatrix}$ $c = \begin{bmatrix} 3 & 1 \\ 4 & 4 \end{bmatrix}$

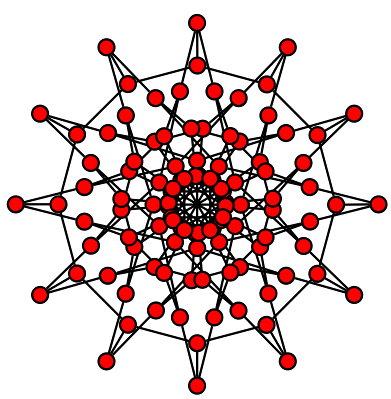
Hamilton cycles for the coresponding cubic Cayley graphs are:

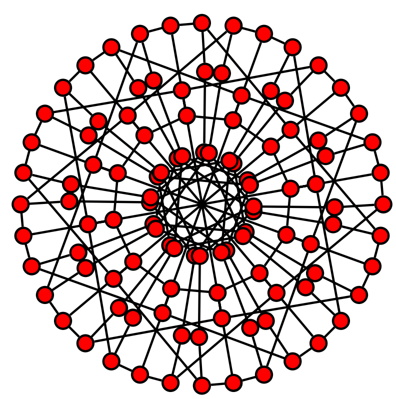
1. $(ab)^2c(ba)^2cbacaba(cb)^2ca(bc)^2ba(bc(ba)^2)^2b(acbc(ab)^2)^2ab(ac)^2$
$abcbac(bc)^2acbcabac((ba)^3bc)^2a(bc)^2bacbc(ab)^3(ac)^3(ab)^2c(ba)^2b(ca)^2cbc$
$a(bc(ba)^2)^2babc(ba)^2cabc(bcba)^2b$

2. $((ab)^3ac)^3(ab)^2ac((ab)^3ac)^2(ab)^2cbab((ab)^2cb)^2ac(ab)^3ac(ab)^2ac$
$((ab)^3ac)^3acabc(ba)^3(bcba)^2bacac((ab)^3ac)^3(abcb)^2cb(ac)^2$

3. $ac(ba)^3bc(ba)^2(bc)^2(ba)^3(bc)^2(ba)^2(bc)^2(ba)^3(bcba)^3(ba)^2(bcba)^2(bc)^2$
$(ba)^2bc(ab)^3ac(ba)^3bcabcb(ab)^2abcbac(bc)^2abcb(ab)^2ab(cba)^2(bc)^2$
$(ba)^2bc(ba)^2bc(ab)^3acbabcac(bc)^2ac(bc)^2ac(bc)^2$

In order to give the reader an idea of how a graph from the Foster Census looks like we present here two different views of the same graph 108. The figures were produced by the system Vega [7].





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