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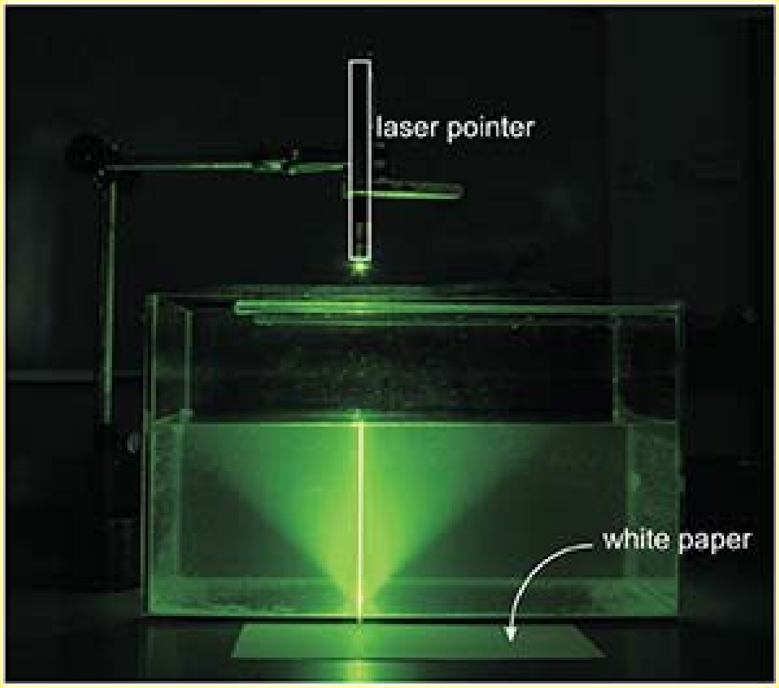

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A simple optics experiment to engage students in scientific inquiry

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A cone of light appears in a tank of water when a laser pointer shines through the water onto a white piece of paper upon which the tank is sitting. We describe how students can understand the origins of this cone by constructing multiple explanations, then proposing and designing experiments to test their explanations. This process is the foundation of the Investigative Science Learning Environment (ISLE) framework, designed to engage students in the reasoning activities similar to those that physicists use to construct and apply new knowledge. We describe typical student ideas and provide a list of equipment and suggestions for facilitating student exploration relating to optics. We also explain the formal physics behind the phenomena that are involved in the experiment. Finally, we suggest how the ISLE framework can be used to help instructors find problems and experiments that engage students in devising and testing multiple explanations.

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I. INTRODUCTION

Perhaps, the heart of physics is the ability to solve complex physical problems that may include elements of experimentation. However, this ability requires students to learn how to think and reason like physicists. In this manuscript, we provide an example of an experimental problem that students can solve using straightforward physics, while simultaneously developing the skill of hypothetico-deductive reasoning[1] and experimental testing of hypotheses. We will use Investigative Science Learning Environment (ISLE)[2] as an educational framework guiding this learning process. ISLE engages students in processes that mirror scientific practice in order to help them learn physics. Specifically, students start learning a new concept by observing a few very simple experiments (called observational experiments). They then identify patterns, develop multiple explanations for those patterns, and finally test their explanations (with the purpose of ruling them out). The first step in testing their explanations involves designing a new experiment, the outcome of which they can predict using their explanations; the second step is to conduct the experiment, and third, they compare their predictions to the outcomes of the testing experiment (see Fig. 1). This purposeful testing of proposed explanations using hypothetico-deductive reasoning is one of the most important features of ISLE, which in turn directly reflects common reasoning in science and, in particular, in experimental physics. Often the unexpected outcome of a testing experiment serves as an observational experiment for a new cycle.

Although the ISLE framework was developed to help students construct new concepts,[1] our research shows that it can be successfully utilized when students apply the concepts that they have already constructed to analyze complex phenomena.[3] In this manuscript, we describe an example of such an application using an experiment involving light. Light phenomena are usually suitable for educational investigation projects as many of these experiments can be correctly explained using only knowledge of basic physics, and most of the equipment is inexpensive.

For the purposes of this paper, it is important to clarify the difference between observational and testing experiments in the ISLE process. Observational experiments serve as "explanation-generating," and testing experiments serve as "explanation-testing." When students perform or observe the former, they do not make any predictions. The goal is to observe and describe the phenomenon in detail, collect any relevant data, and use tools (graphs, force diagrams, energy-bar charts, ray diagrams, etc.) to analyze the data and find patterns. Students subsequently devise explanations for the patterns, keeping in mind that the more explanations they generate, the higher will be the probability that one (or more) of them will "survive" future testing experiments. For the testing experiments, students make predictions before performing or observing them, but these predictions are not based on intuition or "gut feeling"; they must be based on the explanations that the students are testing. When multiple explanations are present, the students have to make a prediction on the outcome of the testing experiment based on each available explanation. The next step is to match the prediction with the outcome of the testing experiment. Another important aspect of the predictions is that in addition to the explanation under test, scientists often use auxiliary assumptions. For example, when one is testing the projectile range equation, some auxiliary assumptions are that the air resistance does not affect the motion of the projectile and the gravitational field strength g is independent of height. If the assumptions are not valid, the outcome of the testing experiment will not match the prediction even if the explanation on the basis of which the prediction was made, was correct. To avoid this situation, one needs to be aware of the assumptions and be able to validate them experimentally.

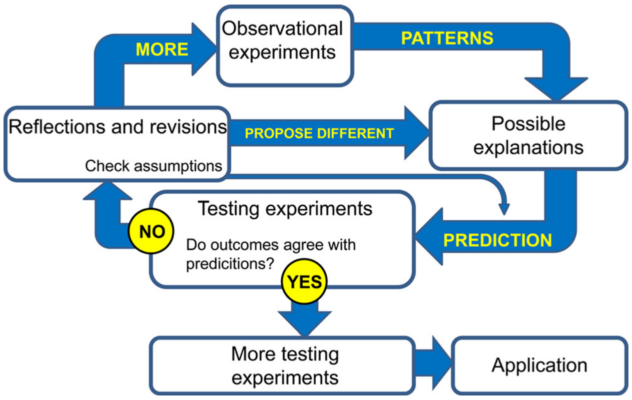

Fig. 1. Investigative Science Learning Environment (ISLE) cycle.

II. STUDENTS APPLYING ISLE IN SOLVING AN EXPERIMENTAL OPTICS PROBLEM

In this paper, we describe an optics experiment that allows the observers to generate multiple explanations for their initial observations and then to test these explanations by performing new experiments. The activity should be conducted after students have learned about geometrical optics phenomena, in particular reflection, refraction, and total internal reflection. It is suitable for physics courses at any level (beginning high school and up to graduate-level classes dealing with wave optics). The difference lies in the level of sophistication of the exploration that the students conduct. The students do not need to be familiar with the ISLE cycle, but they would progress through the problem faster if the course was ISLE based. The paper first describes the experiment and then shares explanations and testing experiments proposed by the participants. This is followed by the "correct" explanation at the elementary level, at which point the subject matter is explored in greater depth. The paper concludes with pedagogical implications and suggestions.

A. The basic phenomena

In the experiment, a light beam (preferably a laser beam), is directed vertically downward into a partially filled glass aquarium tank sitting on top of a flat surface, with a piece of white paper inserted between the table and outer-bottom of the tank. A clearly visible cone with an apex at the bottom of the aquarium can be observed (see Fig. 2).

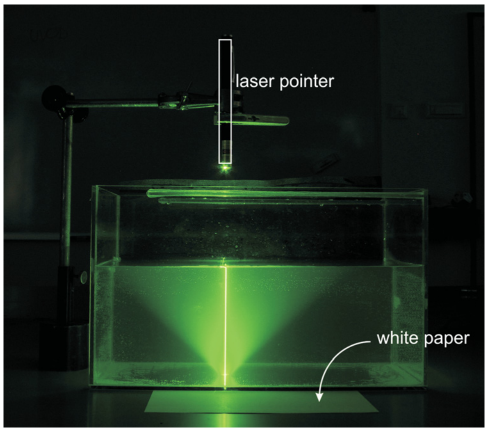

Fig. 2. Initial observation of the light cone.

The students need to first notice the cone, and then attempt to explain how the cone is formed (detailed instructions given to the students are presented in Sec. V). As many explanations are possible, the students need to devise a mechanism to rule some out, i.e., to test the explanations. Since a cone of light is visible in this experiment, we will hereafter use the expression "light cone" or "light cone experiment" (with no relationship to the light cone of special relativity).

B. ISLE procedures

We suggest that students work in groups both when initially observing the light cone and when devising and testing the explanations. Different groups will likely come up with different explanations. The instructor should encourage each group to come up with more than one explanation and use the "seeding" technique[4]—when the teacher focuses the group's attention on a particular idea among several ideas the group may have—to avoid the situation that several groups come up with exactly the same explanations or when other possible explanations do not appear. After participants devise their explanations, they have an opportunity to test them by asking for additional equipment that was prepared in advance. We have run this activity multiple times and have compiled a list of equipment necessary to test several "usual" explanations that might arise. Participants can test their proposed explanations one by one, or make a list and test them all at once. They request the equipment from the instructor and conduct the experiments. It is very important that the students write down the explanations, proposals for testing experiments, predictions of their outcomes based on each explanation, and subsequently, the outcomes themselves. It is crucial that they make predictions based on the explanations before conducting the experiments. Finally, each group needs to make a judgment related to the best explanation.

After group work is done, it is best to bring the groups together and let students present their findings. To avoid repetition, the instructor can circulate among the groups and be familiar with their explanations and testing experiments and then encourage the groups to present "complementary" results, so each group can share a unique idea. Finally, the instructor can summarize the explanations and share the accepted explanation at the level appropriate to the audience. This approach allows the students first to "innovate" and then learn the normative content. This sequence—innovation first, "time for telling" second—was found to be a very effective approach to instruction.[5]

C. Equipment needed for the observational experiment

Students will need to make observations in a darkened room with a glass or plastic tank filled with water (a smaller container will work if each group conducts observations individually). It is important that the sides of the tank (including the bottom) are flat and made of clear, transparent material. The bottom of the tank should be flat, without any dents or edges, so that the whole bottom surface is in contact with the table when you place the tank on it.

Add a few drops of milk or water-based glow-in-the-dark paint (available at any "arts and crafts" store) to the water in order to make the light beams more visible. Note that the presence (or color) of the paint does not affect the outcome of the experiment; it simply improves visibility by increasing the scattering of the light. Next, fix a laser pointer on a stand

POSSIBLE EXPLANATIONS	TESTING EXPERIMENTS	PREDICTION OF OUTCOME OF TESTING EXPERIMENT BASED ON THE EXPLANATION UNDER TEST	OUTCOME OF EXPERIMENT AND CONCLUSION
Explanation E_1 Something in the structure of the paper reflects or redirects the laser beam into the light	Testing Experiment T_1 - Point laser directly and only at the paper. - Use chalk dust to observe light scattered from the paper.	Prediction P_{11} If E_1 is correct, the cone should be visible in the air. Prediction P_{12} If E_2 , E_3 and E_4 are correct, no cone will be visible because of uniform scattering.	No cone is visible ⬇ Reject E_1
	Testing Experiment T_2 - Use two sheets of paper; one having a hole in the middle, the other being untouched. - Place the paper with the hole above the plain paper. - Point laser downward through the hole. - Observe the light on the upper paper.	Prediction P_{21} If E_1 is correct, a circle should be visible on the upper sheet of paper. Prediction P_{22} If E_2 , E_3 and E_4 are correct, the upper paper will be uniformly lit because of uniform scattering.	The upper sheet of paper is uniformly lit ⬇ Reject E_1
Explanation E_2 The paper is not important. The light cone is created due to imperfections on the glass surface that is in contact with water.	Testing Experiment T_3 (For plastic tank only!) - Sand a small area of one of the inner surface of the bottom of the tank with a very fine sand paper. - Point laser directly at the sanded surface.	Prediction P_{31} If E_2 is correct, the cone should become even more visible. Prediction P_{32} If E_1 , E_3 and E_4 are correct, no cone will be visible.	No cone is visible. ⬇ Reject E_2
	Testing Experiment T_4 (For plastic tank only!) - Sand a small area of one of the outer surface of the bottom of the tank with a very fine sand paper. - Point laser directly at the sanded surface.	Prediction P_{41} If E_2 and E_3 are correct, the cone will be visible. Prediction P_{42} If E_1 and E_4 are correct, no cone will be visible.	No cone is visible. ⬇ Reject E_2 and E_3
Explanation E_3 The paper backscatters light uniformly into the glass, in half space. The light cone is the result of total internal reflection at the glass-water boundary.	Testing Experiment T_5 - Repeat original experiment using a different material that scatters less than white paper (e.g. colored paper, wood, plastic etc.).	Prediction P_{51} If E_1 , E_3 and E_4 are correct, the cone will look dimmer or will not be visible at all. Prediction P_{52} If E_2 is correct, the cone will look as bright as in the original experiment.	Cone is dimmer or not visible ⬇ Reject E_2
	Testing Experiment T_6 - Put a thick slab of glass or transparent plastic on the paper and aim the laser vertically down on the slab	Prediction P_{61} If E_1 and E_4 are correct, the cone will be visible in the plastic. Note: you should previously test that the laser beam is visible in the plastic/glass. Prediction P_{62} If E_2 and E_3 are correct, no cone will be visible.	Cone is visible. Measured cone angle is smaller than the original ⬇ Reject E_2 and E_3 Measurements support E_4
Explanation E_4 The paper backscatters light uniformly into the glass, in half space. The layer of air between the paper and the glass is important. The light cone is the result of refraction occurring at the air-glass boundary. Refraction at the glass-water boundary only modifies the cone angle.	Testing Experiment T_7 - Repeat light cone experiment using a liquid with a refraction index similar to that of glass Suggestion: use sugar-saturated water solution.	Prediction P_{71} If E_1 and E_2 are correct, the cone will still be visible and the cone angle should be the same as the original one. Prediction P_{72} If E_3 is correct, no cone or a cone with apex close to 90º should be visible. Prediction P_{73} If E_4 is correct, cone will still be visible but the apex angle will be smaller than in the original experiment.	Cone is visible. Measured cone angle is smaller than the original ⬇ Reject E_1, E_2 and E_3 Measurements support E_4
	Testing Experiment T_8 - Repeat light cone experiment inserting a layer of water between the paper and the bottom of the water tank.	Prediction P_{81} If E_1 is correct, cone will still be visible. Assumption: water does not change optical properties of paper. Prediction P_{82} If E_2 and E_3 are correct, cone will still be visible. Prediction P_{83} If E_4 is correct, no cone will be visible.	No cone is visible. ⬇ Reject E_1 ,E_2 and E_3 Outcome supports E_4

Fig. 3. Common student-proposed explanations, outcomes of testing experiments, and conclusions.

above the tank, pointing vertically down, as shown in Fig. 2. We recommend using a green laser, although the experiment will work with a red laser or even with a flashlight that produces a collimated beam. A clothespin can be used to keep the laser on for a longer time. If this experiment is the first one involving lasers, prepare and review suitable safety instructions. Place a white sheet of paper under the bottom of the tank so that the laser beam is incident on it. The light cone should be clearly visible from the side.

D. Equipment needed for the testing experiments

The equipment will depend on the testing experiment ideas that students propose. Short descriptions of common testing experiments are summarized in the second column of Fig. 3. Typically, most of the students' explanations will fall into the categories described in Fig. 3. Additional notes on the equipment are provided in Ref. 6.

III. LIGHT CONE REVEALED

In this section, we will explain the physics of the light cone. We start with the basic explanation, suitable for high-school students and then gradually address more complex phenomena that are typically treated at the graduate level.

A. Basic explanation

We assume that the light from the laser beam, which is incident on the white paper, scatters in all directions. It is important to note that there is a thin layer of air between the paper and the bottom of the tank. When scattered light re-enters the tank it undergoes two refractions, one at the air-glass interface and one at the glass-water interface. Light rays that scatter from the paper and are parallel to the air-glass interface are refracted into the glass at critical angle α_{gl} (Fig. 4); all other rays are refracted at smaller angles (in the basic explanation we neglect the fact that some light is always reflected at the boundary between two media). Thus, the apex angle of the cone is determined by the rays that

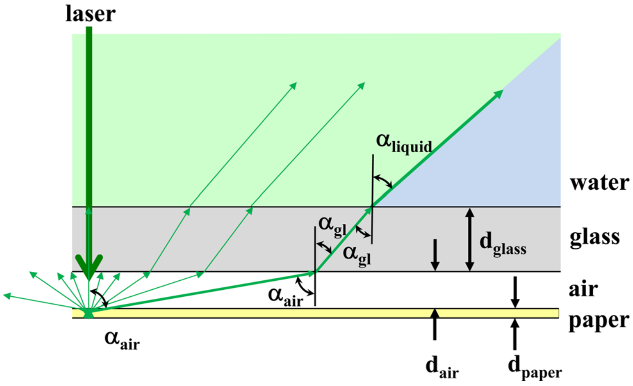

Fig. 4. Schematic diagram showing how the light cone is formed (dimensions not to scale). We use the notation α_{gl} for the critical angle even though the respective schematic light ray still has a $\alpha_{air} < 90^{\circ}$ in the sketch.

undergo refraction first from air to glass at the critical angle α_{gl} and second from glass to water at angle α_{liquid}.

For refraction at the air-glass interface the critical angle is given by

$$\sin\alpha_{gl} = \frac{n_{air}}{n_{glass}}, \tag{1}$$

and Snell's Law for refraction at the glass-liquid interface is

$$n_{liquid}\sin\alpha_{liquid} = n_{glass}\sin\alpha_{gl}. \tag{2}$$

Combining these two equations for the critical condition, we find that the cone angle α_{liquid} observed in the water satisfies

$$\sin\alpha_{liquid} = \frac{n_{air}}{n_{glass}}. \tag{3}$$

From this simple analysis, we can derive the following conclusions:

- The cone angle α_{liquid} is determined only by the index of refraction of the liquid in the tank n_{liquid} (water in this case) and the index of refraction of the medium between the paper and the tank n_{air} (air in this case). The cone angle α_{liquid} is independent of the index of refraction of the tank wall material.
- The cone angle α_{liquid} is independent of the wall thickness.
- The larger the index of refraction of the liquid in the tank, the smaller the cone angle.

Using the values $n_{air} = 1.0$, $n_{glass} = 1.5$, and $n_{liquid} = 1.33$ for water, we find $\alpha_{gl} = 41.8°$ and $\alpha_{liquid} = 48.6°$. The calculated value of the total cone apex angle $2\alpha_{gl}$ is therefore 97.2°, which agrees with the observation (see Fig. 2.)

B. Going deeper

In order to analyze the phenomenon in detail, we first list potential parameters:

- Properties of the light source (spectrum of light, degree of divergence of the light beam),
- properties of the light scattering material (dry or wet paper, plastic, etc., color of the material),
- quality of walls of fish tank (polished or rough surfaces),
- geometry of the set up (illumination from top or side, widths and thicknesses of materials), and
- properties of other materials involved (air gap, tank walls, type of liquid, scattering particles within liquid).

Due to space limitation we will only discuss a few of these. For a laser, we can safely assume collimated, mostly parallel incident light. We start with regular paper ($80\,\mathrm{g/m^2}$) as light scattering material and assume polished flat glass walls of the fish tank. The liquid will initially be water.

Paper is a complex medium that consists mainly of fibers mutually connected by hydrogen bonds and of various substances used as fillers on a macroscopic level. Various theories have been used and experiments performed to understand light scattering in paper (see Refs. 7 and 8, for examples), and the photon mean-free-path between interaction positions is estimated to be[7] $2\,\mu$m. Therefore, for a standard sheet with paper weight of $80\,\mathrm{g/m^2}$ and a thickness of 0.1 mm, one may expect multiple scattering to occur. Figure 5(a) shows schematically two light paths, one leading to backscattering and one leading to transmitted light through the paper.

Naturally, there are many different light paths possible as a macroscopic result of the individual multiple scattering paths of photons. There will be a small but noticeable enlargement of the width of the light beam on both sides of the paper. If the incident spot had a diameter of around 1 mm the enlargement factor would be around two or so. Compared to the much larger dimensions of the observed optical cone within the fish tank, this light beam diameter can still be considered very small. In other words, we treat scattered light as originating more or less from a single point source. In the following, we only discuss the backscattered light. Similar arguments, though including light attenuation depending on paper thickness, can be applied when discussing respective features for transmitted light.

After multiple scattering events, the angular distribution of backscattered light closely resembles that of a Lambertian radiator. Such a radiator has an emitted radiant power per projected source area per solid angle that is independent of the scattering angle with respect to the surface normal. Therefore, its radiant intensity varies with the cosine of the incident angle. As a consequence, the backscattered laser light from the paper is emitted at all angles in the hemisphere above the paper [Fig. 5(b)].

This behavior is the basis for the initial assumption of the basic explanation, namely that light is uniformly scattered by the paper in all directions. We note that in reality, most

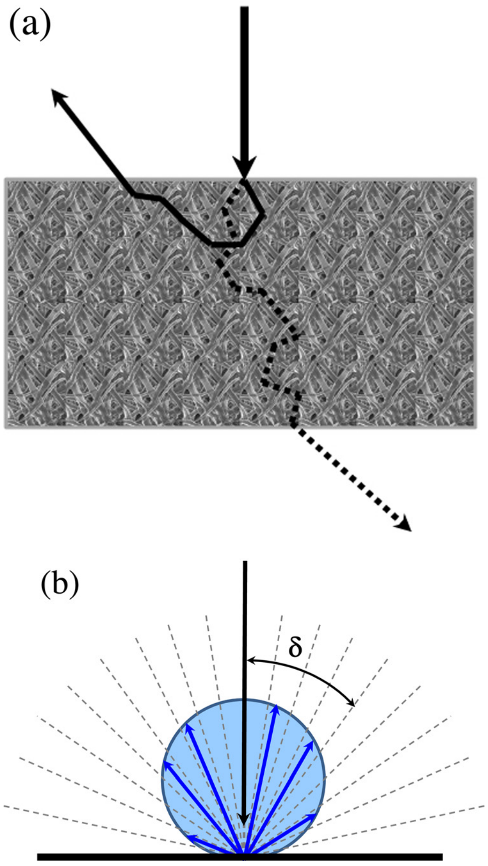

Fig. 5. (a) Schematic ray paths for backward and forward scattering of light within paper, suffering multiple scattering events; (b) Polar plot for angular distribution of backscattered radiant intensity for a Lambertian radiator.

surfaces which are considered Lambertian radiators only obey this law up to angles around 60°, at which point the radiant intensity decreases for larger angles (e.g., see Ref. 9). This will not appreciably affect our results here as the cone will remain, but there could be a somewhat different brightness distribution within the cone.

Next, we discuss how much light can still enter the glass at very large incident angles. Reflection and transmission of light at the boundaries between two materials are governed by the Fresnel equations (e.g., see Ref. 10) that take into account the polarization of the light. For an air-glass interface, we expect (Fig. 6) that at 80° (in Fig. 4 the largest angle shown is around 81°) the average reflectivity R of unpolarized light amounts to 40%, whereas at 88°, it is clearly above 80%.

A suitable (though arbitrary) measure for the small diameter of the truncated cone is found if we assume an air angle $\alpha_{\text{air}} \approx 88°$, which gives a distance of the interaction point from the axis to be $d_{air} \tan\alpha_{\text{air}}$ Since $\tan\alpha_{\text{air}} \approx 28.6$, a small air gap of less than 0.05 mm leads to a lateral distance of only around 1.4 mm. In addition, there will be the lateral widening in glass that roughly coincides with the glass thickness, leading to a lateral distance on the order of several mm. In case the air gap distance is purposely enlarged to say, 1 cm, we would find a lateral distance of 29 cm. Some students indeed suggested increasing the gap to test whether the cone is already formed due to scattering by the paper alone.

Finally, we want to discuss whether the cone angle really refers to a sharp boundary defined by the critical angle. In particular, we want to investigate whether it is possible to easily understand the angular dependence of radiant intensity within the cone. The observed spatial distribution of light intensity is mainly determined by the transmission properties of the air-glass interface, by the angular redistribution of light due to refraction at the glass-water boundary, and by the light scattering particles within the liquid.

On the one hand, we know from the Fresnel equations that the transmission T of light from air into the glass, defined by

$$T = 1 - R, \tag{4}$$

decreases with increasing angle of incidence α_{air} [upper curve on Fig. 7(a)]. In addition, if the scattered light from the paper follows a Lambertian characteristic, radiant intensity will be further attenuated by the cosine factor [middle graph in Fig. 7(a)], with deviations from a Lambertian radiator possibly enhancing this effect. The combined effect is shown in the lower graph in Fig. 7(a), and enlarged for α_{air} above 80° in Fig. 7(b); it resembles a rather steep decrease of radiant intensity with angle of incidence α_{air}.

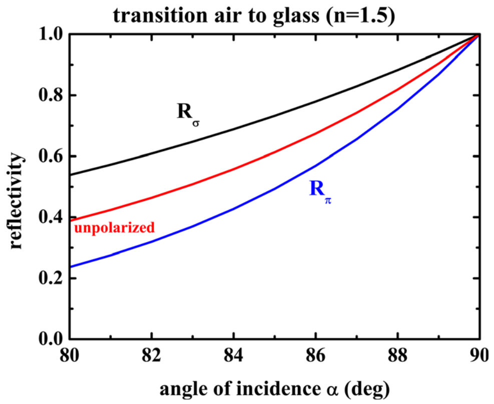

Fig. 6. Reflectivity R at an air-glass interface as a function of the angle of incidence for unpolarized light (middle curve), and for light polarized perpendicular (R_σ, upper curve) and parallel (R_π, lower line) to the plane of incidence, defined by the direction of incidence and the normal to the interface. At larger angles of incidence there is a smaller amount of transmitted light (e.g., at 88° less than 10% of the incident light remains).

On the other hand, the refraction at the air-glass interface results in a cone angle that does not vary dramatically with the angle of incidence [Fig. 7(c)]—it amounts to 48.6° for $\alpha_{\text{air}} = 90°$ but is only one degree smaller (47.6°) for $\alpha_{\text{air}} = 80°$.

Since the refraction at the glass-water interface occurs for incidence angles of at most 42°, the respective additional reflection losses are small (<1%) and the transmission factor of the air-glass interface is also a good first measure for the transmitted light into the liquid. Figure 7(d) shows the respective plot of transmitted light as a function of the angle of refracted light within the liquid. Normally, the human eye can easily detect such changes of light. For example, if we assume that perception of the light scattered to the observer would start to occur for a signal increase of 2%[11] (which should be easily detectable), we would detect a cone angle that is about 0.15° smaller than the theoretical value of 48.6°.

However, the actual consequence for observable changes in radiant intensity is a little bit more complex, as illustrated in Fig. 8. Because the distance between paper and glass is small compared to the distances travelled in glass, the main separation of the various angular contributions happens while light is travelling in glass, and later in water. Figure 8 depicts how light, which is scattered by the paper into two selected angular ranges in air, is transformed into two different angular ranges in glass and in water. Obviously, light that strikes the glass-water interface at larger incidence angles will have a larger angular spread in water, which leads to an additional attenuation. Moreover, the transition from glass to water gives rise to another transmission factor according to the Fresnel equations. However, because the involved angles are below 41.8°, the effect is not as severe as for the air-glass interface.

Finally, we must keep in mind that the observer looks perpendicularly onto the cone. Therefore, the observed changes in brightness at a given height [side view of Fig. 8(b)] are due to scattered light originating from a segment of the respective circular cross section of the cone [top view of Fig. 8(b)]. At the edge of the cone, only a small number of scatterers will direct the light to the observer whereas at the center, more scatterers are available. Therefore, the actual brightness variation across the cone sensitively depends on the angular spread of transmitted light as well as the number of scatterers in the liquid that scatter in a given direction.

Overall, the combined effects of the Fresnel equations, Lambertian radiator, and angular spread should give rise to a soft boundary—there is no step-like increase of radiant intensity at the cone angle but rather one very similar to the one shown in Fig. 7(d). The respective shift of measured cone angle should only be around 0.15°. This must be compared to typical measurement accuracies of 2% in introductory student experiments that refer to an angular accuracy of about 1° This means that n_{liquid} would have to be exactly 4/3 of the actual expected value for the cone angle $\alpha_{\text{liquid}} = 48.6°$ For perception at 2% signal change, the angle would be 48.45°. However, due to the measurement errors in the

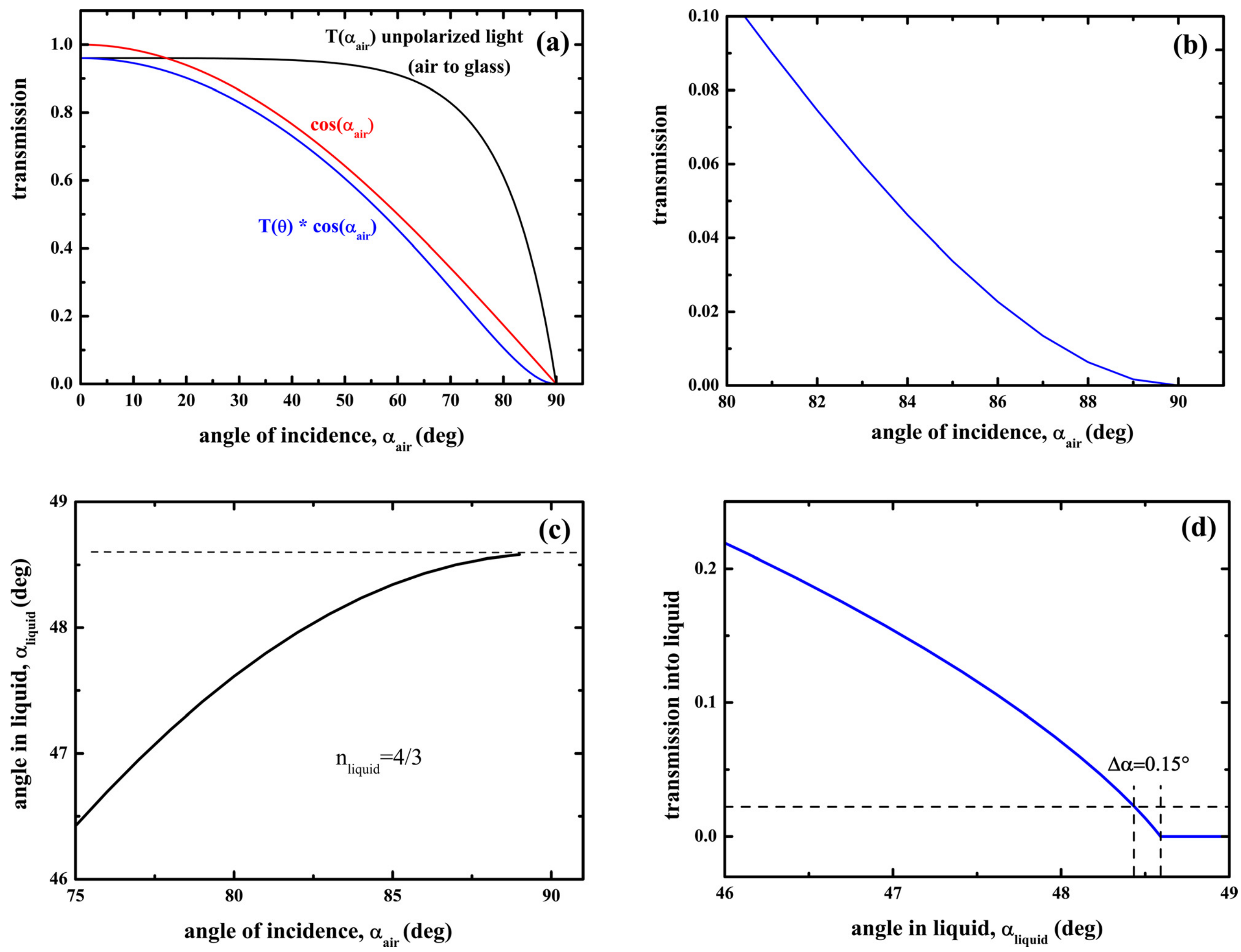

Fig. 7. Theoretical plots demonstrating the strong decrease of radiant intensity with angle of incidence α_{air}. (a) Transmission of radiant intensity as a function of α_{air} for unpolarized light from air to glass (upper curve), modified by the cosine factor of the Lambertian paper surface (lower curve); (b) enlarged section of (a) for α_{air} above 80°; (c): change in cone angle with angle of incidence; (d) transmission of (a) re-plotted versus refraction angle α_{gl} in liquid.

student lab, the measured value could be as low as 47.6° [see Fig. 7(a)].

Without a doubt, a detailed quantitative analysis of the soft boundary would require some more detailed modeling, taking into account the angular scattering phase functions of the particles in the liquid that direct the light to the eye of the observer. These aspects are beyond the scope of this paper.

IV. VARIATIONS, ADDITIONAL EXPERIMENTS, AND APPLICATIONS

A. Perpendicular arrangement with glass

The whole experiment is conceptually simpler if we get rid of one medium; for example, if we only need to deal with air and glass and skip the water.[12] The simplest experimental set up consists of a laser that hits a piece of paper on top of a glass plate and the light cone is projected onto a second piece of paper below the glass. The cone angle is then easily seen as an illuminated circle of radius *R*. Figure 9 shows a photo of the actual set up as well as the projected cone when operating the laser. The main difference to the observation with the fish tank is that there is almost no internal scattering within the glass. Consequently, the cone is seen only after projection.

For a glass thickness of 4 cm, we obtained the illuminated circle of a diameter of 7.2 cm. The latter value corresponds to an angle of 42.0°. From theory, with n=1.5, one would have expected an angle 41.8°, which is considerably close to the measured angle.

B. Horizontal arrangement with fish tank or glass

The same phenomenon can also be observed if the laser beam is incident on the paper from outside the fish tank (see Fig. 10). In this case, the paper absorbs some of the light, but the light that emerges from the paper is uniformly scattered in the half space, just as in the previous experiment. Since this variant involves penetration of the light through the paper, students are more concerned about the interaction between the paper and the light beam. This concern may encourage them to propose additional hypotheses—such as that the light cone is the result of diffraction—but it can also redirect the debate in non-productive directions with regard to the explanation of the cone angle.

C. Related example from nature

The light cone in this experiment corresponds exactly to the cone defining the directions in which an underwater observer (be it fish or diving person) can just still see the world

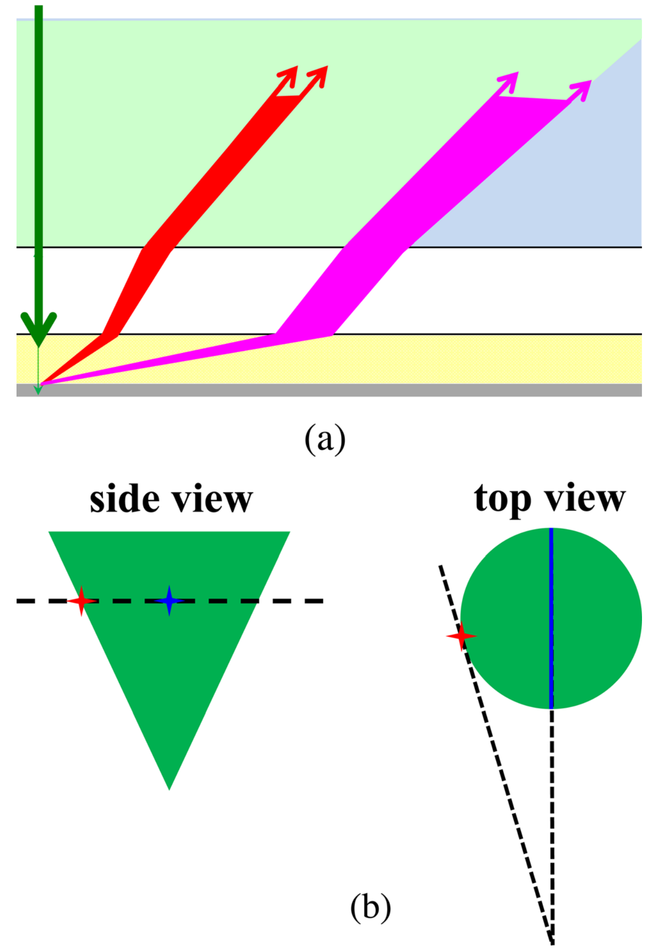

Fig. 8. (a) Strongly magnified view of interaction region. Light being scattered by the paper into equal solid angles is attenuated at larger angles due to a larger angular spread. (b) Top view and side view of observed light cone.

above an unperturbed water surface. If a submerged observer looks in a direction outside this cone, they will see the reflected image of a pool or a lake bottom due to total internal reflection.

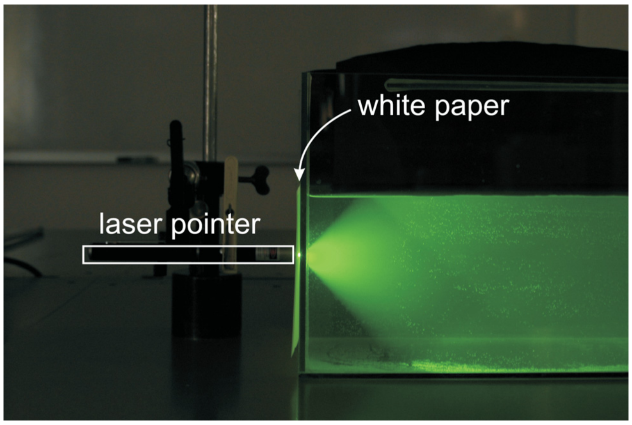

Fig. 10. Alternative horizontal version of the experimental setup. This version is more simple to construct but may redirect the debate in nonproductive directions because of a poorly understood interaction of light passing through the paper.

One can demonstrate that the light cone angle is indeed the critical angle for a water-air interface by using the same experimental setup as above. If you fix the white paper on the bottom of the fish tank (on the outside surface) and tilt the fish tank you will see the total internal reflection on the part of the light that is incident to the water surface at angles larger than the critical angle (see Fig. 11.)

V. LIGHT CONE IN THE CLASSROOM

In this section, we focus on when and how to use this ISLE experiment in your classroom, what to anticipate, and how to react. As suggested in the introduction, if you are teaching an introductory physics course students need to learn geometrical optics before they attempt to explain the light cone experiment. For advanced courses, the problem can be used at any point, possibly even during the first class meeting as an introduction to the reasoning process that physicists employ when solving problems and constructing new knowledge. In any case, the instructor should invite the students to reflect on the process they used to devise the solution to make sure that the students appreciate the difference between the original experiment and the experiments that were designed to test their ideas. Independent of the audience, several issues might arise. For example, the participants might not focus on the cone initially but could instead start exploring other aspects of the experiment, such as studying double reflections of the laser beam from the bottom of the tank. If time is limited, the instructor can bring the cone to their attention at the very beginning.

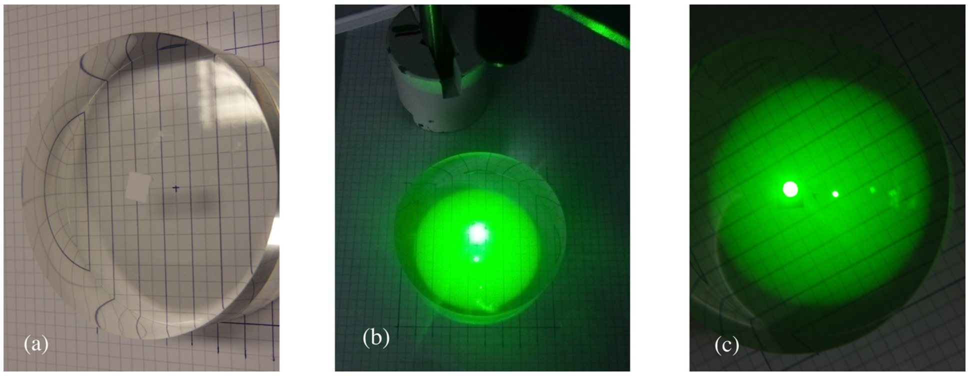

Fig. 9. (a) Experimental set up with a small piece of paper on top of a glass cylinder (diameter 9 cm, height 4 cm) sitting on a piece of graph paper (grid size 5 mm); (b) Overview photo of illuminated circular area using a green laser pointer shining perpendicularly from the top; (c) enlarged view of (b) to facilitate measuring the circle diameter.

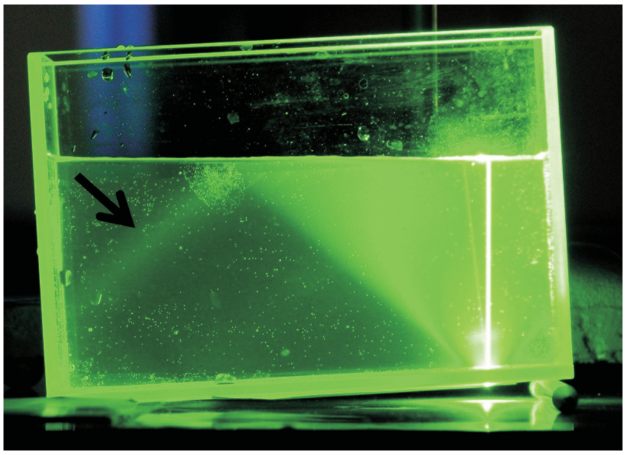

Fig. 11. Light that is incident to water surface at angles larger than the cone angle is totally (internally) reflected.

We have performed this activity in different settings, including with university freshmen, with junior physics majors, and with pre-service and in-service high school physics teachers (over 150 participants in total). In all cases, we gave the participants directions on how to set-up the experiment followed by questions that guided them through the ISLE steps. The sequence is described below:

1. Perform the observational experiment and draw a clear picture of what you see.
2. Propose different explanations for how the light cone is formed and suggest experiments to test each of your explanations.
3. For each of the testing experiments use, the explanation being tested to predict the outcomes of the testing experiment.
4. Perform the testing experiments, record the outcomes, and make judgments about the explanations you proposed.
5. Based on the results of your testing experiments and using a ray diagram, explain how the cone is formed.
6. Prepare group report about your investigation.

For all of these elements to be successful, the instructor needs to monitor group work closely, seeding groups with different ideas by giving small hints, and choosing the best sequence of presentations that allows the weakest groups to present first and the more sophisticated groups to share last. Finally, it is beneficial if the instructor summarizes and enriches group findings. This sequence of monitoring, seeding, ordering, and summarizing was found to be very effective for discourse management.[4]

We have performed this activity with participants from different countries including the Czech Republic, Mexico, Slovenia, and the United States. The number of participants per session varied from 10 to 60. The activity requires about 1.5 h of class time plus time for a discussion, which can vary from 15 min (quick report of the group results followed by the teacher's comments and explanations) to 1 h (group reports followed by a guided discussion that finally converges on a common explanation). The time required also depends on the number of participants, and if the activity is performed with more than 20 participants we suggest having two facilitators.

VI. CONCLUSIONS

The light cone experiment not only allows learners to devise multiple explanations and to test them immediately, but it can also be adjusted to the level of sophistication of the audience. In addition to this experiment, there are other simple experiments that can be used in different level courses to achieve the same epistemological goals (see Refs. 13 and 14, for examples).

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[6] A few notes about the equipment are important: (a) In the chalk dust experiment you rub the sponge with white chalk and then produce fine dust clouds by clapping the sponge in the vicinity of the place where the laser beam hits the white paper; (b) Experiments with sandpaper are suitable only if you are using a plastic tank (note that rubbing the plastic surface will leave permanent scratches on it). An alternative way to make the same testing experiment is to use dull tape (such as Scotch tape) and press it firmly on the tank surface so that there is no air layer between the glass and the tape; (c) Thick slabs of Plexiglas proved to be one of the most frequently requested pieces of equipment, allowing students the opportunity to test what happens inside the transparent wall. We used slabs made from Plexiglas (dimensions 10 cm × 10 cm × 2 cm) with all sides polished to make them clear (transparent); (d) Sometimes students want to repeat the experiment using a liquid with a refraction index greater than that of water. The cheapest way to do this is to use a highly concentrated water-sugar solution or to use prepared sugar syrup (Karo syrup works well). For the more expensive Karo syrup, we used a smaller tank (10 cm × 6 cm × 4 cm glass tank). If using a concentrated sugar solution, you can use the same large tank.

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