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Teaching motion with the Global Positioning System

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Abstract

We have used the GPS receiver and a PC interface to track different types of motion. Various hands-on experiments that enlighten the physics of motion at the secondary school level are suggested (visualization of 2D and 3D motion, measuring car drag coefficient and fuel consumption).

(Some figures in this article are in colour only in the electronic version)

M This article features online multimedia enhancements

Introduction

The need to present the Global Positioning System (GPS) in school and its potential use in student practical work have recently been reported in several places [1–3]. Our work started as a proposal for a thesis by a fourth-year student on a Pedagogical Physics course (also the first author of this paper). Initially we wanted to use GPS data to show how to handle errors in measured data (study of error propagation and filtering of noisy data) but we soon realized that the data measured by the GPS today are 'too good' to be useful for this purpose. However, the accuracy and stability of GPS data proved to be good enough to follow curved motion in several outdoor hands-on experiments that can enlighten kinematics and dynamics in three dimensions. Later the same equipment was used by third-year Pedagogical Physics students (future secondary school physics teachers) as a part of their practical work. The response was very positive and proved that the experiments deepened the understanding of the kinematics and dynamics of a rigid body in curved motion and also demonstrated the utility of GPS and ICT.

In this article we report on the various hands-on experiments that can be done using a standard GPS receiver and a PC. The GPS receiver collects exact time data from the signals of satellites orbiting the Earth. Using the time delays it calculates its position with astounding accuracy. The system of satellites has to account for several effects including relativistic effects [4].

The GPS system currently allows the position measurements to be accurate to about 5 metres horizontally and 25 metres vertically in stable weather and with a good view of the sky, at least according to our experience with the Magellan GPS 315 receiver. Accuracy drops when trees or buildings surround us. However, as these position errors are not randomly distributed through time but rather slowly drifting, the velocity vector of the receiver can be measured to about 0.1 m s^{-1}.

Connecting the GPS to a computer

The generally affordable hand-held GPS receivers are usually equipped with an RS-232 port for communication with other navigation devices or with a computer. Every such contemporary receiver can be set to continuously transmit its position inside NMEA 0183 formatted messages (a type of data protocol) at a suitable baud

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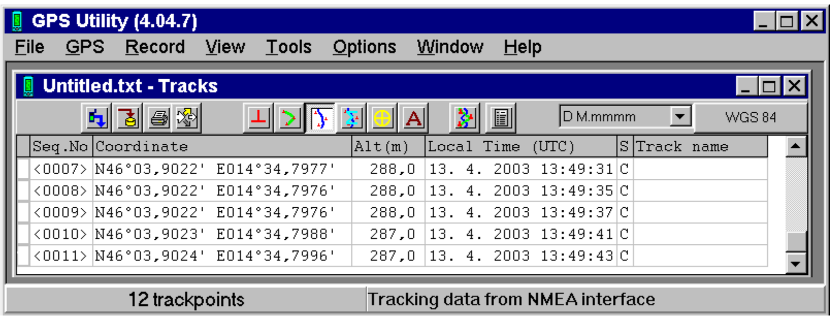

Figure 1. The freeware GPS Utility [5] enables users to track and log their positions into a file, which can be later imported into a spreadsheet for analysis.

rate (usually 4800 bps). The position data are normally available every two seconds. The receiver transmits a whole set of data, depending on its settings and capabilities. We concentrated on a single sentence or line inside the NMEA message, which might read:

```
$GPGGA,190239.62,4603.9040,N,
01434.7951,E,1,05,2.4,00292,M,,,,*2B
```

Every NMEA sentence starts with a '`$`' character, followed by the address field and data fields, separated by commas. Just before the end of the line there is also an optional checksum. The first two characters of the address field '`GP`' show that the message was generated by a GPS receiver. The next three characters indicate the standard in which the data are transmitted. In the `$--GGA` sentence there are 14 data fields, separated by commas. The first field is the Universal Time Coordinate (formerly known as Greenwich Mean Time) of position, the first two digits of which are used for hours, the next two for minutes and the rest for the seconds to hundredths. The second field is the latitude, the first two digits for degrees, the rest for minutes to ten-thousandths. The following field can be either '`N`' for places north of the Equator or '`S`'. The fourth field is the longitude, the first three digits are for degrees, the rest for minutes, also to ten-thousandths. The following field is either '`E`' or '`W`', indicating that the measured position is either east or west of Greenwich. The quality indicator in the sixth field is normally equal to one, but in case no position has been computed from the GPS signals it is zero. The seventh field indicates the number of satellites used in the solution for calculated position. The horizontal dilution of precision is given in the next field. It depends on the distribution of the satellites above the location of the GPS. The field with altitude follows it. The field with `M` denotes the units of altitude, metres in the example above. Unused fields are left blank (no spaces). The checksum is calculated by using the bit-wise XOR function on all characters between '`$`' and '`*`'.

We have seen that any device with a standard RS232 port, even one with only three wires, can be connected to the GPS receiver, can listen to it and record both position and time. It can be a desktop computer, a laptop or a handheld. The desktop PC is rather impractical to use for outdoor experiments. The laptop is more suitable, and for data-logging there is also a freeware program called GPS Utility [5]. It is able to log either whole NMEA messages sent by any receiver, or just time and positions in geographic coordinates, and it is up to the user to extract and interpret the desired data later (see figure 1).

When comparing the prices for different data acquisition systems we realized that a homemade portable device is the best choice for us. For this purpose a portable GPS interface was made by electronic specialists in our department (figure 2). The interface is based on an 8-bit micro controller with a built-in RS232 port and has 512 KB of external memory. The casing including batteries is suitably small for carrying in a small rucksack or handbag. During the measurement the interface stores the position data that are sent from a GPS as part of the `$--GGA` sentence. When the experiment is completed, the data are transferred to a PC via the same RS232 port. Each record consists of the Universal Time Coordinate, the latitude, the longitude, the altitude and the number of satellites used. Each position point is compressed into 16 bytes, so 512 KB of flash memory is sufficient for 18 hours of straight recording. The LCD shows

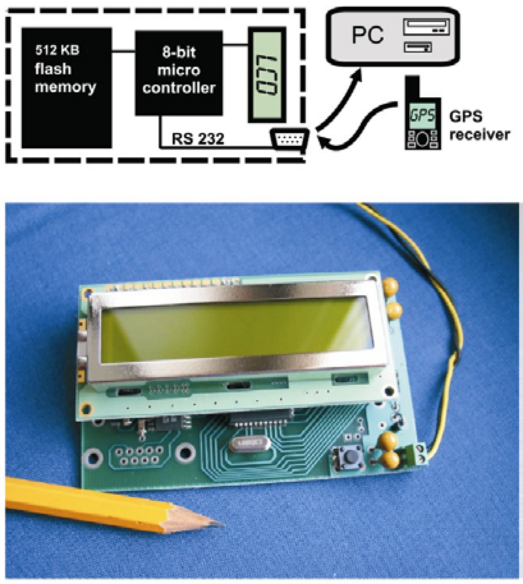

Figure 2. Schematic picture of the home-made interface. The photo shows the completed interface module (without the batteries and connectors).

the status of the memory and how many bytes have already been recorded.

Using GPS in school

Describing two-dimensional motion with vectors

Experienced teachers are aware of the difficulties most students have with the description of motion in two or three dimensions. There are conceptual difficulties such as non-collinearity of the velocity and the acceleration, but also the lack of suitable hands-on experiments that make this topic hard for students. The GPS offers a possibility to measure the type of motion that we are most familiar with and which can be directly experienced by our senses. We believe that the formal description of curved motion will be better understood if it is related to our personal experience.

The obvious use of GPS data is to show positions as dots on a digitized map. Using an ortho-photo where each pixel corresponds to 1 m^2, we were able to show positions for motion on roller-blades, on a bicycle and in a car (figure 3). The data analysis and presentation has been done on a PC using a Visual Basic program written for this purpose. The 2 km trip with the roller-blades took about 8 minutes. The position points are close together because the speed never exceeded 5 $m\,s^{-1}$. In an urban area (lower part of the photos) the error increased, but there was no sudden jump. The error just drifted slowly to its maximum value and then decreased.

The bicycle trip took about 5 minutes at an average speed of about 10 $m\,s^{-1}$. The car trip took 3 minutes to get around at a speed of up to 20 $m\,s^{-1}$. The short time of the experiment resulted in very little drift in position error. When planning a student practical activity with GPS one should carefully choose the most suitable means of transport (normally bicycle is the best choice) and a quiet neighbourhood with a low traffic density.

But why stop at position dots? Let us calculate the velocity and acceleration vectors. Graphical

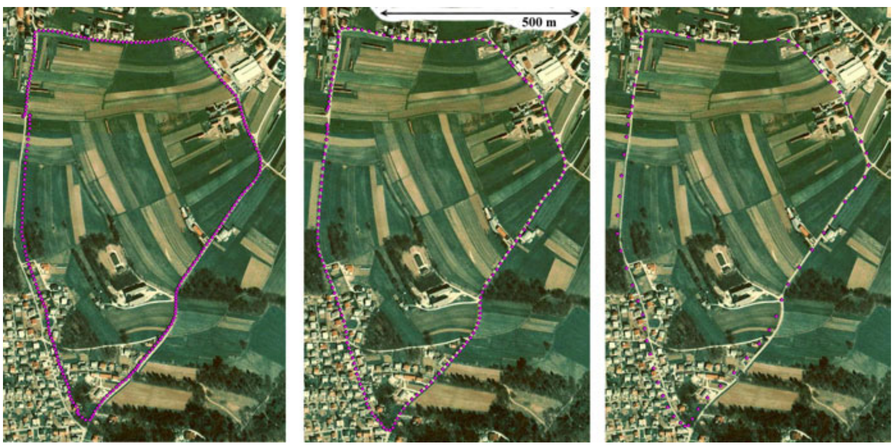

Figure 3. Comparing the motion on roller blades (left), on a bicycle (middle) and in a car on the same 2 km path. All travel was in a clockwise direction and a position was measured every 2 seconds. The ortho-photo was obtained from the Slovenian telephone company homepage.

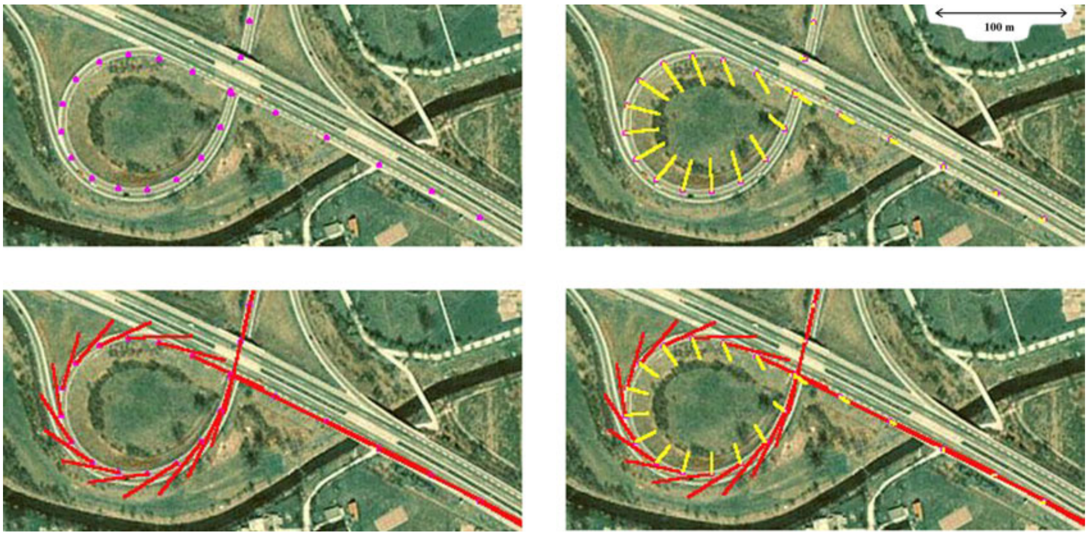

Figure 4. The same motion of a car joining the highway represented in different ways (vehicles drive on the right in Slovenia). Top left: positions only (magenta); bottom left: positions with corresponding velocity vectors (red) for each point (no arrow heads are drawn, to prevent overlapping); top right: positions with acceleration vectors (yellow); bottom right: all of the above.

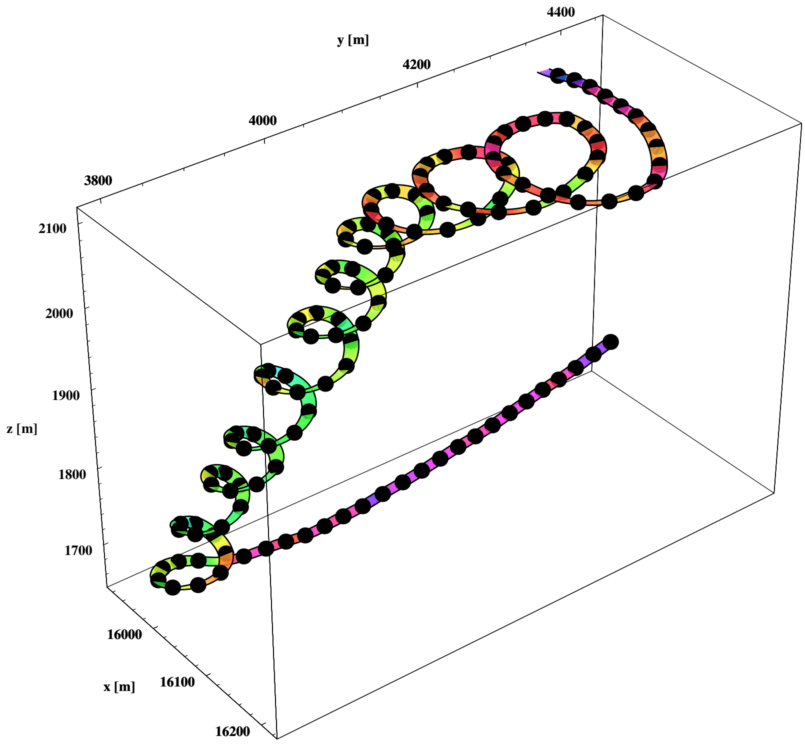

Figure 5. The graph shows a visualization of the 3D motion of a hang-glider flown upwards in a column of rising air. Each point corresponds to a GPS position reading. The time interval between the readings is 2 seconds. Colour shows the vertical component of velocity: greens are used for the climbing hang-glider, reds as it descends. (This measurement was taken by a skilled hang-glider pilot. Picture produced by *Mathematica.* By courtesy of Matija Lakner, Faculty of Civil and Geodetic Engineering, University of Ljubljana.)

representation of several vectors on the same picture can soon become unclear; therefore we decided to show the velocity and acceleration vectors as originating from the measured points $\boldsymbol{r}(t)$. Therefore the vectors were calculated as follows:

$$\boldsymbol{v}(t) = \frac{\boldsymbol{r}(t+\delta t) - \boldsymbol{r}(t-\delta t)}{2\delta t}$$
$$\boldsymbol{a}(t) = \frac{\boldsymbol{r}(t+\delta t) + \boldsymbol{r}(t-\delta t) - 2\boldsymbol{r}(t)}{\delta t^2}. \quad (1)$$

Showing the velocity and acceleration vectors on the same image offers an opportunity for the discussion about what we felt at a particular moment during the trip and how this reflects on the vectors that are describing the motion (figure 4).

Three-dimensional motion

It is also possible to reconstruct 3D motion from GPS data. Although the vertical component is not very accurate, in good conditions (such as when flying in a hang-glider, see figure 5) it can be useful. In order to avoid messy pictures the positions are shown as dots on a 2D surface (resembling ticker-tape) inside a 3D block. Shading was used to denote the vertical component of velocity. The image was produced with *Mathematica*.

Measuring the aerodynamic drag coefficient of a car

On a windless day the car was driven on a fairly straight road. When reaching about 22 m s^{-1}, the transmission was put into neutral and the car was left to slow down until it reached the end of the straight part of the road. The experiment was repeated several times on the same part of the road and in both directions.

In the simplest model the retarding force on the car (or bicycle) can be written as a sum of two contributions [6]: the velocity-independent term F_r (due to the rolling friction and the slope of the road) and the air drag proportional to v^2:

$$F = F_r + F_d = F_r + \tfrac{1}{2}\rho A C_d v^2. \quad (2)$$

ρ is the density of air, A the cross-sectional area of the car (estimated from a photo), C_d the aerodynamic drag coefficient and v the speed of the car.

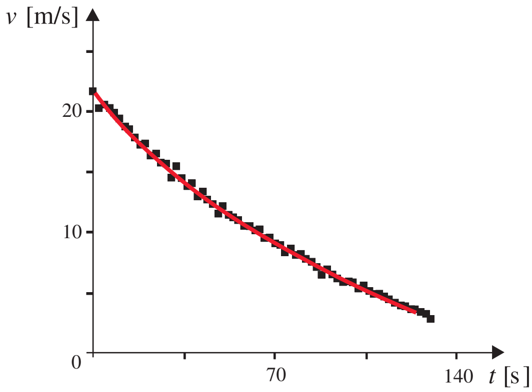

Figure 6. Graph $v(t)$ for the car which was slowing down only due to the air drag and the friction (squares: the measured values; full curve: best fit to equation (3)).

Using Newton's second law we get an equation of motion, which after integration gives the speed as a function of time, which can be written in the following form:

$$v(t) = \tilde{v} \tan\left(\frac{t_0 - t}{\tau}\right), \qquad 0 < t < t_0 \quad (3)$$

where

$$\tilde{v} = \sqrt{\frac{2F_r}{\rho A C_d}} \quad (4)$$

$$\tau = \sqrt{\frac{2m^2}{F_r \rho A C_d}} \quad (5)$$

$t_0 = \tau \arctan(v_0/\tilde{v})$ is the time the car would need to stop completely and m is the mass of the car.

Fitting the measured GPS data to a tan curve (figure 6) yields $C_d = 0.37$ for our Fiat Tipo (the nominal value is 0.32 [7]).

From the same fit we also get the value of F_r. The road where the experiment was performed seemed to have no slope at first glance. But when comparing the measured F_r values we noticed that they were systematically larger when going in one direction. This indicated that F_r was combined with the dynamic component of weight due to the non-zero slope of the road. As the experiment was repeated in both directions several times we averaged out a value of 115 N for the rolling friction. This could also be an interesting way of measuring the slope of the road. F_r can also be measured simply by pulling a car with a spring balance at a constant speed (with the gear in neutral).

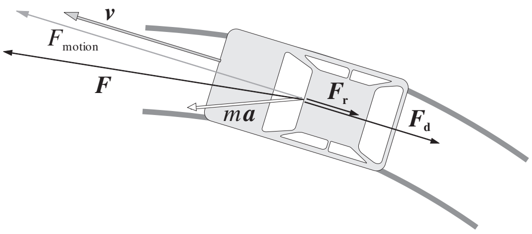

Figure 7. Forces on a car when turning.

Estimating fuel consumption

Having measured the drag coefficient and the rolling friction force, we can estimate the car's fuel consumption from the GPS measurements. Our assumption was that the road is flat and that the car engine has a constant efficiency of 0.25.

To calculate the work done by the engine, we have to consider the forces that act on the car (figure 7). The road pushes the car with a force $\boldsymbol{F}$, which appears as a reaction to the force exerted by the engine. This force has in general a component in the direction of motion, F_{motion} and a component perpendicular to this direction—a centripetal force, when the car is turning. To estimate the work of the motor, we need to know F_{motion}. It is reasonable to assume that F_{motion} is positive when the car is accelerating or driving at a constant speed, zero when the engine does no work (car is decelerating due to air drag and friction) and negative while braking or shifting to a lower gear.

Writing Newton's second law as

$$m\boldsymbol{a} = \boldsymbol{F} + \boldsymbol{F}_{\text{r}} + \boldsymbol{F}_{\text{d}} \tag{6}$$

one can express the force component in the direction of motion $(\boldsymbol{v}/v)$ as

$$F_{\text{motion}} = F_{\text{r}} + \tfrac{1}{2}\rho A C_{\text{d}} v^2 + m\boldsymbol{a} \cdot \frac{\boldsymbol{v}}{v} \tag{7}$$

where we used the expression for F_{d} from equation (2). We can 'justify' the use of the dot product by reminding the students that the work is done only by the force component that is parallel to the direction of motion.

The work done by the engine was estimated by calculating the area under the $F_{\text{motion}}(s)$ graph where $F_{\text{motion}} > 0$. On the 12 km highway trip with speeds of 90–130 km h^{-1}, in a single gear (figure 8), we got 8.6 MJ for the work of the engine force. Considering the constant efficiency of 0.25 and combustion heat of 36 MJ l^{-1} for gasoline, the consumption was 7.5 litres of gasoline per 100 km. The result is in accordance with practical experience to within 10%.

Note. An Excel spreadsheet calculation is available in the online version of this article which we believe will answer many practical questions that a novice teacher may have when trying to use the GPS as described in this article.

Acknowledgments

We wish to thank Dr Jurij Kotar from the Faculty of Mathematics and Physics, UL for designing the PC interface and Dr Matija Lakner from the Faculty of Civil and Geodetic Engineering, UL for

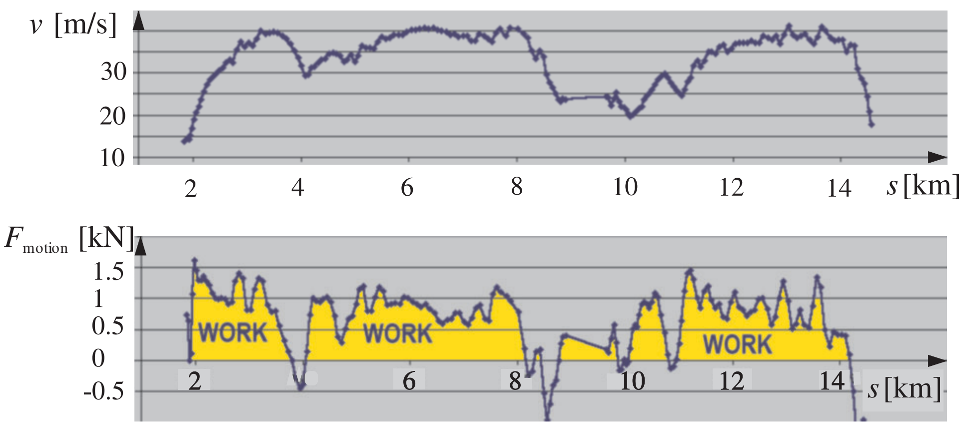

Figure 8. Graphs $v(s)$ and $F_{\text{motion}}(s)$ for a highway trip in a car. Only the intervals where $F_{\text{motion}} > 0$ were included in the calculation of the fuel consumption.

allowing us to use the hang-glider GPS data and their representation with *Mathematica*.

Received 7 November 2002, in final form 24 April 2003
PII: S0031-9120(03)55717-3

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Gorazd Planinšič received his PhD in physics from the University of Ljubljana, Slovenia. Since 2000 has led the undergraduate Pedagogical Physics course and postgraduate course on Educational Physics at the University. He is co-founder and collaborator of the Slovenian hands-on science centre 'The House of Experiments' and has also been secretary of GIREP since 2002.

Marko Budisa has just graduated to become a high-school physics teacher. He is currently unemployed. He has much experience in using computer technology and writes fiction as a hobby.