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Sketching graphs—an efficient way of probing students' conceptions

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Abstract

This paper describes a teaching method that allows for the fast and early detection of students' conceptions, misconceptions and their development. The empirical study of two examples where the method was applied is reported. The prerequisites for the efficient use of the method are discussed and results of the pilot study of its effectiveness are briefly presented.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Students' understanding of concepts and its development is one of the most important issues in physics education [1–4]. To facilitate the development of conceptual understanding, existing misconceptions have been identified, new experiments designed, introduction of everyday phenomena to increase motivation has been suggested, special tests to discover the effect of the teaching process have been designed and much more [5–9]. In addition, the identification of conceptual understanding development which threatens to lead into misconceptions and false understanding at its early stages is very important for the lecturer [2, 10, 11]. Although teachers, lecturers and educators are aware of this necessity, syllabus limitations, curricula requirements and many other factors often hinder the identification of difficulties at early stages.

In this paper, we suggest a fast and efficient method which allows for identification of misconceptions at early stage and is not time consuming. The method is appropriate for the identification of problems in mechanics, especially for more complex topics as oscillations and waves. Once students get used to the method, it can be extended to other complex phenomena in various fields.

The paper is organized as follows. In section 2, we describe in detail the method and circumstances in which the method could be used. In section 3, we shortly describe the Slovenian school system and curriculum, to define the framework in which the method was introduced and tested. We present the method and its application for the two examples, report the results of the pilot study and suggest how the lecturer might intervene when the different types of misconceptions described in the paper start to appear. In the last section, we discuss necessary prerequisites and report the results of the pilot study.

2. The method

New topics and concepts are traditionally presented by the lecturer's verbal explanation of the phenomenon, derivation of equation(s) (or simply by writing equations), and finally experimental demonstration of the phenomenon introduced during the lecture [12, 13]. This 'traditional' way of teaching has been highly criticized but is often still used as many lecturers tend to teach in the same way that they were taught [14, 15]. A step forward in traditional teaching is a reversed order of the teaching plan; first, the experiment is shown as a motivation and illustration of the phenomenon which is a goal of the lecture; the explanation and derivation of equation(s) follow [16].

Literature suggests a number of different ways to increase the understanding and/or motivation: inquiry-based learning [17, 18], peer instruction [19], interactive lecture demonstrations and hands-on experiments [7]. Unfortunately, efficient methods are often time consuming and, in curricula with a tight schedule, hard to realize. Therefore, one or another way of traditional teaching is still widely used, perhaps with minor modifications. During traditional lectures the detection of misconceptions and their development in early stages is hard and is generally missing. The main obstacle for getting feedback from students during the traditional lecture is their fear of embarrassment in front of their colleagues [20]. Although such uneasiness is not reported as a general issue in the literature, to our knowledge at least, it was studied in detail at the Ljubljana Nuclear Information Centre in Slovenia during visits of students [21], and also our experiences show that it is widely present among our students. We believe that the problem is present in every society to some extent.

To obtain the feedback from students' activities during the lecture, we suggest the method called 'sketching graphs'. The activity needs a few prerequisites, which students can develop at early stages. Graphical presentations are known to be types of presentations which are less comprehensible to students [22, 23]. However, if graphical description, reading and sketching graphs, is intentionally encouraged, demanded and also tested from the very beginning of the programme, students become competent in 'graphical' language in the discussion of problems.

Detection of problems in understanding is generally a difficult task. Students may stop the lecture, pose questions, require additional explanation, etc. However, our studies have shown that students often blame themselves for the lack of understanding or they count on last-minute information before the test in the form of a recipe given by colleagues. A lecturer's attempt to detect problems using verbal communication requires a few minutes for every student and is therefore very time consuming. As alternatives, written questions, comments or short post-tests can be performed after the lecture [24, 25]. The lecturer checks a few questions for each student in a class and starts the next lecture a few days later with an idea of the problems. However, parts which students understand poorly are already forgotten and the lecturer is tempted to repeat the whole explanation. The method of written answers to questions is also very time consuming and mostly not considered as an option by less enthusiastic lecturers. The possibility for early detection of problems during the lecture is therefore lost. We propose 'sketching graphs' as a method which allows for the detection of existing, persisting or newly developed misconceptions at the early stages of a new concept introduction. The method is more efficient than a verbal or written post-test. The method is also not time consuming and can be used for detection of problems very early in the lectures.

The lecture room should be organized to allow the lecturer to pass by every desk. Each place and student has to be accessible at a distance that allows the graph in the student's notebook to be seen. The arrangement of students' desks in rows is best as the lecturer's approach is easy and all students have a similar view of the demonstrations and experiments at the front of the classroom. Another alternative is the U-shaped desk arrangement.

During the time spent for sketching graphs (only a minute or two) the lecturer passes by each student. A short glance at the sketches provides feedback. Furthermore, details in the graph reveal students' conceptual knowledge which is often not recognized in their verbal responses. For instance, students report on the rolling motion over an inclined plane that the distance from the origin increases; however, the graphs also show awareness of increasing speed which they clearly indicate as a change in the steepness of the curve.

During the sketching period, the lecturer usually recognizes three to five types of graph. After the sketching period the lecturer asks students with representative graphs to draw them on the blackboard. The discussion starts with one of the incorrect graphs to stimulate discussion about the concepts involved. The student is encouraged to explain the meaning of the graph. The lecturer asks for the detailed meaning of parts of the graphs. The class is encouraged to join the discussion. The teacher asks questions, encourages alternative explanations, and stimulates discussion about the suggested changes in graphs. He/she stimulates the change of students' opinion with respect to the graphs drawn on the blackboard; however, he/she does not offer suggestions, but only poses questions. Finally, when the class reaches an agreement, the lecture continues. The whole procedure is open-ended and cannot be well defined with respect to time requirements. However, personal experiences from classes taught during the last seven years show that almost always the whole procedure requires up to 10 min, frequently less. This time requirement is still acceptable considering the acquired information and early detection of the development of misconceptions.

3. Results

The teaching method is applicable from high school to university level, providing the classes are not too big and the space is organized as suggested. However, due to the small size of the Slovenian population, we were not able to guarantee the proper conditions for empirical study at the university level (a control and experimental group within the same programme). Therefore, we decided to empirically test the method in the high school. In the continuation, we describe the high school system in Slovenia in order to define the testing framework.

The high school in Slovenia (also called a gymnasium) is a four-year school for students aged 15–19 with a programme that gives a general background and is completed by the final 'matura' exam. Almost all students continue their education by enrolling at universities. This general programme has become more and more popular in recent years, and nowadays 40% of young people attend this type of high school; the percentage increases up to 60% in urban areas. The overpopulation results in classes with 30–36 students in the classroom, which was the case in the classes tested. The students attend science classes (physics, biology and chemistry) in the first three years, two sessions of 45 min per week for each subject. In the fourth year, the students elect two subjects for the final exam in addition to compulsory mathematics, and native and foreign languages. The elective subjects need not include a science subject. On the other hand, science-oriented students can choose both elective subjects within science (for instance physics and chemistry) for the final exam if they prefer. If science subjects are chosen, students have four 45 min sessions allocated per week for each elected subject. This includes a substantial amount of laboratory work.

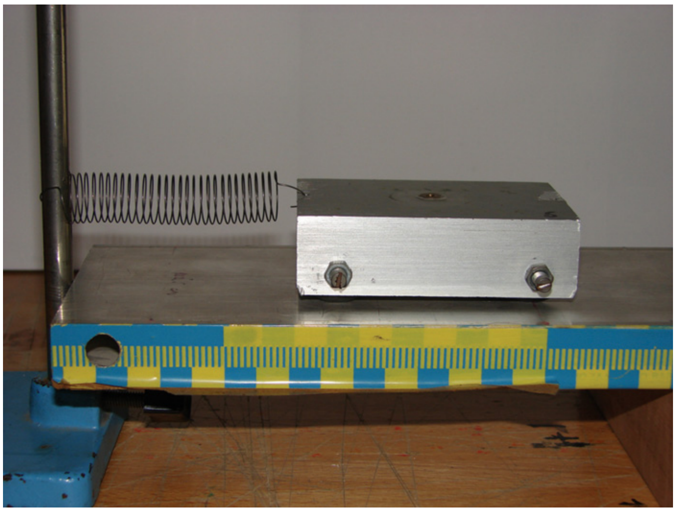


Figure 1. The massive cart is attached to a single spring, which can be compressed or extended. The motion of the pendulum is in one dimension and horizontal. The horizontal tape measure is added to facilitate the observation of position.

The ‘sketching graphs’ activity was performed in the second year (age 16–17) during compulsory physics lectures on oscillations. The Slovenian curriculum expects that students are able to describe oscillations verbally and graphically, calculate oscillation periods, and determine energies and amplitudes of position, velocity and acceleration for different oscillations. The students’ ability to describe oscillations is studied in the first example (section 3.1).

Students also meet a cognitively more demanding topic—resonance—where they are challenged to observe and describe amplitude dependence on the frequency of the oscillating driving force. Arising difficulties are nicely shown by the ‘sketching graph’ activity and are presented within the second example (section 3.2).

3.1. The oscillating cart

Before the study students were already familiar with two different types of oscillators: the string (mathematical) pendulum and the spring pendulum. They were able to describe the time dependence of the position for a pendulum’s bob and to present the dependence graphically using sinusoidal curves.

Description of velocity is always a difficult subject especially for motions where reversal of direction and non-constant acceleration is present [26]. To describe variable direction and magnitude of velocity precise wording has to be used, which students find very demanding. Therefore, ‘sketching graphs’ can help students to formulate their observations and to become aware of a number of important subtleties in the observed experiments as well as to better formulate their observations verbally.

The lecture starts with the observation of the cart oscillations (figure 1). Students are asked to sketch the time dependence of the cart velocity within a time period in which damping does not reduce the amplitude of oscillation significantly. For the sketch of the graph a time slot of approximately 2 min is allocated. The teacher walks through the classroom and observes the drawings without any comment. In a typical class of 34 students, usually four to five types of graphs can be identified. If the number of different graphs is larger,

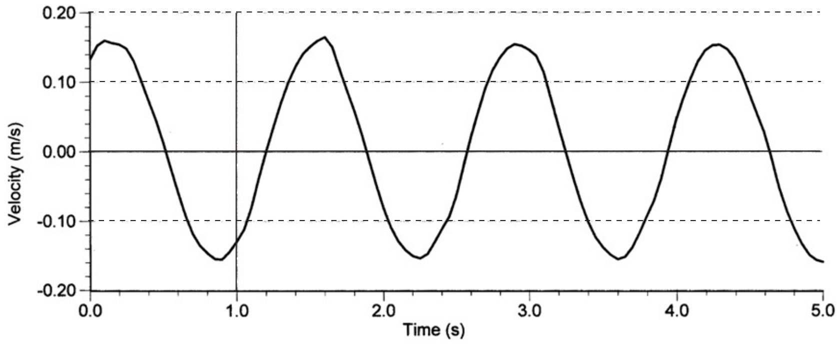


Figure 2. The measurement of the cart velocity obtained by computer-assisted measurement.

the method is not suitable for discussion anymore. When more than five different graphs are present, students are encouraged to discuss the graph with their neighbours. The 'peer instruction' procedure [19] usually reduces the number of various graphs to the acceptable number.

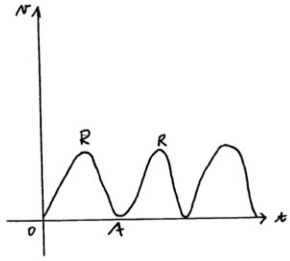
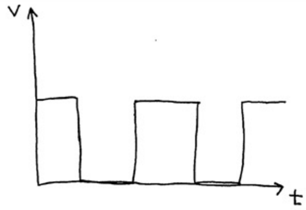
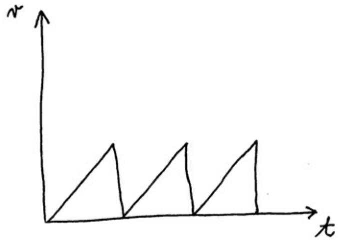
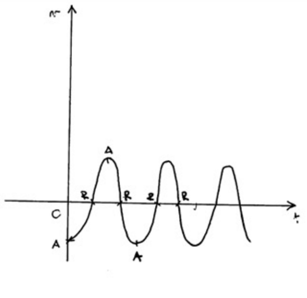
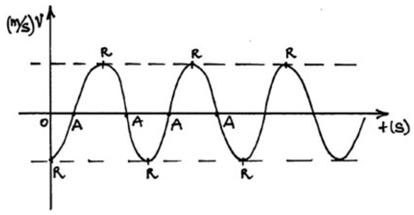
A typical set of graphs with their frequencies is given in table 1. Graphs (a), (b) and (c) report the oversimplified presentation of the time-dependent velocity. Surprisingly, the most frequent graph type (a) does not report the reversal of the motion direction. The result is consistent with earlier reports that in everyday experience one typically considers only the magnitude of the velocity as the person rarely moves backwards in the meaning of the expression (facing in the opposite direction of movement) but always follows the sight direction [27]. From this point of view graphs (a) are semi quantitatively correct if the mark *A* indicates the zero velocity and the mark *R* indicates the maximal velocity, although there is no deceleration approaching the equilibrium point. Students who sketched graphs (b) and (c) have difficulties understanding the graphical description of motion. They reproduce either moving or non-moving body (b) or connect velocity with the position in the sense the longer it moves the faster it is (c). One of the students who has sketched graph (c) explained the graph as 'the cart starts to move, then it stops and everything repeats'. The student is asked to denote the equilibrium (*R*) and amplitude (*A*) positions and to compare the velocities at two different points in the sketched graph. One of the students who sketched graph (d) recognized that he sketched the time dependence of displacement from the equilibrium position and not the time dependence of velocity. Correct graphs (e) are extremely rare. Finally, the student is asked to show the sketched velocity dependence (its linear increase) by hand or by cart. In general, these satisfy to persuade the student to consider other graphical options. The questions also allow students with stepwise graphs to see that horizontal lines have the meaning of either moving with constant velocity or staying still, which is certainly not the description of the observed continuously changing velocity.

In addition, whenever is possible, the sketches should be verified by an experiment. The computerized measurements enable simultaneous observation of the experiment and its graphical presentation (figure 2). Therefore, the teacher finishes the lecture with the computer-assisted measurement of the velocity with the result in a similar form to graph (d).

3.2. Forced oscillation

The second example presents the observation of students' understanding of resonance. Again, the phenomenon is introduced experimentally by a simple mathematical pendulum which

Table 1. Different types of students' graphs found after the observation of an experiment (oscillating cart). Letter *A* denotes the amplitude and *R* the equilibrium position.

Types of graphs sketched by students		Number of students
a.		20
b.		4
c.		4
d.		2
e.		2

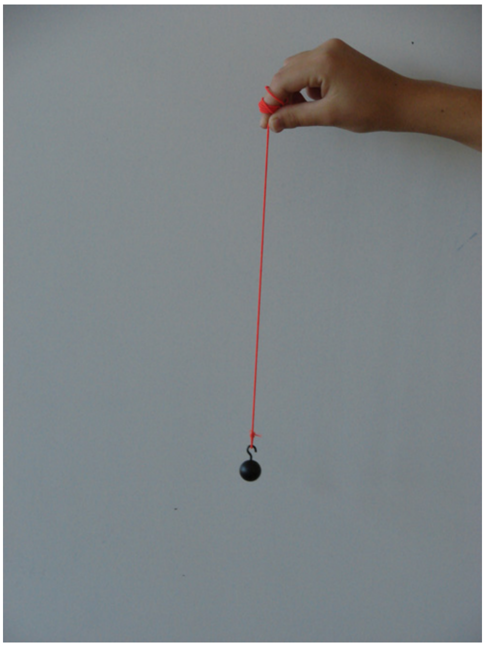


Figure 3. The simple mathematical pendulum used as a preliminary experiment in example 2.

undergoes a forced oscillation. The mathematical pendulum with the length of a few tenth centimetres and an approximate period around a second is hold by hand.

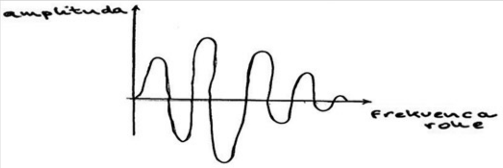
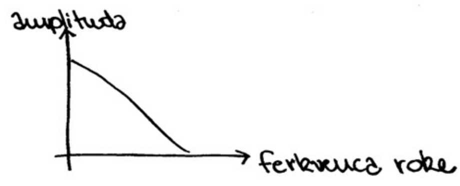
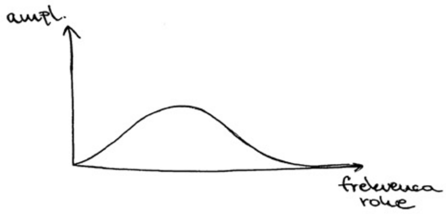
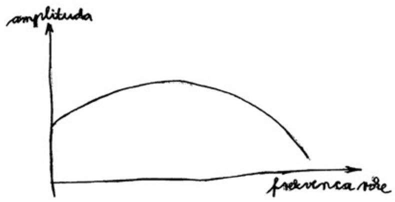
The typical experiment showing the resonance is performed. The hand is moving back and forth with a fixed frequency and the oscillation of the pendulum's bob is observed. The experiment can be performed in a number of more or less sophisticated variations using various objects as oscillators. The described simple variation of the experiment (figure 3) was chosen purposely as it allows for observations of all resonance phenomena during the lecture and is simple enough that students can repeat the experiment at home. The experiment is qualitative; students observe the movement of the hand, its frequency and the response of the pendulum, i.e. the frequency of the pendulum and the amplitude of its oscillation. They do not measure quantitatively any of the observed variables.

After observation of the experiments students are asked to sketch the graph which shows the dependence of the amplitude of the pendulum's on the hand's frequency. Simple expressions like the hand's frequency instead of the frequency of the forced oscillation, etc were intentionally used.

The task is rather difficult. The students have to concentrate on a number of different variables: the frequency and sometimes the amplitude of the hand, the frequency and the amplitude of the pendulum. They have to extract all the relevant data from the 'vividly oscillating' pendulum where the oscillation itself actually obstructs the extraction of details. It is not surprising that the number of graph types found in the classroom is often higher than in the oscillating cart example. A typical set of graphs is presented in table 2.

Again, graphs are plotted on the blackboard and authors describe their reasoning. It is not surprising that graph (a) gives a mixture of requested dependence and additional reflection on oscillations. The part of the tested audience which produced this graph was the largest. The students' intention was to point out the oscillating behaviour of the pendulum even if

Table 2. Dependence of the amplitude of the pendulum (vertical axis) on the hand's frequency (horizontal axis) as presented in the students' sketched graphs.

<table>
<tr><th colspan="2">Types of graph sketched by students</th><th>Number of students</th></tr>
<tr><td>a.</td><td>
</td><td>11</td></tr>
<tr><td>b.</td><td></td><td>9</td></tr>
<tr><td>c.</td><td>
</td><td>5</td></tr>
<tr><td>d.</td><td>
</td><td>4</td></tr>
<tr><td>e.</td><td></td><td>2</td></tr>
</table>

they were asked to concentrate on other properties of the oscillation (amplitude, frequency). Therefore, the first task of the teacher is that he/she increases the students' awareness of the phenomenon they are asked to observe and the relation between the graphical presentation and the experiment. The teacher asks students what the meaning of positive and negative values of the amplitude is and whether they observe at certain hand frequencies that the pendulum does not oscillate (zero points in graph (a)). If necessary, the teacher repeats the experiment. Next, the teacher concentrates on the rest of the graphs. For example, one of the authors of graph (e) commented that an increase in the hand's frequency resulted in an increase in the pendulum's amplitude. When the teacher asked if the amplitude would continue to increase, the student commented that he sketched only the dependence for small frequencies. Authors of graph (b) commented that they had missed the first part of the graph. The classroom finally realizes that the combination of two graphs (b) and (e) gives most of the details. Finally, the teacher repeats the experiment and points out details which enable the audience to observe also subtleties in the amplitude of the oscillations at very low and very high frequencies, and the difference between the two situations.

If the teacher had performed an experiment only and sketched the graph by him/herself, a large percentage of students would most probably have failed to get the meaning of the graph. The students who sketched a mixture of the oscillation and the amplitude needed more time to extract the important message from this complex phenomenon. The discussion which accompanies different graphs helps students to realize the nonmonotonous dependence of amplitude versus frequency. Such dependences are rare in high-school topics and present the problem by themselves. However, discussion of partial graphs (i.e. (b) and (e)) and realization of different behaviour in different regimes help the construction of student understanding.

4. Discussion

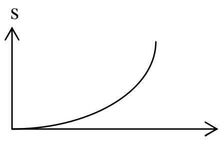
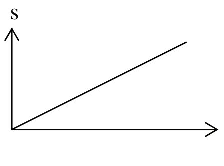
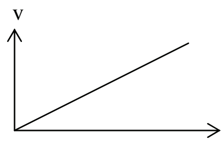
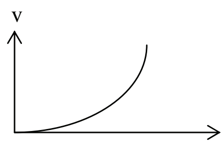
The two examples presented above show the application of the 'sketching graphs' method. However, one question has to be answered in addition. Are the sketches which were wrong a consequence of the graphical illiteracy of some students or do they correctly detect misconceptions? As pointed out earlier, the use of the 'sketching graphs' method is legitimate only if the students are graphically literate. Therefore, the teacher has to teach and build the ability to communicate graphically from the very beginning. Graphical presentations of motions and all laboratory work where measurements are presented graphically provide the best opportunities for this.

The development of the ability to use graphical presentations of observed phenomena is one of the main general scientific goals of the physics curriculum. Training in graphical thinking and reporting occurs in lower secondary school, is especially stressed in high school during lectures and laboratory work, and its development continues to be supported at university level. Students are trained to graphically present and read graphical presentations of various studied phenomena. Graphical presentations are also always used in tests. However, strong training does not necessarily mean that students are graphically literate in general. In order to check the graphical literacy of students, students involved in the empirical study were additionally tested by comparing their verbal and graphical literacy. The tested group consists of the first-, second- and the third-year students.

A ball rolling on an inclined slope was shown. Students were asked to describe and graphically present the time dependence of the distance and the velocity for the ball rolling down the inclined plane. The results are shown in table 3.

The results clearly show that students are rather sloppy in verbal communication but report more details graphically. The results support the use of sketched graphs to assist concept

Table 3. Summary of students' verbal descriptions and graphical presentations for the time dependence of distance and velocity for a ball rolling down an inclined plane.

Verbal description: distance vs time	**First year (N = 25)**	**Second year (N = 25)**	**Third year (N = 30)**
The distance increases with time.	68%	96%	70%
The distance increases with the time quadratically.	32%	4%	30%
Graphical description: distance vs time			
	96%	76%	79%
	4%	24%	21%
			
Verbal description: speed vs time			
The speed increases with the time.	46%	82%	65%
The speed is proportional to the time.	54%	18%	35%
Graphical description: speed vs time			
	100%	40%	80%
	0%	8%	20%

development. One would expect that students' graphical literacy would increase with time but one has to bear in mind that kinematics, with an extensive use of graphical representation, is taught in the first year. Therefore it is not surprising that first-year students had more detailed comprehension about phenomena related to motion with constant acceleration.

Students were also asked to describe a given complex graph of a time-dependent velocity. Similarly as in this study, graphs included more information than verbal descriptions, especially with respect to acceleration which was clearly indicated graphically but not even mentioned in answers to open questions. The percentage of students whose graphical reports were completely correct was high (close to 80%) and increased from first- to third-year students for more complex motions. However, the extent of difficulties with the graphical presentation of velocity, which changes direction, was still significant (more than half of the answers were

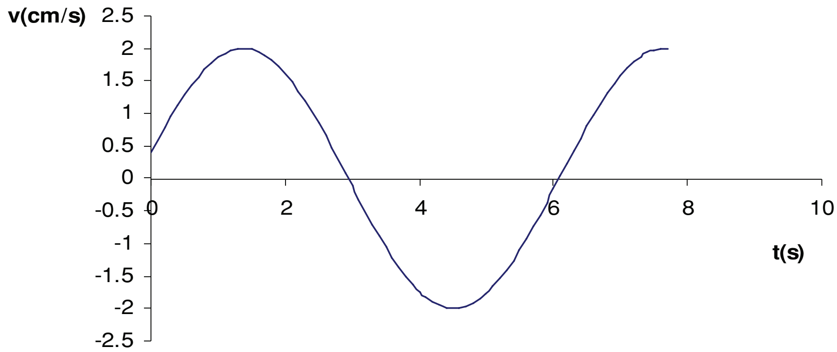


Figure 4. Graph $v(t)$ used for testing the concept of velocity.

wrong), a fact that was reflected in graphical presentations of oscillations. As has already been pointed out several times [28] the teacher has to be careful when commenting on graphs in this respect.

As the vast majority of students described the motion graphically in detail, one can conclude that students are able to describe their perceptions about the observed motion correctly. In addition, the teacher tested the consistency of the graphical and verbal descriptions of motion during the discussion accompanying presentations of sketched graphs.

The efficiency of the proposed method was tested by post-testing and comparison of the test results for two groups of students. One (experimental) group was taught by the presented method and the other (control) group was taught traditionally using demonstration experiments and graphical presentations given by the teacher. The post-test was designed by the teacher who taught traditionally. The tested concepts were consistent with the official syllabus for physics in the third year of a general Slovenian high school.

The comparison of the results achieved by both groups shows that the basic concept of resonance has been attained by 30% of students from the experimental group and by only 8% of students taught traditionally (control group).

The concept of velocity was tested by a specific question: students were asked to draw graphs $s(t)$ (displacement from equilibrium position versus time) and $a(t)$ (acceleration versus time) for given graph $v(t)$ of a general oscillation (figure 4). The testing should indicate the awareness of the relation between the three dependences: distance, velocity and acceleration for the general situation where the observation starts at a general position. Such a question is considered as cognitive and more demanding. The results showed that one fifth of students using the sketching graphs method answered completely correctly, but there were no completely correct answers from the traditionally taught students.

Although the drawing of the correct graph could be done by the teacher him/herself, the copying of the graph to the notebook would lead to the memorization of the graph by less enthusiastic students. In addition, graphical representations are obvious for physicists: they help express ideas, visualize processes and are the language used in discussions. For students the situation is not the same. Therefore, graphical literacy can be widely improved by requiring students to express their observations in graphical presentations and their comparison with direct observations. When literacy is established the 'sketching graph' activity enables the teacher to detect efficiently pre-existing or newly developed misconceptions, to follow-up with knowledge construction and also to stimulate and accelerate conceptual understanding.

5. Conclusions

To conclude, in this paper we present the 'sketching graph' method as an efficient method which is not time consuming to detect students' conceptual difficulties at an early stage. The teaching method is applicable at different levels, from high school to university. The method also allows for a constructivistic type of teaching process and enhances the student's discussion and communication in the classroom. The 'sketching graphs' method is presented in detail, and practical realization is presented using two examples. In addition, the results of the pilot study show the efficiency of the method on a small scale. The results show that the method is promising and worth consideration.

Besides mechanics, the method can be used in other topics such as thermodynamics (the description of temperature dependences, for example, development of temperature profiles under various circumstances), electromagnetism (the drawing of fields, especially for more complex charge densities, dependences of magnetic fields for various combinations of permanent magnets and/or currents etc) and geometrical optics. The method is especially useful for quantum mechanics, where misconceptions are very common [29]. It can be used for probing the development of concepts related to wavefunctions in various potentials as well as for studies of relations between wavefunction and probability density. The method is not necessarily limited to graphical sketches of dependences but can also be used for other graphical representations of various phenomena as sketches of observations, constructions of experiments, Feynman diagrams and in many other circumstances.

Although the method was initially developed in the high school, the authors successfully use the method at the university level during lectures of physics and its didactics. Our advice is that students—future teachers—get acquainted with the method and trained in it using various examples during lectures of special didactics.

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