Conservation of mechanical and electric energy: simple experimental verification

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Abstract

Two similar experiments on conservation of energy and transformation of mechanical into electrical energy are presented. Both can be used in classes, as they offer numerous possibilities for discussion with students and are simple to perform. Results are presented and are precise within 20% for the version of the experiment where measured values are available over the complete time of the experiment, and within a few percent for the simpler implementation of the same experiment where only final results are shown.

(Some figures in this article are in colour only in the electronic version)

Introduction

Though the formulation of the energy conservation law looks very simple, it is well known that it is one of the most problematic laws when required to be applied in various situations at primary and secondary school levels [1, 2] and also at higher level [3]. Perhaps the best illustration of the principle of energy conservation, described as a short story about a mother and a child whom she leaves alone in a room with 28 absolutely indestructible blocks, has been given by Feynman [4]. The story illustrates the law of conservation of energy: whatever the child does with the blocks, the number of blocks remains the same. Once the mother realizes this law she is able to predict how many blocks have been added or hidden during the day when the child has been playing with them. Knowing the properties of the blocks (such as volume) she is also able to find the missing blocks. Stories like this are an important part of instruction, especially when the subject is abstract, but the ability to apply knowledge in new situations comes through the active engagement in concrete examples. In this respect, computer-based data acquisition systems brought an important improvement in teaching physics. Such systems enable quantitative verification of energy conservation in real time. This type of measurements can be part of the lectures [5] or student laboratory activities [6].

Table 1. A list of the materials needed for the experiment and relevant technical data.

Magnets	5 neodymium magnets (diameter 7 mm, thickness 5 mm), stacked into a 25 mm long bar with total mass 9.1 g $\pm$ 0.05 g
Coil	inner diameter 20 mm, outer diameter 45 mm, length 25 mm, measured resistance $R_L = 81\Omega \pm$ 0.1 Ω, approximately 3000 turns of copper wire (diameter 0.25 mm)
Resistor	$R = 10\ \Omega$, in our case measured as 9.7 $\Omega \pm 0.1\ \Omega$
Picket fence	total length 25 cm; black stripes: length 5 mm each, distance between adjacent stripes is 10 mm
Plastic tube	length approximately 200 mm, inner diameter 9 mm

In the literature, one can find several reports on experiments that demonstrate or prove energy conservation in cases where mechanical energy is transformed from one form to another. On the other hand, examples that show quantitatively the conservation of energy for a combination of mechanical and electric energy are rare [7, 8].

In the present paper, we describe simple experiments that allow one to demonstrate the conservation of energy where the initial gravitational potential energy is transformed into kinetic energy and electric potential energy, which in turn transforms into thermal energy. Two versions of the experiment are described. The first experiment deals with the time dependence of the energy balance and is particularly suitable as an interactive lecture demonstration. The second experiment enables verification of energy conservation with higher precision and is more suitable as a student laboratory experiment at introductory physics level.

In the following treatment the term electric energy will be used for electric potential energy. In our case the transformation of the mechanical work into electric energy is enabled by the magnetic field. A closer look at the conversion from mechanical to electric energy in the cases that involve magnetic fields can be found elsewhere [9].

Experiment 1: the time dependence of the energy balance

The basic idea of the experiment is the following. A bar magnet is released from a certain height above the centre of the vertically oriented coil. The time dependence of the vertical position of the bar magnet as well as the induced current (i.e. voltage drop on a series resistor) in the coil are recorded simultaneously. From these measurements, a time dependence of kinetic energy, gravitational potential energy and electric energy that is dissipated as heat is determined, compared graphically and discussed with respect to energy conservation.

The experimental setup ready for the measurement is shown in figure 1. A cloths peg at the top of the stand holds the picket fence ribbon made from a transparency. A bar magnet is attached on the lower end of the picket fence ribbon and inserted in the plastic tube. Above the plastic tube is a photogate detector, which is adjusted (in our case tilted) so that the picket fence ribbon passes right over the photodiode. A piece of cardboard is glued on the photogate holder in order to guide the ribbon through the narrow slit in front of the photodiode. Basic materials and relevant technical data are summarized in table 1.

All measurements and data analysis have been done using commercially available school equipment for real-time data acquisition (Vernier equipment in our case). Time dependence of the vertical position of the bar magnet has been measured using a photogate detector and a picket fence ribbon attached to the magnet (see table 1 for details). The picket fence ribbon was printed on a transparency and glued on the bar magnet. A transparent plastic tube that passed through the coil was used to guide the fall of the magnet and the ribbon and prevented its tumbling.

Figure 1. Experimental setup.

The position z of the bar magnet was measured from the initial position with the positive z-axis oriented upward. From $z(t)$ measurements the velocity–time dependence is calculated as well as the time dependence of kinetic energy $\frac{1}{2}mv^2(t)$ and gravitational potential energy $mgz(t)$.

The experiment can be performed in two steps:

First, the coil terminals are left open in the air (not connected to voltage probes); second, a resistor is added in series with the coil terminals. In the first case the $z(t)$ and $v(t)$ graphs show that the magnet falls through the coil as a free-falling body with average acceleration close to g (figure 2).

The energy graph in figure 2 shows that the energy is transformed from gravitational potential energy (W_pot) to kinetic energy (W_kin) and that the total mechanical energy (W_mech) is nearly conserved. The observed small decrease of W_mech can be attributed to friction and air resistance. It is instructive to repeat the measurement but this time with voltage probe clips attached to the coil terminals (i.e. measuring time dependence of the induced voltage U_i) to show that though a significant voltage appears between the coil terminals (figure 3), the energy–time graph does not change, proving that there is no detectable energy dissipation associated with the probes, since the current through the voltage probes is negligible. This step can also be used as a predict–observe–explain teaching sequence [10] in which students are first asked to predict the shape of the voltage–time graph in the case when the bar magnet is dropped through the coil, explain their reasoning and after observing and discussing the result reconcile their original explanation.

The graph of the induced voltage is given in figure 3. As expected, the duration of the second negative peak is shorter than the duration of the first positive peak, which indicates the
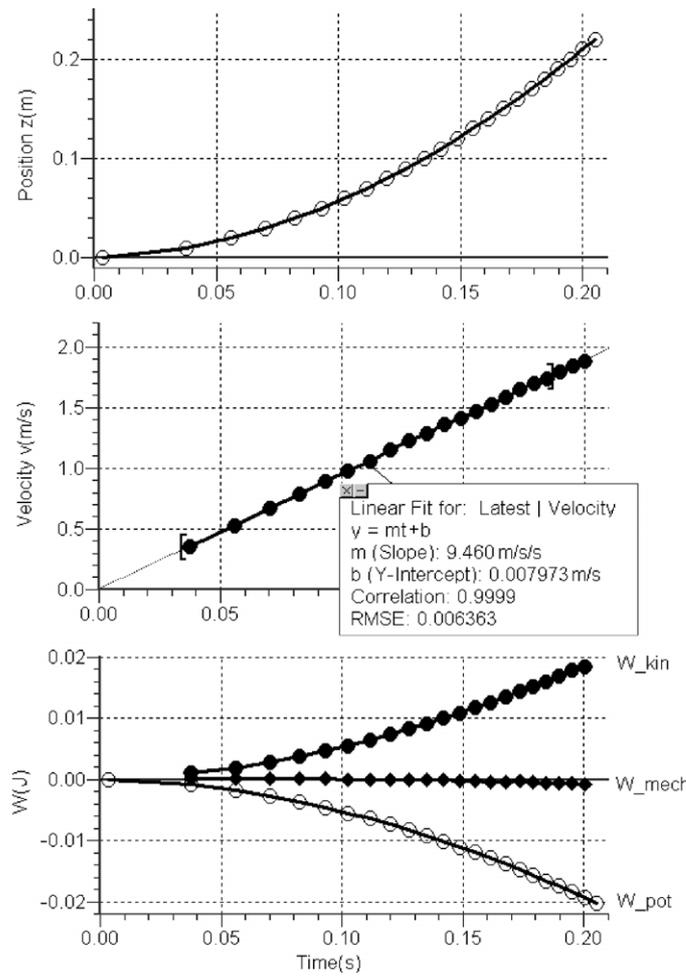
Linear Fit for: Latest | Velocity
y = mt + b
m (Slope): 9.460 m/s/s
b (Y-Intercept): 0.007973 m/s
Correlation: 0.9999
RMSE: 0.006363

Figure 2. The graphs of position, velocity and energies at free fall.
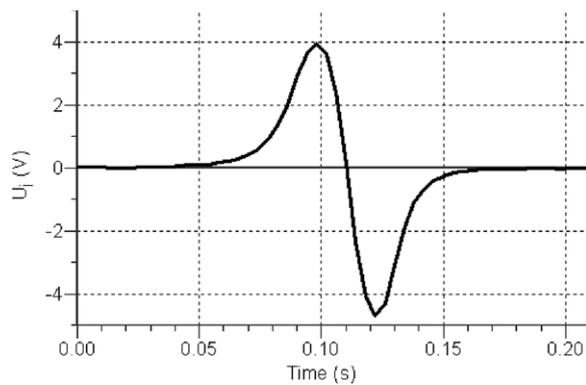

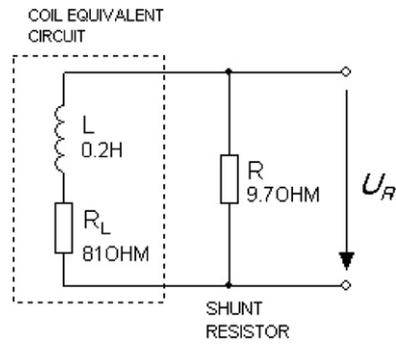**Figure 3.** The induced voltage due to the fall of the magnet.

Figure 4. The equivalent circuit of the loaded coil.

increase of the velocity of the falling magnets. Since the induced voltage is proportional to the speed of the falling magnets, the amplitude of the second negative peak is larger.

In the second experiment with this setup, a resistor R is added in series with the coil terminals (so-called shunt resistor) to form a closed circuit as shown in figure 4. The experiment with the falling magnet is repeated. This time the $z(t)$ dependence and the voltage drop on the shunt resistor $U_r(t)$ are recorded simultaneously.

The velocity–time graph (figure 5) shows a notable deceleration of the magnet during the time it passes through the coil (note that the final velocity in this case is about 25% smaller than in the first experiment). In general, a magnet would decelerate only during the entrance to and exit from the coil, but since the length of 25 mm of the magnet in our case is about the same as the length of the coil, these two regions are merged together and the free fall of the magnet through the inner part of the coil cannot be observed. The kinetic (W_kin), gravitational potential (W_pot) and mechanical energy (W_mech) are calculated as in the previous case, showing that the latter undergoes a substantial decrease during the time that the magnet passes through the coil.

The 'missing' part of the mechanical energy is transformed into electric energy (W_el), which in turn is dissipated as heat by the resistance of the coil and the shunt resistor. The dissipated electric energy can thus be expressed as

$$W\_el = \int_0^T P_{el}(t)\,dt = \int_0^T I^2(t)\cdot(R_L + R)\,dt = \frac{R_L + R}{R^2}\int_0^T U_r^2(t)\,dt,$$

where $I(t)$ is the current that flows through the coil and shunt resistor and is calculated from a voltage drop U over the shunt resistor R. In our case the integration has been done with the same computer program as used for data acquisition (see figure 6).

Adding all the contributions of the energies one can demonstrate that the total energy is conserved within a certain accuracy (figure 5). The change of the potential energy W_pot matches the sum of the kinetic energy W_kin and the dissipated electrical energy W_el within 20% or better. The reasons for this error lie mainly in problems associated with the use of the picket fence ribbon. A part of the energy is transformed into heat due to the friction between the ribbon and the cardboard or photogate box. In addition some error may come from bending of the ribbon (for example during the deceleration), what may cause false measurements of the ribbon position. One can reduce these problems to some extent by using a transparent tube with black stripes instead of the ribbon, though one should take into account that making such a tube can be substantially more difficult than making the ribbon.
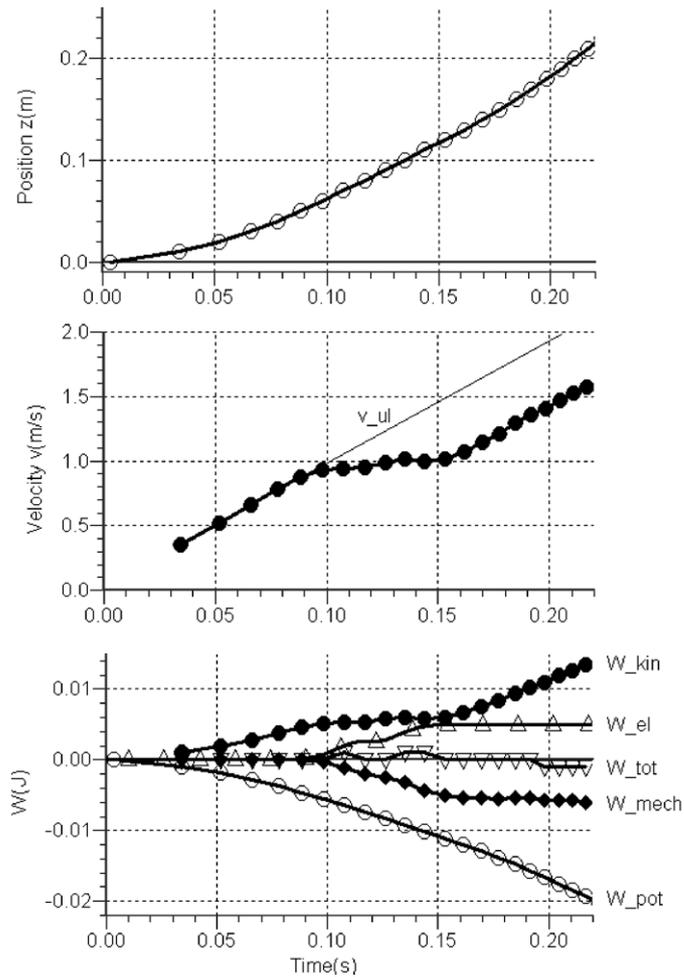
Figure 5. The graphs of position, velocity and energies during the fall through the loaded coil, the velocity v_ul of the free-falling magnet is given in the velocity graph for comparison.

Experiment 2: the verification of energy conservation with higher precision

When a significantly better accuracy of results is required, one has to avoid the formerly proposed measurement of the speed of the falling magnets. The same experiment was re-made without the picket fence ribbon, which is the major source of errors. Only final energies were compared.

The photogate was moved just below the coil, as shown in figure 7.

The cloths peg was removed and a short piece of copper wire inserted through a small hole in the tube is used to hold the magnets in place prior to the experiment. The magnets are released by pulling the wire out of the tube. The distance h from the resting point of the magnets to the photogate is measured as $170\,\text{mm} \pm 0.5\,\text{mm}$ in our setup. The speed of the falling magnets at the photogate is calculated from the known length l of the magnets and the time-of-flight T through the photogate. The experiment was repeated several times and results were averaged to obtain better accuracy. An example of calculation is given below.
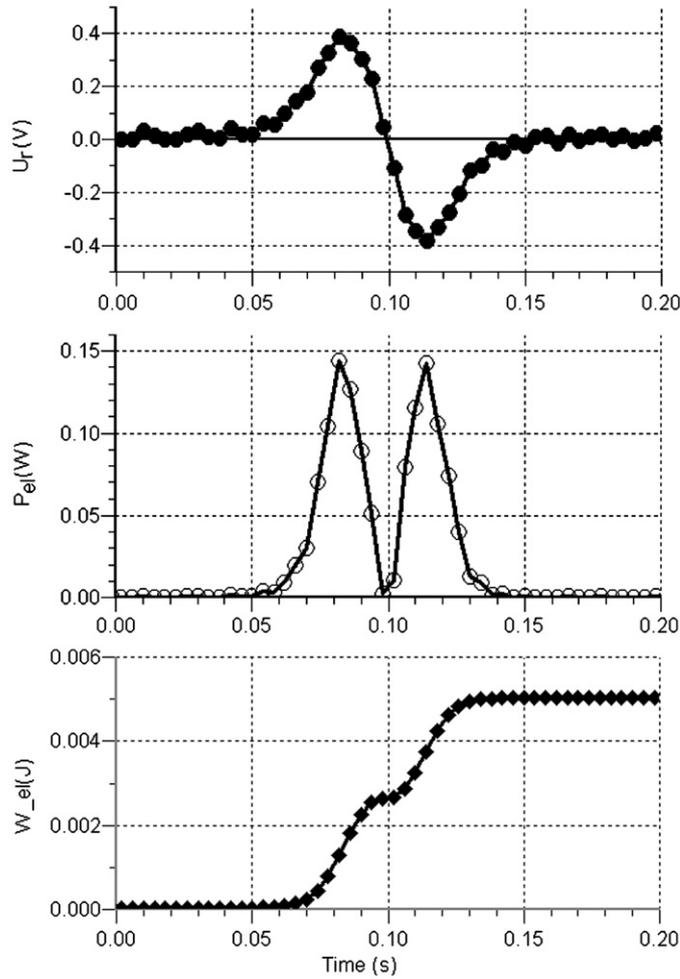
Figure 6. Voltage drop on the shunt resistor and the corresponding power and electrical energy dissipated by the circuit.

- Part 1: the potential energy W_P of the magnets is calculated as

$$W_P = mgh = 0.0091 \text{ kg} \cdot 9.8 \text{ m s}^{-2} \cdot 0.17 \text{ m} = 15.2 \text{ mJ}.$$

When taking into account the uncertainty of the mass m and the uncertainty of the length of the fall l, the result becomes

$$W_P = 15.2 \text{ mJ} \pm 0.13 \text{ mJ} = 15.2 \text{ mJ} \pm 0.8\%.$$

- Part 2: the kinetic energy W_{K1} of the free-falling magnets at the position of the photogate is calculated as

$$W_{K1} = \frac{mv_1^2}{2} = \frac{m\left(\frac{l}{T_1}\right)^2}{2} = \frac{0.0091 \text{ kg} \cdot \left(\frac{0.025}{0.0139}\right)^2 \text{ m}^2 \text{ s}^{-2}}{2} = 14.7 \text{ mJ}.$$

The uncertainty of the measured variables is reflected in the result as

$$W_{K1} = 14.7 \text{ mJ} \pm 0.2 \text{ mJ} = 14.7 \text{ mJ} \pm 1.3\%.$$
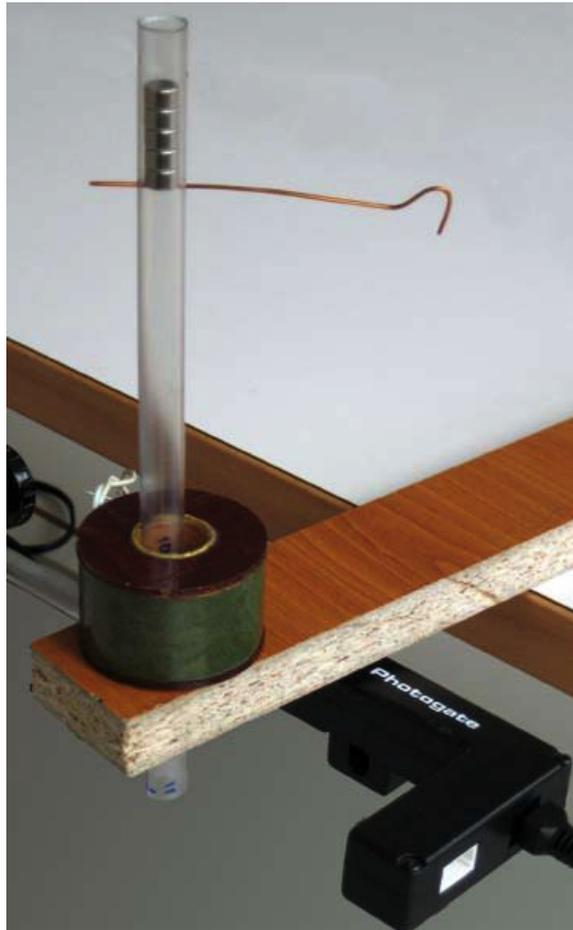
Figure 7. Experimental setup.

This verifies a conversion from the calculated potential energy W_P into the kinetic energy W_{K1} of the free-falling magnets, the difference is approximately 3%

- Part 3: the kinetic energy of the falling magnets when the coil is loaded.
  The coil was loaded by a shunt resistor of 9.7 Ω and part 2 was repeated. The kinetic energy W_{K2} is calculated as

$$W_{K2} = \frac{m\left(\frac{l}{T_2}\right)^2}{2} = \frac{0.0091 \text{ kg} \cdot \left(\frac{0.025}{0.0183}\right)^2 \text{m}^2 \text{ s}^{-2}}{2} = 8.54 \text{ mJ}.$$

  With the uncertainty of the mass m and length l taken into account, the former becomes

$$W_{K2} = 8.54 \text{ mJ} \pm 0.1 \text{ mJ} = 8.54 \text{ mJ} \pm 1.3\%.$$

  It can be seen that the shunted coil slows magnets down and therefore removes $\Delta W_K = W_{K1} - W_{K2} = 6.16 \text{ mJ} \pm 0.3 \text{ mJ}$ energy from the falling magnets.

- Part 4: the electrical energy W_{el} during the fall of the magnets.
  The voltage U_R over the shunt resistor is measured and used to calculate the electrical power dissipated in the resistance of the coil R_L and shunt resistor R. The diagram of the
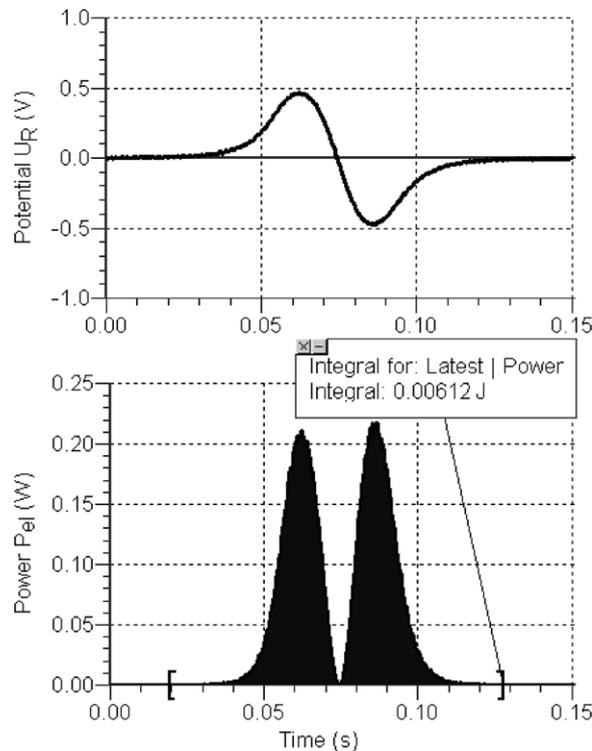
Figure 8. The voltage drop over the shunt resistor and the dissipated electrical power.

voltage U_R is given in figure 8, upper half. It can easily be calculated that the maximum current is about 50 mA. The lower diagram in figure 8 shows the electrical power P_{el} dissipated in resistances R_L and R. This is also the stopping power for the magnets which increases to 200 mW maximum.

The total electrical energy W_{el} in the coil is calculated using the software package of the interface as $W_{el} = 6.12$ mJ ± 0.06 mJ $= 6.12$ mJ $\pm 1\%$. Since the electrical interface can be calibrated using a high precision reference voltage, the uncertainty is mainly caused by the shunt resistor and the coil resistance.

This result confirms the conversion of the difference of mechanical energies ΔW_K into the electrical energy W_{el} within a few per cent.

Conclusions

A simple experiment has been presented that enables the demonstration of the conservation of energy in a system that involves transformation of mechanical energy into electric energy, which in turn is dissipated as heat. Two versions of the experiment have been described. The first experiment, particularly suitable for interactive lecture demonstration, shows time dependence of energy balance. The conservation of energy in this case is confirmed with the precision of 20% or better. The second experiment enables the verification of the conservation of energy within a few per cent by comparing the energy contributions at the beginning and

at the end of the experiment. This version is more suitable for a laboratory. Both experiments provide grounds for discussion of the energy and its conversion.

Appendix

A coil for this experiment was prepared following the specification given above. If such a coil cannot be obtained, then the experiment can be made using a primary winding of an old iron-core transformer for 220 VAC and about 20 W. The core has to be removed and the secondary winding of this transformer should not be connected. The cross-section of such a coil should be about 20 mm × 20 mm and should fit onto the plastic tube. The resistance R_L should be measured to obtain the correct value. The inductance is not important for this experiment. The quality of results depends on the coil, which should have low resistance, high number of turns and diameter matching the diameter of the falling magnets.

One has to be aware of a potential problem with the use of a stack of magnets. The experiment has to be performed every time with the same stack of magnets oriented in the same way, like north-side up. Each magnet in a stack does not produce the same magnetic field, and results depend on the strength of the field.

A lower number of magnets in the stack is not recommended due to possible tumbling of the stack within the plastic tube, which at least increases the friction and worsens the results. A higher number of magnets with the same length as the coil will effectively merge the two peaks obtained for the induced voltage (figure 3), which is didactically not recommended.

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