Measuring depth takes a lot of bottle

On a weekend break in September with my family in Savudrija – a small village on the northern Adriatic coast in Croatia – I spent some time on the beach. There are a number of physical phenomena on the beach that offer up the opportunity for exciting physics experiments. This time I was looking for an experiment in the field of static fluids.

First I started with a simple problem: how do I measure water depth? The obvious answer is by using a rope with a stone tied to its end etc. But there isn't much physics in this approach, and besides, this method is useful only for relatively shallow depths and in very calm seas.

I then remembered the problem about diving with a rubber balloon. Assuming the water temperature does not change much at the depths that I can reach, I can determine water depth from a change in a balloon's volume using Boyle's law. However, there are problems with this approach too. It is very difficult to take measurements of a balloon's volume at the bottom of the sea, and by bringing the balloon up to the surface the information about the change in volume would be lost. Could I somehow 'lock' the volume of the air in the balloon at the seabed and bring it out to dry land where I could then measure the change? Here is how I did it.

I took an empty 0.5 l drink bottle with a screw cap. I dived to the desired depth (the bottle may get flattened a little bit), turned the closed bottle upside-down (with the cap towards the sea bottom) and I opened the cap. At this moment some amount of sea water quickly moved into the bottle, compressing the air that was trapped in it. I kept the bottle in an upside-down position so that no air could escape from it. I carefully closed the bottle, swam up to the surface and gave the bottle to my 10-year old daughter Tina who measured the height of the water column (figure 1). (It is a good idea to open the upright bottle before taking measurements, since the pressure inside the bottle is now greater than the atmospheric pressure and so the bottle may get slightly deformed.) In the meantime I used the 'rope and stone' method to take another measurement of the same water depth. We repeated the measurements at five different depths.

Figure 1. *Tina marks the height of the column of water in the soda bottle (yes, we have forgotten to take a ruler to the beach again).*

Later, using the marked heights on the bottle, we measured the exact volumes of the water in the bottle with a measuring cylinder. We also determined the total volume of the bottle (table 1).

Table 1. Water volume and depth measurements

V (ml)	V_0/V	h_{calc} (m)	$h_{rope} \pm 0.05$ (m)
65	8.3	1.4	1.1
85	6.3	1.9	2.0
127	4.2	3.1	3.0
149	3.6	3.8	3.8
187	2.9	5.3	5.3

V = volume of water in the bottle; V_0 = total volume of bottle (539 ml); h_{calc} = depth determined by Boyle's law; h_{rope} = depth measured by 'rope and stone' method.

The values for depth determined using volume measurements and Boyle's law closely match those found using the 'rope and stone' method.

The change in air volume in the bottle is related to water depth. The pressure (p) at the distance h under the sea surface is the sum of the atmospheric pressure (p_0) and the hydrostatic pressure

$$p = p_0 + \rho g h$$

where ρ is sea water density (1.028 g/ml for north Adriatic sea) and $p_0 = 10^5$ N/m^2 is atmospheric pressure at sea level. The initial volume of the air in the bottle measured at p_0 is V_0. At increased pressure p the volume of the air in the bottle decreases to $V_0 - V$, where V is the volume of the water in the bottle. Assuming a constant temperature of the water and using Boyle's law,

$$p_0 V_0 = p(V_0 - V)$$

one can express the depth h in terms of V in the following way. From

$$p_0 V_0 = (p_0 + \rho g h)(V_0 - V)$$

one finds

$$h = \frac{p_0}{\rho g} \frac{1}{V_0/V - 1}.$$

Using this expression I was able to determine h_{calc} (listed in the third column of the table). Though I never doubted that physics works, I admit that I felt proud when I showed Tina how closely the two sets of depth measurements corresponded.

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