Prism foil from an LCD monitor as a tool for teaching introductory optics



2011 Eur. J. Phys. 32 601

(http://iopscience.iop.org/0143-0807/32/2/029)

IOP Publishing European Journal of Physics
Eur. J. Phys. **32** (2011) 601–613 doi:10.1088/0143-0807/32/2/029

# Prism foil from an LCD monitor as a tool for teaching introductory optics

**Gorazd Planinšič and Mihael Gojkošek**

Faculty of Mathematics and Physics, University of Ljubljana, Jadranska 19, Slovenia

E-mail: gorazd.planinsic@fmf.uni-lj.si

Received 3 November 2010, in final form 17 January 2011
Published 18 February 2011
Online at stacks.iop.org/EJP/32/601

**Abstract**
Transparent prism foil is part of a backlight system in LCD monitors that are widely used today. This paper describes the optical properties of the prism foil and several pedagogical applications suitable for undergraduate introductory physics level. Examples include experiments that employ refraction, total internal reflection, diffraction and image formation in a nontrivial way and are therefore particularly useful for active learning strategies.

## Introduction

Many items that we meet in everyday life are the result of modern technology. They may contain materials or parts with special properties that were not known or widely accessible even as little as 10 years ago. Having high-technology, low-cost items at hand opens new opportunities also for teaching. Apparatus and materials that were in the past available only in specialized laboratories are now accessible to everyone. Reports on various types of applications of such items in teaching physics at all levels can be found in numerous papers. Examples include microwave ovens, compact disks, light-emitting diodes, lasers, neodymium magnets, giant magneto-resistors and many more.

What are the benefits of using such items in teaching? Everyday high-tech items can be very useful in designing context-rich teaching materials that are often used in active learning approaches. Research showed that students view context positively both in terms of stimulating interest and in being helpful to learning [1]. The same study also showed that contexts of particular interest are those that explain something about what the student wants to find out, relate to everyday experience or centre on contemporary technology. In addition it should be noted that such items can be interesting for physics education research, for example, for probing students reasoning or evaluating their abilities to build explanatory models.

In this paper we explain the basic optical properties of a prism foil and present some pedagogical applications that utilize the prism foil and are suitable for undergraduate level[1].

[1] Basic physics of prism foil and some pedagogical applications have been presented in oral presentation by one of us (MG) at GIREP Conference, August 2010 in Remis, France (Proceedings book is in preparation).

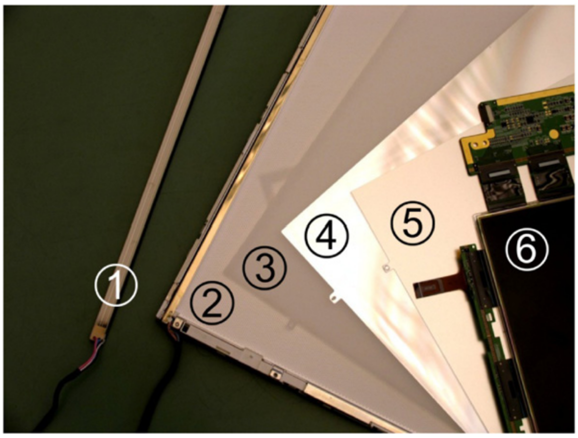


**Figure 1.** Structure of a typical LCD monitor screen (starting from the back of the screen): 1—fluorescent lamp; 2—light guide plate; 3—first diffusive foil; 4—prism foil; 5—second diffusive foil; 6—LCD panel with part of the electronics.

Prism foil is part of common LCD monitors. It employs basic concepts of optics in an intriguing way. Though the application of the prism foil in LCD monitors is related to interesting physics in itself, our primary goal in this paper is not to describe the role it plays in LCD monitors but rather to describe various examples how it can be used in teaching physics at undergraduate level. The price of LCD monitors today is low enough that buying a new monitor is often cheaper than repairing a faulty one. Though this is not in line with sustainable development, it gives easy access to LCD monitor parts, including the prism foil described here. But before we continue with the examples, let us describe briefly how to get the prism foil and what its main function is in LCD monitors.

## Prism foil

Vaguely speaking, a LCD monitor consists of a power supply, an electronic circuit and a screen. The screen consists of an LCD panel, where the image is formed and a backlight module, which further consists of a light source (usually one or two tubular fluorescent lamps) and several layers that optimize light for viewing the image on the LCD panel [2, 3]. Prism foil (also called backlight enhancement film*)* is one of the foils in backlight module. The main task of this foil is to let through the light in the direction appropriate for the LCD panel and reflect most of the remaining light back into the display, where it is recycled. Using the prism foil the viewing angle of light that emerges from the screen is compressed in one direction, increasing the display brightness. More information about the prism foil including material safety data can be obtained from [4].

Apart from the prism foil and the fluorescent lamp the backlight module consists of other diffusive foils and a light guide plate made from Plexiglas (see figure 1). Though disassembling the used LCD monitor is rather a straightforward task, one should be careful when removing and handling the LCD panel because it consists of glass plates that can easily break.

So far we deliberately did not show any details of the prism foil or explain any physics behind it. This is done in the remaining part of the paper in an investigative way. An investigative approach is found to be successful as a basis for teaching strategies that engage students in learning that mirrors scientific practice, such as Investigative Science Learning

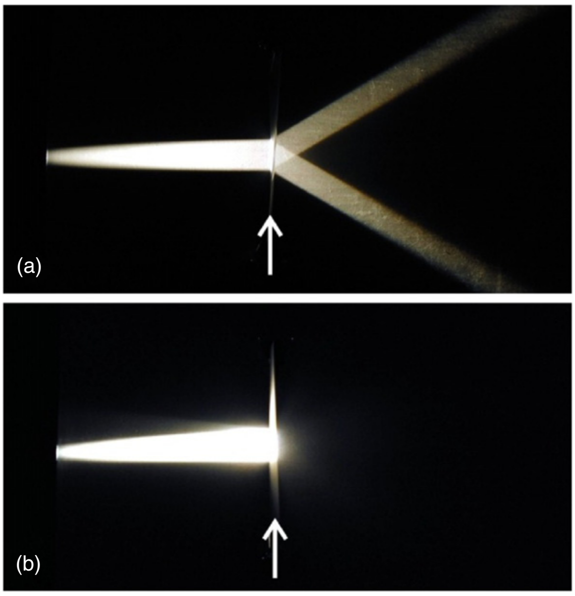


**Figure 2.** (a) Light beam incident perpendicularly on one side of the foil splits into two symmetrical beams; (b) if incident on the other side of the foil, the light beam gets reflected. The arrows indicate the position of the foil.

Environment (ISLE) [5, 6]. In ISLE students construct new ideas themselves by first observing carefully selected phenomena, proposing multiple explanations for their observations and then designing experiments to rule out those explanations. Explanations that they fail to rule out are then used for further investigations and practical applications. If students are expected to participate actively the name 'prism foil' should be initially replaced by another name (for example 'a magic foil' or simply 'a special foil') that does not suggest a construction of the foil.

## Experiments with prism foil

### *Perpendicular incidence*

Let us start with a simple observation. A beam of light (e.g. from flashlight, slide projector or overhead projector) incident perpendicularly on one side of the foil splits symmetrically into two outgoing beams at angles of about 30° with respect to normal (figure 2(a)). If the foil is turned around so that the light beam strikes its other surface, almost all the light is reflected back, leaving the space behind the foil dark (figure 2(b)).

The structure of the foil is revealed if it is observed under the microscope. Figure 3(a) shows the top view of the foil with 0.2 mm copper wire placed on top of it to set the scale. Figure 3(b) shows the side view of the foil (a thin strip has been cut and placed with cross-section facing up under the microscope). Observations under the laboratory microscope suggest that the foil consists of prismatic ridges with angles of about 90° at their apices and with distance of about 0.05 mm between the neighbour apices. The thickness of the prism foil is about 0.15 mm. Using a better microscope the distance between the ridges was determined to be $48.2 \pm 0.6\ \mu$m.

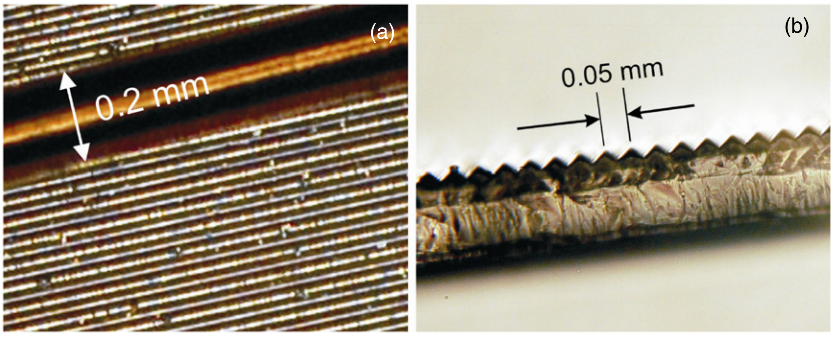


**Figure 3.** Prism foil: (a) top view and (b) side view under the laboratory microscope. A 0.2 mm copper wire has been placed on the foil to set the scale.

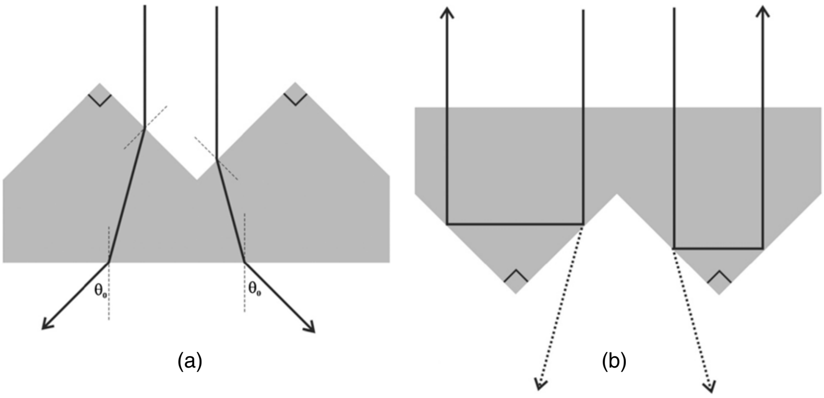


**Figure 4.** Light beam incident perpendicularly on (a) the prism side and (b) the flat side of the foil.

From now on we shall use the name prism foil. The next step is to build and analyse the theoretical model and then verify the theoretical predictions against the experimental results. All these steps are suitable for the first year university level.

We assume that the prism foil consists of prismatic ridges with angles of 90° at their apices. In the analysis we focus on the main optical phenomena that give easy observable results. First let us study the simple case when the parallel light beam is incident perpendicularly to the flat surface of the prism foil. If the index of refraction of the foil is larger than $\sqrt{2} \approx 1.414$, then the light beam undergoes total internal reflection and returns to the original direction (figures 2(b) and 4(b)). If the index of refraction is smaller than 1.414, then the light beam undergoes simple refraction at the prism surface (dotted lines in figure 4(b)).

If the parallel beam of light is incident perpendicularly on the prism side, the beam undergoes two refractions and emerges at angles $\pm\theta_0$, depending on which side of the prisms the beam strikes (figure 4(a)).

The angle $\theta_0$ can be calculated using simple geometry and Snell's law at both boundaries. After short calculation the following expression can be found:

$$\sin\theta_0 = \tfrac{1}{2}(\sqrt{2n^2 - 1} - 1), \tag{1}$$

which can be rearranged to express the index of refraction as a function of the emerging angle

$$n = \sqrt{2\sin^2\theta_0 + 2\sin\theta_0 + 1}. \tag{2}$$

As with any prism, dispersion is also present in the prism foil. An attentive observer will note that two outgoing symmetrical beams have reddish hue on the inner side and bluish on the outer side. To avoid problems with dispersion we measured the angle $\theta_0$ using green laser light with wavelength 532 nm. We measured $\theta_0 = 30^\circ \pm 1^\circ$, which gives $n = 1.58 \pm 0.02$ and a corresponding critical angle of 39.3°. In this case the accuracy of the measured index of refraction is restricted also by the interference that cannot be eliminated. Measurement error can be estimated with an angle between two neighbour maxima in the interference pattern, which is in our case approximately 0.7° (see the section on interplay between refraction and diffraction). Anyway, the measured index of refraction matches reasonable well with 1.5750, the index of refraction for polyethylene terephthalate (also known as Dacron), which is the reported material that prism foils are made of [7]. An interesting upgrade of this problem is the study of light beam refraction on a wavy surface [8], which is suitable for introductory optics.

### *General case*

In the general case, a parallel beam of light is incident on a prism foil at an arbitrary angle. We assume that the index of refraction of the foil material is 1.58. We focus only on the major effects that can be easily observed and give equations in the appendix. Let us adopt the convention about the signs of the angles: the angles of incident and emerging beams are measured from vertical, which indicates 0°; clockwise angles are taken as positive and counterclockwise as negative. Experiments described in this section can be done with a laser ray box and angular scale, which enable easy and accurate measurement of angles of incident and outgoing light beams.

If the light beam is incident on the flat side of the prism foil the following scenarios may happen (figure 5(a)). Part of the light beam always reflects from the top flat surface (C). The remaining part of the light beam gets refracted at the top surface and then can either refract once again and leave the foil (B) or it can first undergo a total internal reflection and then refract and leave the foil (A), depending on which side of the prism the refracted beam is incident on.

For the light beam incident at angle $\varphi$ on the prismatic side of the foil the following scenarios happen (figure 5(b)). Let us study first the light beams that emerges from the prism side (i.e. incident side). At one set of prism sides single reflection occurs and the emerging beam appears at angle $90^\circ - \varphi$. This beam can either leave the foil (case D) or it hits the opposite prism and mostly reflects in the direction of the incident beam (dotted line on the right from the case D in figure 5(b)).

The light beam that reflects from the other set of prism sides (cases E and C) cannot emerge after single reflection. Part of it reflects again on the opposite prism and emerges in the direction of the incident beam (not shown in the sketch) while the remaining part undergoes refraction, total internal reflection and another refraction and emerges from the prism side at angle $\varphi - 90^\circ$. Part of this beam emerges from the foil (case E) while the remaining part hits the opposite side of the prisms, reflects and then emerges at angle $-\varphi$ (case C).

Explanations for the cases E and C can be supported by a simple experiment. If an opaque self adhesive tape (such as Scotch tape) is firmly pressed on the flat side of the prism foil all light beams except beam D disappear or get significantly dimmed (see figure 6). The tape represents a third medium with high enough index of refraction that total internal reflections

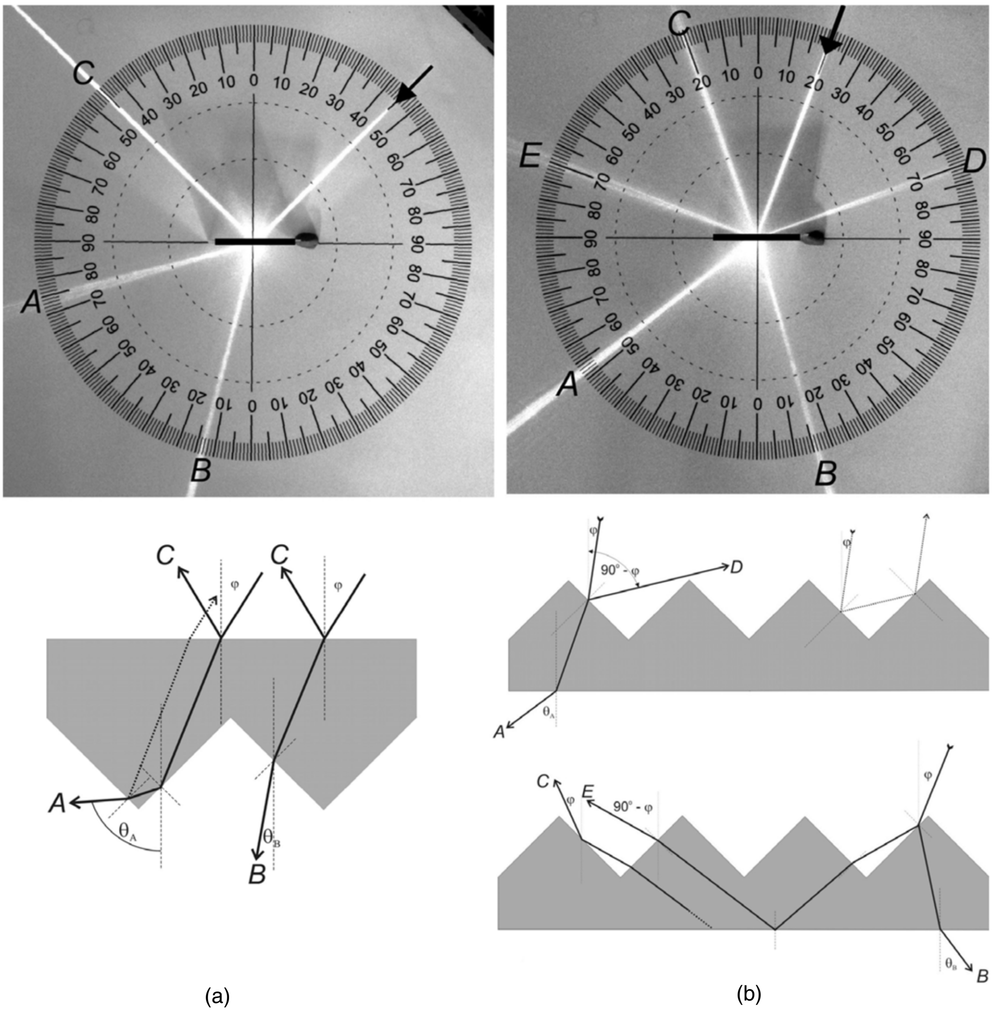


**Figure 5.** Light beam incident on the prism foil at arbitrary angle: (a) on the flat side and (b) on the prism side of the foil.

on the flat side of the prism foil are frustrated. Due to the opaque surface of the tape the light that passes through it is diffusely scattered. This affects all scenarios except D.

Pedagogically interesting is a special case when the light beam is incident on the prism side at 45° (i.e. perpendicularly to one set of prism sides). After the first refraction, which does not change the direction, the light beam undergoes total internal reflection at the flat side of the foil and emerges from the foil at angle −45°. In this case the prism foil behaves like an ordinary mirror. The case is pedagogically interesting because it can be predicted from a simple model without any calculations.

### *Third medium*

Let us return to the simple case where the light beam is incident perpendicularly on the flat side of the foil and study what happens if a third transparent medium (a liquid for example)

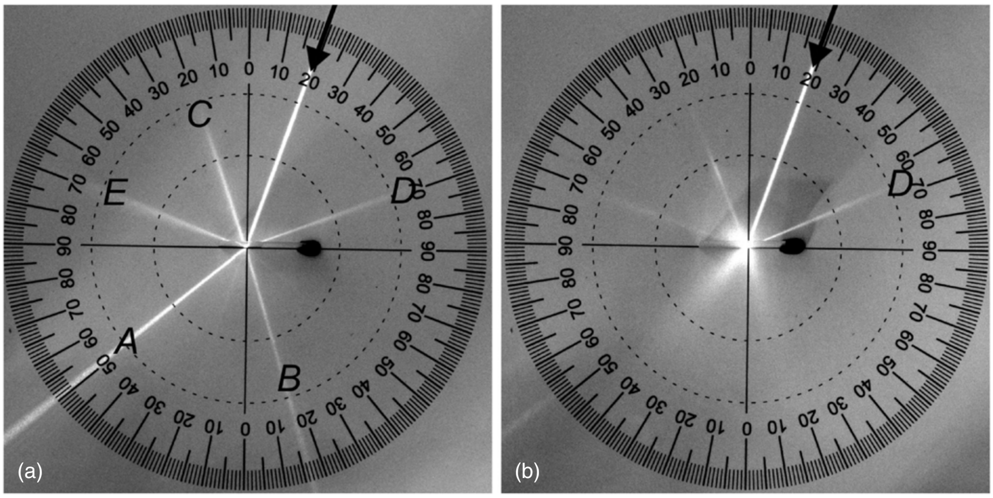


**Figure 6.** (a) Light beam incident on a prism side of the foil; (b) same as (a) but with Scotch tape covering the flat side of the foil.

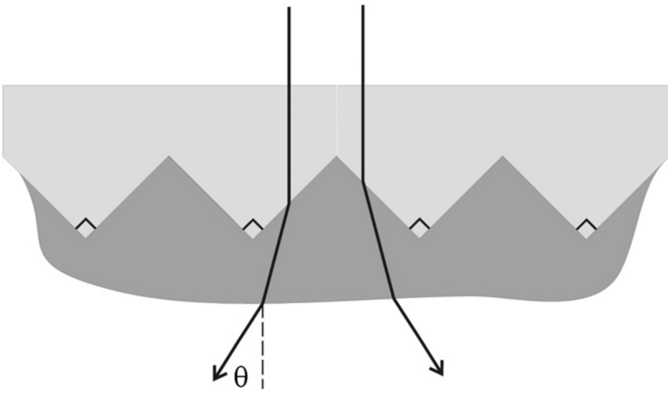


**Figure 7.** Refraction of light incident perpendicularly on the flat side of the foil with liquid deposited on the opposite side of the foil.

covers the prism side of the foil. In case this third medium has high enough index of refraction $n_l$ that the light beam does not undergo total internal reflection at prism sides but is rather refracted into the new medium. If the third medium surface on the air boundary is flat, two symmetrically emerging light beams at angles $\pm\theta$ are observed (figure 7). To ensure that the liquid–air boundary is sufficiently flat, one must use enough liquid to fill the prism ridges and cover an area that is much larger than the cross section of the laser beam (in our case a few drops of liquid were sufficient). The laser beam with diameter of about 2 mm will illuminate more than 40 prism ridges at the same time. For this reason small variations in flatness of the liquid surface turn out to have only little effect on the sharpness of the outgoing beams.

Using simple trigonometry and Snell's law the following expression relating $n_l$ and $\theta$ can be found:

$$n_l = \sqrt{\frac{(n - 2\sin\theta)^2 + n^2}{2}}, \tag{3}$$

**Table 1.** Indices of refraction for some common liquids as measured with research quality refractometer and with the prism foil.

| | $n$ (Refractometer) | $n$ (Prism foil) |
|---|---|---|
| Water | 1.332 | 1.33 |
| Vinegar | 1.337 | 1.34 |
| Olive oil | 1.467 | 1.44 |
| Ethanol | 1.362 | 1.36 |
| Glycerin | 1.452 | 1.45 |
| Paraffin oil | 1.466 | 1.46 |

where $n$ is the index of refraction of the prism foil. By measuring the angle $\theta$ and knowing the index of refraction of the prism foil this expression can be used to determine the index of refraction of the medium $n_l$. The range of indices $n_l$ that give observable effect depends on the value of $n$. In our case $n = 1.58$, which gives the range for $n_l$ from 1.11 to 1.57. This range is wide enough to cover most liquids used in everyday life. Comparison between the indices of refraction for some common liquids measured with the prism foil and values measured with research quality refractometer are shown in table 1. In prism foil measurements a green laser beam with a wavelength of 532 nm has been used. Once again, the accuracy of these measurements is limited by interference phenomena. The method proved to be useful as a first year student project task or home experiment for students.

### *Image formation*

A prism foil can produce virtual images of objects that are placed behind it. The same images could be obtained with a single prism [9] but in practice this requires a prism of unusually large size (and considerable weight).

Place a flat object (such as pocket calculator or mobile phone) on the table on its long narrow edge. Bring the prism foil close to the object and observe the image. If the prism side is facing the object, only the reflected light can be seen but if it faces the observer, a double-view image that reveals both sides of the object appears behind the foil (see figure 8(a)). Image formation can be qualitatively explained by tracing the rays from two extreme points on the object (figure 8(b)).

Each of the two images is showing the object as rotated approximately 30° and −30° from the original position. This can be easily explained assuming the observer is very far from the prism foil. In this case the rays entering observer's eyes are nearly parallel. In our case emerging parallel rays can only be obtained if the incident rays are forming the angle ±30° with normal to the prism foil (see figure 4(a) with reverse directions of the light beams). The situation is schematically shown in figure 8(c). If the size of the object is $b$, then a distant observer can see the projection of the object which is equal to $a = \cot(60°)b$. The apparent tilt angle of the object can be estimated also from the ratio of the dimensions on the photograph. The estimated value for the tilt angle in our case is 27°.

If two objects are placed symmetrically further behind the prism foil so that the viewing angle of them measured from the foil is around 60°, their images will overlap straight in front of the observer behind the foil. Figure 9(a) shows two texts and their combined image as seen through the prism foil. Figure 9(b) shows a qualitative explanation of image formation in this case. Note that the image of the foil of the paper which is placed between the texts perpendicularly to the board is not visible from the point of observation.

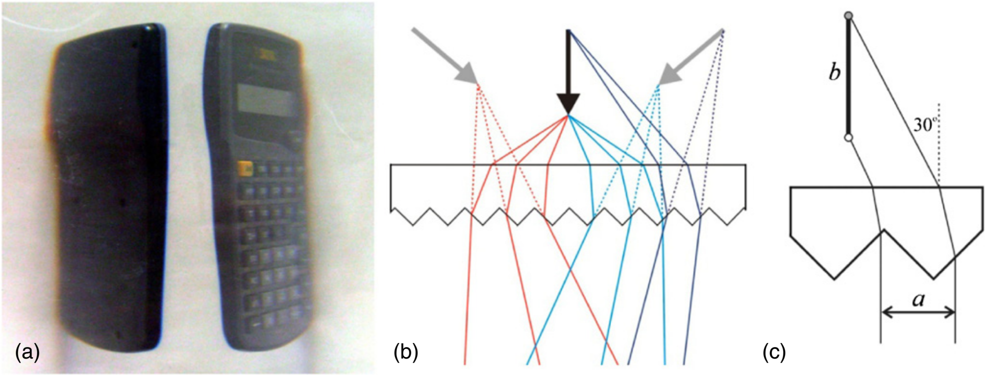


**Figure 8.** Double-view image of the calculator that is placed near the prism foil: (a) photo of the image; (b) image construction; (c) sketch for the simple derivation of the image tilt ($a$ is the size of the object and $b$ is the size of the image as seen by the very distant observer).

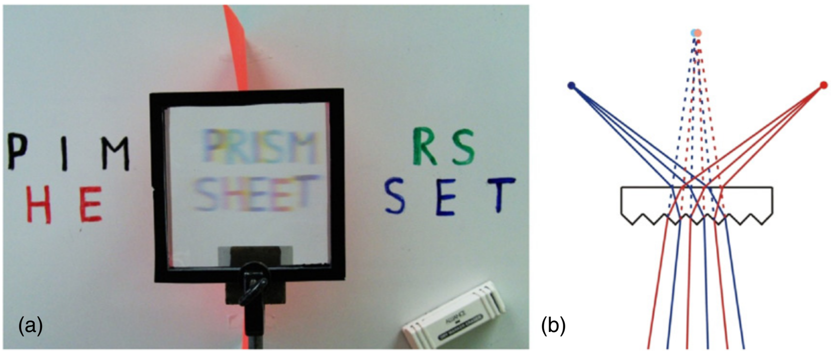


**Figure 9.** (a) Overlapping image of two texts that are written on the whiteboard (note that the image of the foil placed between the texts, perpendicular to the board, does not appear in the image); (b) image construction.

### *Interplay between refraction and diffraction*

It is interesting to examine the foil with a laser light more carefully. In the previous section we described experiments with a laser from the ray box, but since we observed the light beams very near to the prism foil, the far-field diffraction pattern was not observed. In the experiment described in this section we used a green laser pointer with wavelength of 532 nm and observed the light pattern on the screen which was about 2 m away from the prism foil. A laser beam incident perpendicularly on the prism side of the foil produces on the screen a symmetrical pattern with brightest points in the directions at about 30° with respect to incident beam direction (figure 10). If incident perpendicularly on the opposite side of the prism foil, again almost all laser light is reflected. We noted that the sharpness of diffraction pattern may depend on the quality of the prism foil. Foils with slightly rounded apices of prisms tend to produce more blurred maxima.

The observed diffraction pattern is the result of combined effect of diffraction and refraction. In the incident laser beam wavefronts can be treated as plane waves. At the boundary with prism foil wavefronts bend due to refraction but since the distance between

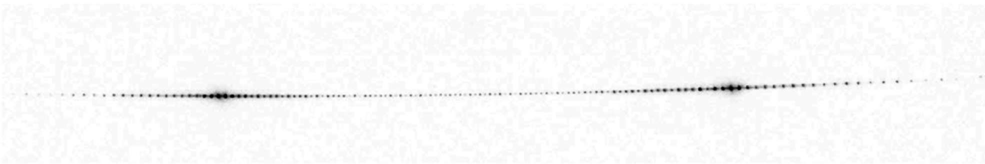

**Figure 10.** Pattern obtained on the screen when a laser beam is incident perpendicularly to the prism side of the foil. The photograph has been inverted.

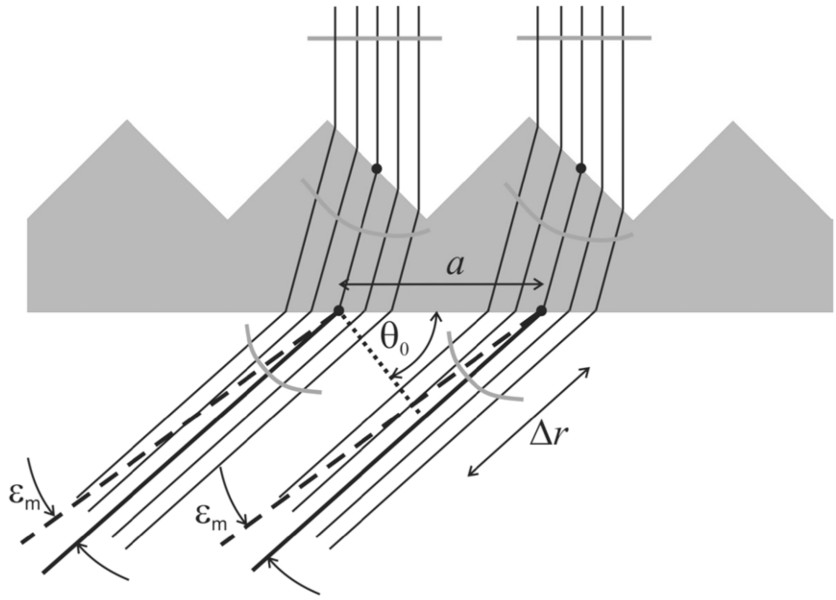


**Figure 11.** Diffraction of laser light after passing through the prism foil. For clarity only incidence on two right sides of prism ridges is shown.

neighbour prism ridges $a$ (in our case about 50 $\mu$m) is comparable to the wavelength of light the wavefronts are no longer planar but curved due to diffraction. At the rear side of the prism foil wavefronts bend again due to refraction and interfere when they meet.

At large distance from the prism foil interference can be treated in a similar way as is usually done for diffraction grating, bearing in mind that the direction of propagation of wavefronts has been changed due to double refraction (figure 11).

The approximate path length difference from the two adjacent prisms is

$$\Delta r = a \sin\theta_0. \tag{4}$$

To obtain maximum in the direction of $\theta_0$ the following condition should be fulfilled:

$$a \sin\theta_0 = N_0\lambda \tag{5}$$

(note that in our case $N_0$ is of the order of 50). Other maxima in directions $\varepsilon_M$ with respect to $\theta_0$ are given by the following condition:

$$a \sin(\theta_0 + \varepsilon_M) = N_0\lambda + M\lambda; \qquad M = \pm 0, 1, 2, \ldots \tag{6}$$

**Table 2.** Measured and calculated angular positions of diffraction maxima.

| $M$ | $\varepsilon_M$ (calculated) (deg) | $\varepsilon_M$ (measured) (deg) |
|---|---|---|
| 8 | 5.97 | 5.9 |
| 7 | 5.2 | 5.2 |
| ... | ... | ... |
| 3 | 2.19 | 2.2 |
| 2 | 1.46 | 1.5 |
| 1 | 0.71 | 0.7 |
| −1 | 0.71 | 0.7 |
| −2 | 1.44 | 1.3 |
| −3 | 2.15 | 2 |
| ... | ... | ... |
| −7 | 4.95 | 4.8 |
| −8 | 5.64 | 5.6 |

from which the expression for angular positions of maxima can be derived:

$$\varepsilon_M = \arcsin\left(\sin\theta_0 + \frac{M\lambda}{a}\right) - \theta_0. \tag{7}$$

For the maxima close to the angles $\pm\theta_0$, $|\varepsilon_M| \ll \theta_0$ and therefore the approximate angular positions are given by the following expression:

$$\varepsilon_M = \frac{M\lambda}{a\cos\theta_0}. \tag{8}$$

This expression is similar to the small angle approximation of the well-known diffraction grating formula, except for the cosine factor. Its presence reminds us that interference pattern is not formed around the direction of the incident light beam, but rather around directions of $\pm\theta_0$, which are determined with refraction of the incident light beam. Note that the theoretical expression also takes into account the observed asymmetry in diffraction pattern. As shown in table 2 measured values agree reasonably well with values calculated from equation (7). In that case accuracy of measurement is better because positions of maxima $\varepsilon_M$ were measured relative to the angle $\theta_0$. In order to minimize the error of projection we rotated the experimental setup, so that one of the main outgoing beams was perpendicular to the screen.

This experiment has important pedagogical value even if we perform no measurements. In textbooks refraction and diffraction are always treated as separate phenomena and to our knowledge no example has been described where these two phenomena occur simultaneously. The presented experiment with prism foil offers an excellent opportunity to demonstrate the interplay between refraction and diffraction in a simple but efficient way. Alternatively, the experiment can serve for testing students' ability in applying acquired knowledge in a new situation.

## Conclusions

We have described several pedagogical applications of the prism foil that can be found in common LCD monitors. If the parallel light beam is incident on the foil the outcomes can be explained using refraction and total internal reflection. If liquid is spread on one side of the prism foil and the light beam is aimed perpendicularly at the foil the index of refraction of the liquid can be determined from angular deflection of outgoing beams. If laser light is used, an interesting diffraction pattern can be observed that is the result of combined effects of diffraction and refraction. Prism foil can also be used to obtain unusual images, such as simultaneous views of an object from two angles or combination of two images. The beauty of this high-tech easily available device is that it employs basic optical phenomena in nontrivial way, which makes it perfect to be used in context-rich teaching materials that are often used in active learning approaches.

## Acknowledgments

The authors wish to thank Josip Slisko, Mojca Čepič and Aleš Mohorič for valuable discussions and comments that helped improve the paper.

## Appendix

Note that the angles of incident and emerging beams are measured from vertical, which indicates 0°; clockwise angles are taken as positive and counterclockwise as negative. Also note that due to the symmetry of the problem incidence at $-\varphi$ should give negative value of incidence at $\varphi$.

### *A.1. Light beam incident on the flat side of the prism foil (figure 5(a))*

The angles of the emerging beams $\theta_A$ and $\theta_B$ can be calculated by using some trigonometry and Snell's law at both boundaries. The angle $\theta_B$ can be expressed in the following form:

$$\theta_B = \arcsin\left[\frac{\sqrt{2}}{2}\left(\sqrt{n^2 - \sin^2\varphi} - \sin\varphi\right)\right] - \pi/4, \tag{A.1}$$

and it can be easily shown that

$$\theta_A = \pi/2 - \theta_B. \tag{A.2}$$

Equations (A.1) and (A.2) give physically meaningful solutions that can be verified through observations under the following conditions.

- In cases A and B the internal incident angle at the exit from the foil should be small enough for refraction to occur. In our case this implies $\varphi > 9.1°$. If the incident angle is smaller than this value, the light beam undergoes two total internal reflections and is redirected back into the direction of the incident beam (dotted line in figure 5(a)).
- $\theta_A$ should be smaller than 90° for the outgoing light beam to emerge from the foil without hitting the opposite prism side. In our case this requires $\varphi > 24.5°$.

These conditions restrict the incident angle to the interval $24.5° < \varphi < 90°$ $(-90° < \varphi < -24.5°)$ for case A to be observed and to $9.1° < \varphi < 90°(-90° < \varphi < -9.1°)$ for case B to be observed.

### *A.2. Light beam incident on the prismatic side of the foil (figure 5(b))*

Now let us study the beams that emerge from the bottom side of the foil. This part can undergo two successive refractions and emerges at angles $\theta_A$ (case A) or $\theta_B$ (case B), depending on which set of the prism sides the first refraction occurs.

Again the angles of the emerging beams can be calculated using a similar approach as in the previous case leading to the final expressions for the angles of the emerging beams:

$$\theta_A = -\arcsin\left[\frac{\sqrt{2}}{2}\left(\sqrt{n^2 - \sin^2(\pi/4 - \varphi)} - \sin(\pi/4 - \varphi)\right)\right], \tag{A.3}$$

and

$$\theta_B = \arcsin\left[\frac{\sqrt{2}}{2}\left(\sqrt{n^2 - \sin^2(\pi/4 + \varphi)} - \sin(\pi/4 + \varphi)\right)\right]. \tag{A.4}$$

Note that in this case clear experimental results are obtained only for incident angles in the interval $-45^\circ < \varphi < 35.9^\circ$ ($-35.9^\circ < \varphi < 45^\circ$). At incident angles larger than $35.9^\circ$ (smaller than $-35.9^\circ$) light beam undergoes refraction and then total internal reflection at the flat side of the foil.

## References

[1] Bennett J 2003 *Teaching and Learning Science* (London: Continuum) p 119

[2] Koyama Y 2001 Ray-tracing simulation in LCD development *Sharp Tech. J.* **80** 51–5

[3] Li C J, Fang Y C, Chu W T and Cheng M C 2008 Design of a prism light-guide plate for an LCD backlight module *J. Soc. Inf. Disp.* **16** 545–50

[4] http://solutions.3m.com/wps/portal/3M/en_US/Vikuiti1/BrandProducts/secondary/optics101/ (retrieved on 29.10.2010)

[5] Etkina E and Van Heuvelen A 2007 Investigative science learning environment—a science process approach to learning physics *Research-Based Reform of University Physics* ed E F Redish and P J Cooney (AAPT) www.compadre.org/per/per_reviews/media/volume1/isle-2007.pdf (retrieved on 29.10.2010)

[6] Brookes D and Etkina E 2010 Phenomena in real time *Science* **330** 605–6

[7] http://www.suntech-web.jp/pdf/suncrysta_2008.pdf (retrieved on 29.10.2010)

[8] Čepič M 2008 Underwater rays *Eur. J. Phys.* **29** 845–55

[9] Galili I and Goldberg F 1996 Using a linear approximation for single-surface refraction to explain some virtual image phenomena *Am. J. Phys* **64** 256–64