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Two-liquid Cartesian diver

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Abstract

It is quite easy to make a version of the well known Cartesian diver experiment that uses two immiscible liquids. This allows students to test their knowledge of density and pressure in explaining the diver's behaviour. Construction details are presented here together with a mathematical model to explain the observations.

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The Cartesian diver is one of the most popular simple experiments. It is always attractive even if you have seen it many times before. In addition, this simple experiment offers various possibilities for interesting inquiry questions for students, therefore encouraging discussion and a search for the right answers. For these reasons we decided to build a large-scale version of the Cartesian diver for our hands-on science centre in Ljubljana. But, as in any other science centre where people build their own exhibits, we also wanted to add 'something new' to our exhibit. The following idea emerged from discussion: let's try with two liquids of different densities that do not mix and two or more divers that will initially float at different liquid boundaries. As usual, the idea was first tested on a simple prototype made from material that one can find easily. While playing with a prototype it appeared to one of us that the two-liquid Cartesian diver may be useful as a demonstration experiment in school. In this article we present the construction of the experiment and some suggestions for how to use it at the advanced level in secondary school.

There are numerous references in the literature and on the web on how to construct a simple Cartesian diver, how to explain its behaviour and how to use it in building students' ideas about the observed phenomena. When searching for previous reports on multi-liquid Cartesian divers we found only a report on the Cartesian diver using continuously varying liquid density obtained with salted water [1]. One drawback of such a diver is that even if we do not touch the bottle the density of the liquid will eventfully become uniform due to convection and diffusion.

How to make it

Fill half of a soda-pop bottle with a liquid for washing car windows (we used Sonax, the ethanol-based, blue-coloured liquid with a density of about 0.91 $\mathrm{g\,cm^{-3}}$ that freezes at -40 °C according to the producer) and the rest with paraffin oil, a colourless, odourless liquid with a density of about 0.86 $\mathrm{g\,cm^{-3}}$. Fill the paraffin oil almost to the top of the bottle. The different colours, similar viscosity and the fact that they do not mix together make these two liquids ideal for our experiment. In addition, the two liquids do not deteriorate with time. However, the simple combination of water and cooking oil also works well, as shown later in this article.

Making the divers

There are several suggestions in the literature and on the web for how to make divers using eyedroppers, test tubes or even ketchup bags. We made ours from folded drinking straws and we

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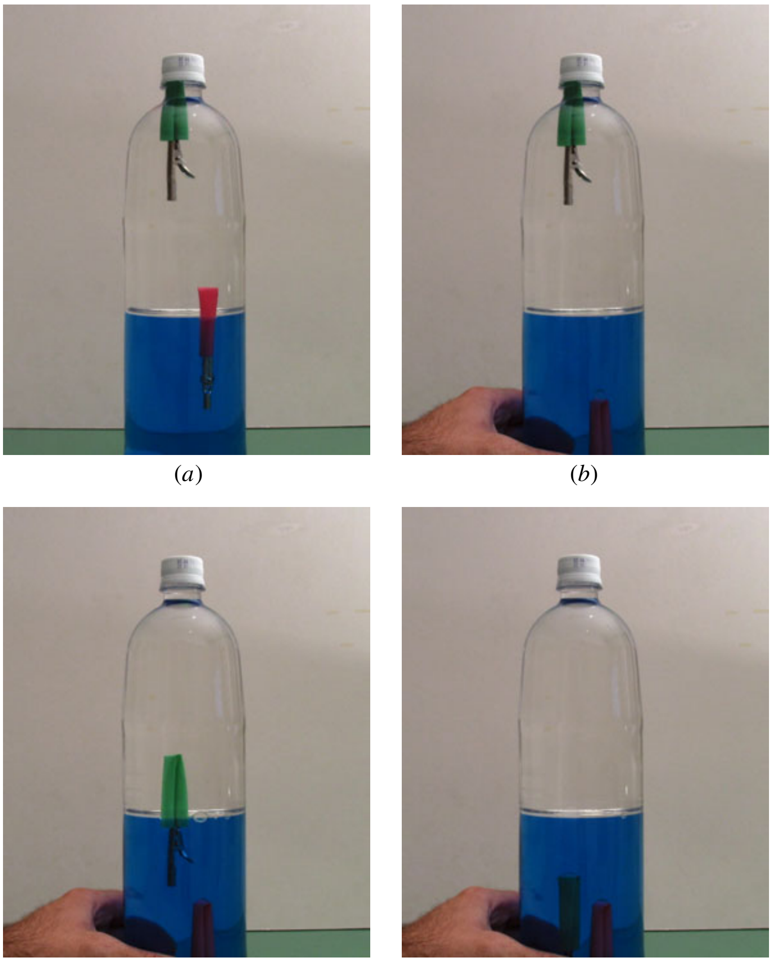

(*a*) (*b*) (*c*) (*d*)

Figure 1. Two-stage Cartesian diver: (*a*) when left alone; (*b*)–(*d*) as the plastic bottle is squeezed harder and harder.

used crocodile clips to adjust their masses (see figure 1).

Adjusting the divers

First take a piece of thick wire that is about 5 cm longer than the height of the bottle, and bend it to make a hook at one end. You will need it for lifting the sinking divers from the bottle. Make a first diver as explained in the previous paragraph and put it into the filled bottle. If you are lucky the diver should float on the upper liquid or on the lower liquid, at the boundary between the two liquids. Depending on this, cut a second diver's straw so that it will float on the surface of the other liquid (if you cut a longer straw the diver will be less dense and vice versa). Be prepared to make fine adjustments by cutting off short pieces of the straws to finally achieve the desired result (figure 1(*a*)). Any diver that sinks to the bottom of the bottle should be replaced by a new one made from a longer piece of straw.

When you have adjusted the divers, close the bottle and start squeezing it with your fingers. In our case, the lower diver sank first (figure 1(*b*)). When the bottle was squeezed further the upper diver started sinking but stopped at the boundary between the liquids (figure 1(*c*)). By varying the force on the bottle walls the diver sank more or less into the lower liquid, as its effective density was changed. By squeezing the bottle even harder the upper diver eventually also sank to the bottom (figure 1(*d*)). Reducing the pressure on the bottle walls caused the divers to rise to their initial

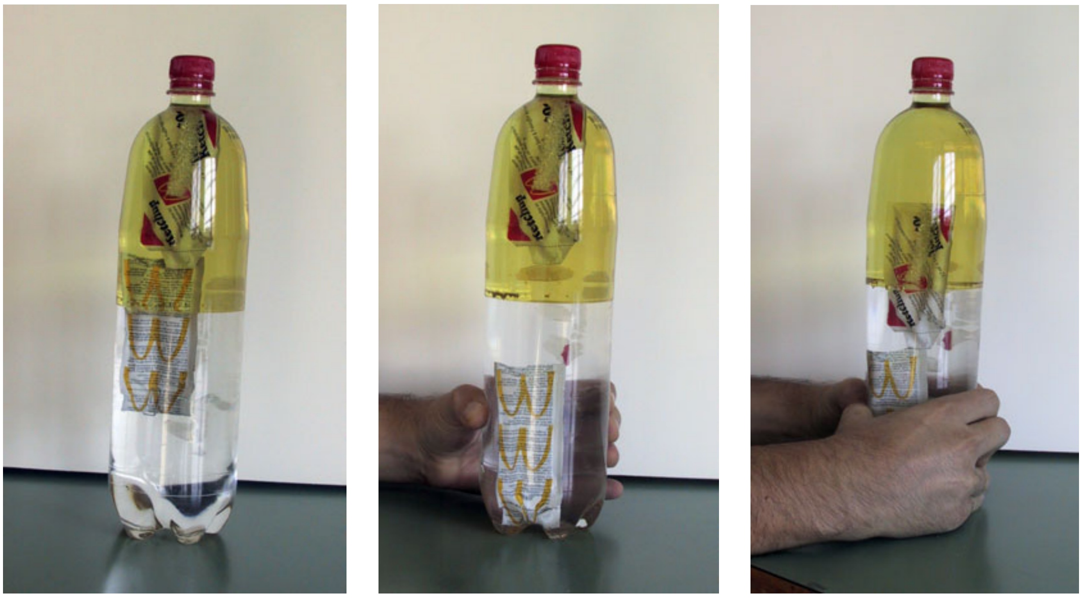

Figure 2. The kitchen version of the two-liquid Cartesian diver: cooking oil at the top, water at the bottom and ketchup bags as divers. See text for details on how to adjust the divers.

positions in the reverse order to that in which they sank.

Kitchen version of the two-liquid Cartesian diver

The two-liquid Cartesian diver can also be made from materials found in the kitchen. Cooking oil and water make good substitutes for the paraffin oil and Sonax. As explained earlier, ketchup bags are ideal divers. The little air bubble trapped in the bag works in the same way as the air bubble trapped in the straw. However, fine adjustment of the ketchup divers takes a little more patience. The lower one can be made heavier by fixing one or two paper clips to it. The upper one should be made a little lighter. This can be achieved by gluing a narrow strip of Styrofoam to it. The kitchen version of the two-liquid Cartesian diver in operation is shown in figure 2.

It is instructive to explore and compare the divers' behaviour when they are both immersed in the upper or the lower liquid with that when they are between the two liquids.

Using the two-liquid Cartesian diver in the physics classroom

Here are some suggestions for inquiry questions that can be asked when showing the two-stage Cartesian diver in school:

Q1. Plot a graph that will show how the density of the liquid in the bottle varies with the depth. How does the graph change when the bottle is squeezed?

Q2. Plot a graph that will show how the pressure in the bottle varies with depth. (Note that there is a small air bubble trapped in the bottle.) How does the graph change when the bottle is squeezed?

Q3. Try to predict how a change of temperature would affect the experiment (recall how the Galileo thermometer works).

The answers to the first two questions are shown in figure 3 but readers are encouraged to find and experimentally check the answer to the last question.

We have demonstrated the two-liquid Cartesian diver and posed questions Q1 and Q2 to a group of 31 first-year physics students (age 19) at one of the first meetings at the beginning of the school year (in Slovenia physics is a compulsory subject at all secondary schools). Students were reminded that the liquids are practically incompressible. About 26% of the answers were completely correct and 45% of the students gave the correct answer to Q1. There were no cases with both the wrong answer to Q1 and the correct answer to Q2. Some typical wrong answers are

3. Density and pressure

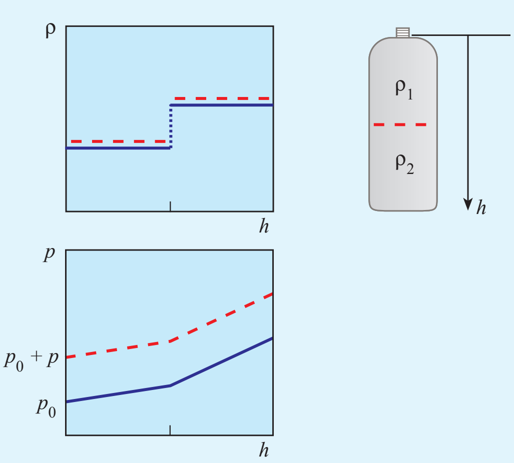

Figure 3. The liquid density and the pressure in the bottle as a function of depth in the closed bottle filled with two liquids that do not mix. The dashed line shows the change after the bottle is squeezed.

shown in figure 4. It was clear that many students had problems transforming the natural vertical axis defined by the direction of g in the experiment into the horizontal axis on the graph. We did not analyse the results in detail but it was interesting that the percentage of completely correct answers (26%) matches very well with the typical percentage of first-year students who manage to complete the first-year exams in our department in the first term. A systematic analysis of student understanding of Archimedes' principle has recently been published elsewhere [2].

Using the two-liquid Cartesian diver in building the model

At the beginning of a well-known textbook one can read, "Models are simple 'artificial worlds' created to give insight into how real systems work, and predict what they might do. We start with the simple models…and go on to model variations of one quantity with another [3]". As explained later in the same book, the evolution of a model goes through several steps before the model is accepted. The derivation of a theoretical model for the Cartesian diver, which floats between two liquids, may be a good example for secondary school students at advanced level.

Observations

Everything starts with the observations. Let's concentrate on the diver that initially floats at the boundary between the two liquids. Part of it is in the upper fluid and the rest is submerged in the lower fluid. If I squeeze the bottle gently (and keep the force on the bottle constant), the diver moves down a little and finds a new equilibrium position. If I continue squeezing the bottle the whole diver eventually sinks into the lower liquid and down to the bottom of the bottle.

4. Wrong answers

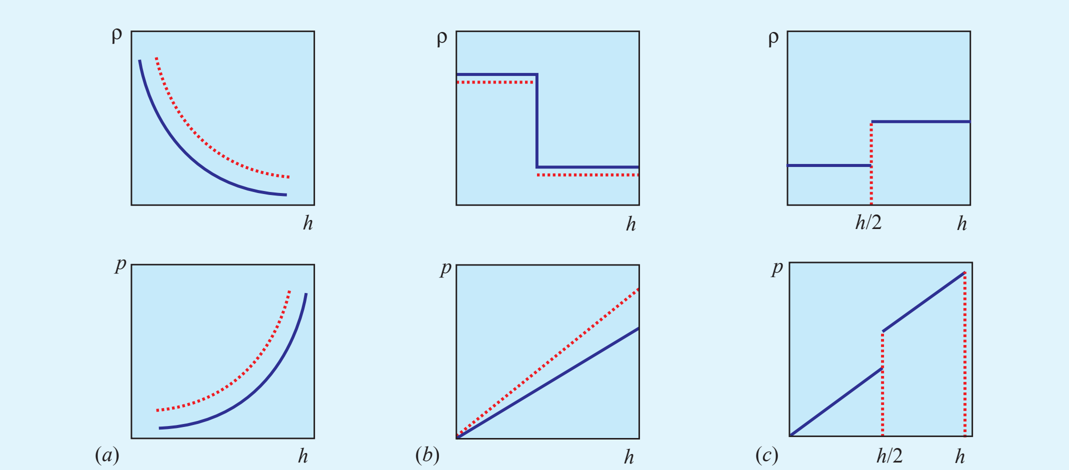

Figure 4. Typical wrong answers to the questions Q1 and Q2 (see the text for details). The dashed line shows the predicted change after the bottle was squeezed. In (*c*) no change after squeezing has been predicted.

Building a theoretical model

We wish to have a theoretical model (a 'formula', if you wish) that will correctly explain the behaviour of the diver between the two liquids. The important part of building the model is to decide what parameters or phenomena make a major contribution to our experiment and what can be neglected. The usual approach is to make the first model as simple as possible and see if it supports the observations. If it doesn't, try to take into account what was neglected in the first model (one parameter or phenomenon at a time, the 'easiest' first) and check the model again.

Using the appropriate questions, students may be guided to come to the following assumptions that will lead them to build the simplest model for the diver between the two liquids (for clarity we list the assumptions first but in practice it is better to bring them up during the derivation of the model):

- The liquids are practically incompressible, so their densities are constant and do not change with depth.
- The diver consists of the straw, the crocodile clip and the air bubble trapped in the straw. It is reasonable to say that the volume of the air bubble is bigger than the sum of the volumes of the crocodile clip and the straw. In our first crude model we will therefore assume that the crocodile clip and the straw (i.e. the plastic) have negligible volumes and that the volume of the diver V is approximately equal to the volume of the air bubble. It is important to emphasize here that the mass of the diver m is also equal to the sum of the three masses but now the mass of the crocodile clip is the largest. However, as the total mass of the diver does not change during the experiment, no approximation needs to be made here.
- The height of the diver is less than 10 cm. The corresponding change in hydrostatic pressure during the diver's excursion is therefore less than $1/50$ of the normal ambient pressure. In the simple model we will assume that the contribution of the hydrostatic pressure to the pressure 'felt' by the trapped air bubble may be neglected[1]. In other words, we assume that the air bubble feels only the ambient pressure plus the pressure exerted on the walls of the soda bottle by our fingers. However, note that the small difference in hydrostatic pressure is essential in explaining the buoyancy force on the diver!
- The temperature of the liquids and the air trapped in the straw is constant during the experiment.

[1] This assumption can be supported by the estimation of the added pressure produced by our fingers needed to sink the diver (measure the force and estimate the contact area).

Let's see what forces act on the diver that floats between the two liquids. Since the diver is at rest, the sum of the forces should be zero. The total buoyancy force can be seen as the sum of two contributions. The first is from the part of the diver with volume V_1 that is submerged in the upper liquid with density ρ_1; the second is from the rest of the diver of volume V_2 that is submerged in the lower liquid with density ρ_2 (obviously $\rho_2 > \rho_1$). The total buoyancy force is balanced by the weight of the diver. The same statements can be formulated mathematically as follows:

$$-mg + F_{\text{buoy}} = 0 \tag{1}$$

$$F_{\text{buoy}} = \rho_1 g V_1 + \rho_2 g V_2 \tag{2}$$

where

$$V = V_1 + V_2. \tag{3}$$

The last equality is one way of saying, "The volume of the diver consists of the two parts V_1 and V_2." Another way of describing the same thing is by writing the following two equations:

$$V_1 = \eta V \qquad V_2 = (1 - \eta)V \tag{4}$$

where η denotes the fraction of the total volume of the diver that is in the upper liquid. For example, if we say, "one quarter of the diver is in the upper liquid and the rest (i.e. three quarters) is in the lower liquid", then $\eta = 0.25$. Obviously $0 \leqslant \eta \leqslant 1$. Note that our main goal is to obtain an expression that will describe how η depends on the pressure in the bottle. Using the equations above one can express the 'dive ratio' η as

$$\eta = \frac{\rho_2 - m/V}{\rho_2 - \rho_1}. \tag{5}$$

Since we assumed that the temperature remains constant through the experiment and that the diver's volume is approximately equal to the volume of the air bubble, we can use Boyle's law to

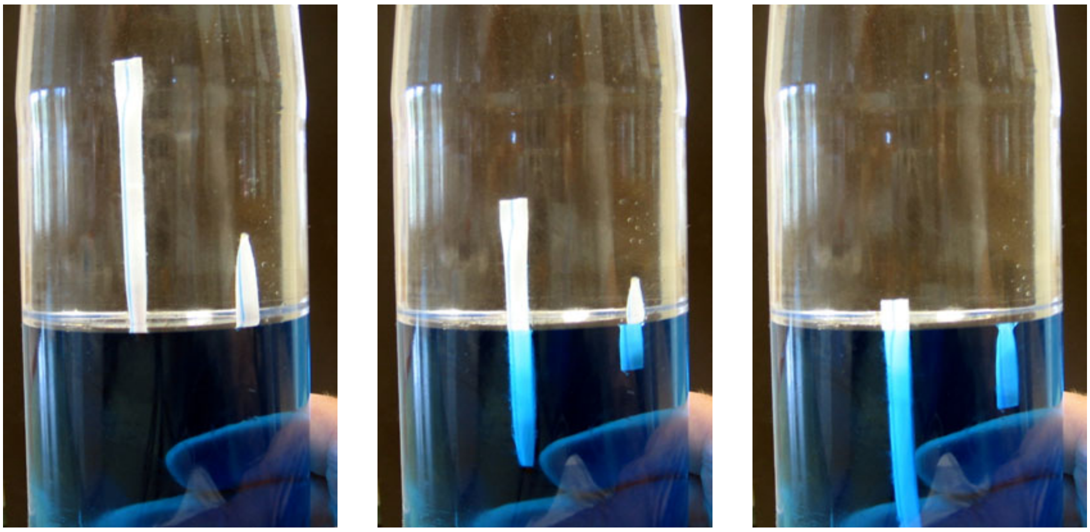

Figure 5. Two divers of different sizes cross the boundary between the two liquids as the pressure in the bottle increases (from left to right). The divers are made from 5 mm diameter drinking straws, glued at the top and weighted at the open end. Initially the divers were barely floating on the top of the upper liquid.

M An MPEG movie of this figure is available from stacks.iop.org/physed/39/58

relate the volume of the air bubble to the pressure in the bottle. In our case Boyle's law reads as

$$p_0 V_0 = (p_0 + p)V \quad (6)$$

where p_0 is the normal ambient pressure (about 10^5 N m^{-2}), V_0 is the initial volume of the air bubble and p is the additional pressure caused by squeezing the bottle with our fingers. Now equation (5) can be written in the final form

$$\eta = \frac{\rho_2 - \rho_{\text{eff}\,0}(1 + p/p_0)}{\rho_2 - \rho_1} \quad (7)$$

where $\rho_{\text{eff}\,0}$ is equal to the ratio m/V_0 and therefore plays the role of the effective density of the diver at the beginning of the experiment. The diver was initially adjusted to sink in the upper liquid and float between the two liquids, so we know that $\rho_2 > \rho_{\text{eff}\,0} > \rho_1$. Equation (7) is supposed to describe in a mathematical way how the position of the diver changes with the pressure in the bottle. In order to trust the equation and justify the model we have to verify whether the equation predicts correctly what has been observed. In school this verification is usually done on a few simple cases as shown in two steps below.

1. Observation:
If the bottle is not touched, the diver floats between the two liquids.

Theoretical prediction based on equation (7):
$p = 0$ gives $\eta = (\rho_2 - \rho_{\text{eff}\,0})/(\rho_2 - \rho_1)$. Taking into account the relationship between the three densities, one finds that indeed $\eta < 1$, which agrees with the observation.

2. Observation:
When I squeeze the bottle harder, the diver moves down into the lower liquid. At a certain pressure the whole diver sinks into the lower liquid.

Theoretical prediction based on equation (7):
When p increases, the numerator in the equation decreases (the denominator is constant) and therefore η decreases, which agrees with the observation. At a certain pressure $\eta = 0$. This value can be calculated from equation (7) and is equal to

$$p' = p_0 \left(\frac{\rho_2}{\rho_{\text{eff}\,0}} - 1 \right). \quad (8)$$

This is the pressure needed to sink the diver completely into the lower liquid, as predicted by the model. If appropriate equipment is available, the calculated value p' can be compared with the measured value.

It is important to note that once the whole diver is in the upper or lower liquid, η becomes constant

(1 or 0 respectively) and is no longer given by equation (7).

That is not quite the end! Sometimes studying and verifying the model can lead to the prediction of a new experimental result that we had not thought about before. Of course, we should know that since the model is only an idealization of the real situation such a prediction might be wrong. But in our case it happened to work well.

Note that in equation (7) all the information about the diver is concentrated in $\rho_{\mathrm{eff}\,0}$. Therefore this equation describes the sinking of *all* the divers on this world that have the same initial effective density! Or in other words, all divers, no matter how big or small, will cross from one liquid to the other together, providing that they have the same initial density. This prediction can be verified experimentally by using two divers of different sizes and adjusting their masses so that the divers initially barely float on the upper liquid. The result of the experiment is shown in figure 5 but can be also watched as a movie on the journal's website (see stacks.iop.org/physed/39/58).

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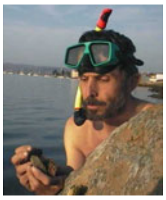

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