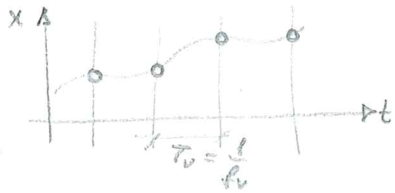
2011/3a

vzorčenje

Zajemanje

① načini: SW, HW direktno, HW/SW prekinitve
- timer, IRQ

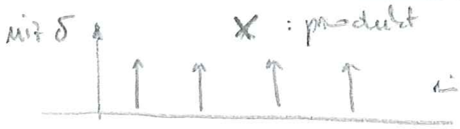
② teorija



izgubljamo informacijo, saj vidimo signal le ob kT_v, $k \in \mathbb{Z}$

demo LV

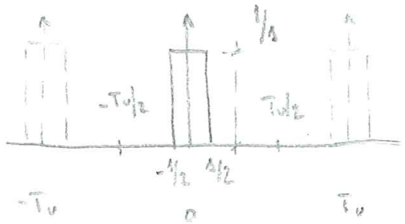
|||



niz $\delta(kT_v)$ $\Rightarrow$ niz $\delta = \sum_{k=-\infty}^{\infty} \delta[kT_v]$

$$x_a(t) \rightarrow x_d[kT_v] = \underbrace{x_a(t) \cdot \sum_{k=-\infty}^{\infty} \delta[kT_v]}_{\text{slabo zapisano}} = x_a(t) \sum_{k=-\infty}^{\infty} \delta(t - kT_v)$$

spekter niza δ:



$$F[\text{niz } \delta] = \frac{1}{T_v} \int_{-T_v/2}^{T_v/2} \delta(0)\, e^{-ik\omega t}\, dt =$$

$$= \frac{1}{T_v} \int_{-\Delta/2}^{\Delta/2} \frac{1}{\Delta} e^{-ik\omega t}\, dt =$$

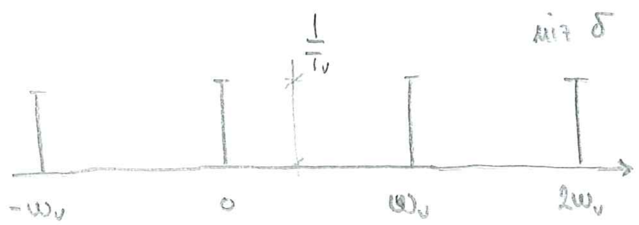
$$= \frac{1}{\Delta T_v} \frac{1}{-ik\omega} e^{-ik\omega t} \Big|_{-\Delta/2}^{\Delta/2} = \frac{i}{\Delta T_v k\omega} \left[\cos k\omega \frac{\Delta}{2} - i \sin k\omega \frac{\Delta}{2} - \cos k\omega \frac{\Delta}{2} - i \sin k\omega \frac{\Delta}{2} \right] =$$

$$= \frac{2 \sin k\omega \frac{\Delta}{2}}{T_v 2 k\omega \frac{\Delta}{2}} \Rightarrow F[\text{niz } \delta] = \lim_{\Delta \to 0} \frac{1}{T_v} \frac{\sin k\omega \Delta/2}{k\omega \Delta/2} = \frac{1}{T_v}$$

⇓

$$\text{niz } \delta = \frac{1}{T_v} \sum_k e^{ik\omega t} = \frac{1}{T_v} \sum_k e^{ik 2\pi \frac{t}{T_v}}$$

↑ velikost!



demo ba'disp

Zato: spekter niza δ sunkov (periodičnih) je sestavljen iz niza harmonskih komponent, vse imajo velikost $\frac{1}{T_v}$

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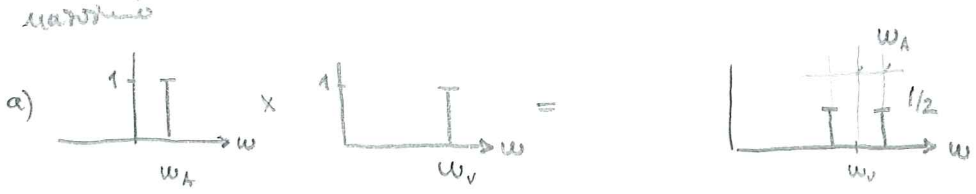
nazorno

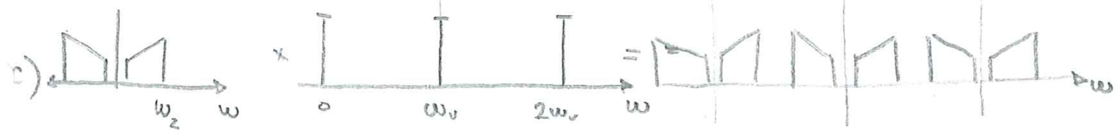
a)



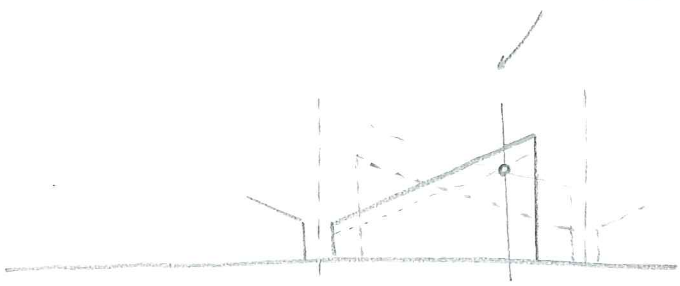
$\cos\omega_A t \times \cos\omega_v t = \frac{1}{2}\left[\cos(\omega_A+\omega_v)t + \cos(\omega_v-\omega_A)t\right]$

b)

c)



pozor! $\omega_2 \leq \frac{\omega_v}{2}$ če $\omega_2 > \frac{\omega_v}{2} \Rightarrow$ prekrivanje spektrov!
iz spektra ne vemo kam spada označena vrednost!

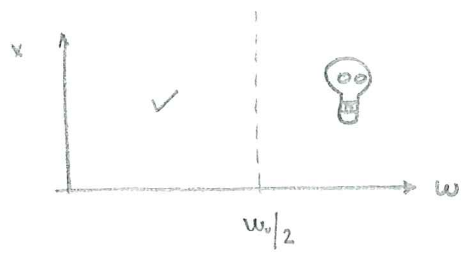


?

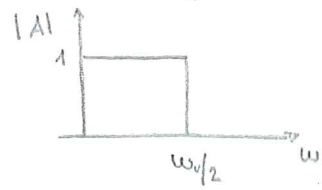
demo VB6 za spektre
demo LV za zajem

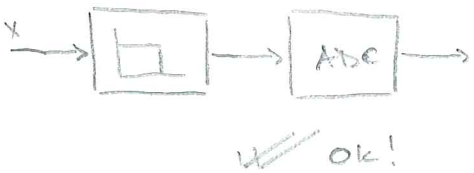
⇓

omeji doseg frekvence signala, ki ga zajemaš!



⇒ potrebujemo filter: analogni



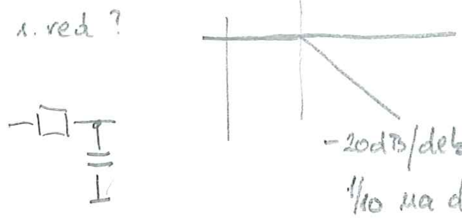


GRRR! žal takega filtra ni!

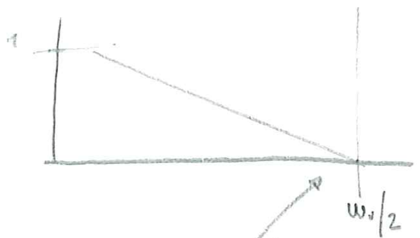
2011/3d

kakšen analogni filter?

1. red ?



} torej



zanemarljiva napaka =
= dušenje na manj od 1LSB
10BITNI ADC → 0.001
−60dB

pozabi ⇐ ! začeti je treba 3 dekade prej! ←

2. red ? začeti je treba 1.5 dekade prej! ⇒ pozabi

6. red ? začeti je treba pri $0.2\ \omega_v/2$ za Bessel
$0.5\ \omega_v/2$ za Čebišev ±1 dB !

⋮

(pozor: Čebišev nima lin. faze!)

nujno je, da enkrat filtriramo pred $(\omega_v/2)$ ADC! FilterPro demo

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nek pojav: kaj pa, če je frekvenčni spekter omejen
na ω_v do $\frac{3}{2}\omega_v$?
na $10\omega_v$ do $10\omega_v+\frac{1}{2}\omega_v$?
} gre ok!

⇓

bandpass sampling

⇓

zelo koristno za sprejemnike

! samo v enem območju naenkrat
✓ → pravi vrstni red frekvenc
X → obrnjeni – " –

prekrivanje spektrov in matematična utemeljitev

$$F(x_D) \quad ; \quad x_D = x_a \underbrace{\sum_k e^{ik\omega_v t} \cdot \frac{1}{T_v}}_{\text{niz } \delta} = x_a \sum_k \delta(t - kT_v) \quad ; \ \omega_v = 2\pi / T_v$$

$$\hookrightarrow F(x_D) = \underbrace{\int_{-\infty}^{\infty} x_D \cdot e^{-i\omega t} \, dt}_{\text{Fourier za neper. f.}} = \frac{1}{T_v} \int_{-\infty}^{\infty} x_a \left[\sum_k e^{ik\omega_v t} \right] \cdot e^{-i\omega t} \, dt$$

$$e^{ik\omega_v t} \cdot e^{-i\omega t} = e^{-i(\omega - k\omega_v)t}$$

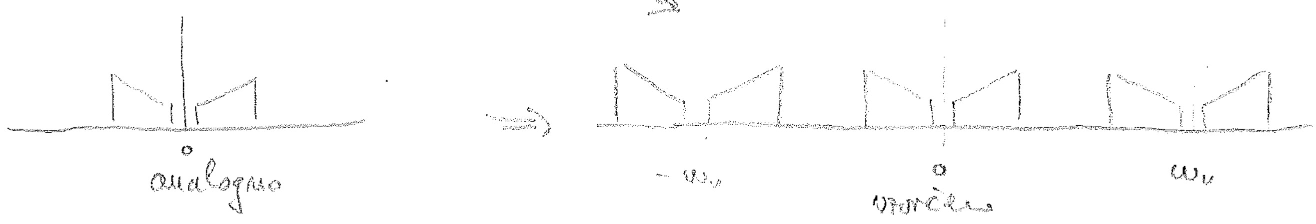
$$= \frac{1}{T_v} \sum_k \int_{-\infty}^{\infty} x_a(t) \, e^{-i(\omega - k\omega_v)t} \, dt \qquad \omega' = \omega - k\omega_v$$

ker $F(x_a(t)) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt$ prepoznamo integrirani del kot Fourierovo trf. s premaknjenim spektrom, premik za $k\omega_v$

$$\boxed{F(x_D) = \frac{1}{T_v} \sum_{k=-\infty}^{\infty} F(x_a, \omega - k\omega_v)}$$

↑ argument ↖ premik

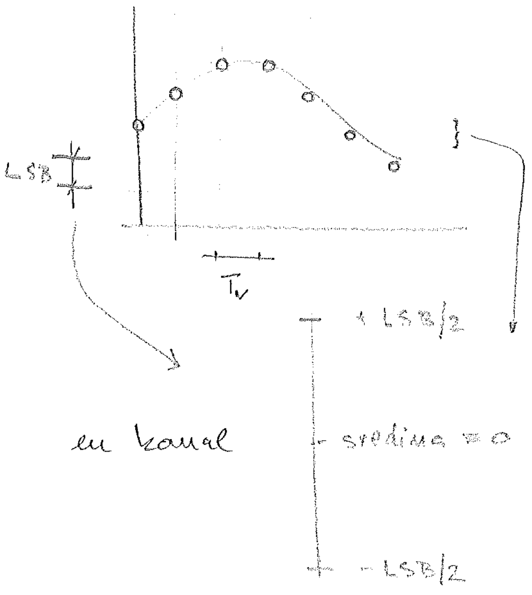
⇓



NI "prekrivanje spektrov"

demo

Težave pri kvantizaciji = šum



včasih preveč, včasih premalo!
⇓
naključno

en kanal

Zanima me $SNR = 10 \cdot \log \frac{U_{SRMS}^2}{U_{NRMS}^2}$ signal/noise

⇓

določi RMS signala } razmerje
določi RMS šuma } 10 log !

a) RMS signala

$$U_{RMS}^2 = \frac{1}{2\pi} \int_0^{2\pi} A^2 \sin^2\varphi \, d\varphi = \frac{A^2}{2\pi} \int_0^{2\pi} \frac{1-\cos 2\varphi}{2} d\varphi =$$

signal: $A \sin\varphi$

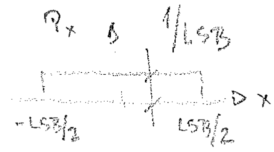
$$= \frac{A^2}{2 \cdot 2\pi} \cdot 2\pi = \frac{A^2}{2} \Rightarrow \boxed{U_{RMS} = \frac{A}{\sqrt{2}}}$$

b) RMS šuma

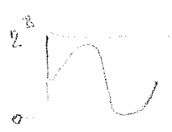
$$\sigma^2 = \int_{-LSB/2}^{+LSB/2} (x - \bar{x})^2 \cdot P_x \, dx = \int_{-LSB/2}^{LSB/2} x^2 \cdot \frac{1}{LSB} dx = \frac{x^3}{3\,LSB} \Big|_{-LSB/2}^{LSB/2} =$$

$\bar{x} = 0$

vse napake so enako verjetne }



$$\int_{-\infty}^{\infty} P_x \, dx = 1 \Rightarrow P_x = \frac{1}{LSB}$$



$\uparrow 2^{B-1} = \frac{2^B}{2}$

$$= \frac{1}{3\,LSB} \left[\frac{LSB^3}{8} + \frac{LSB^3}{8} \right] = \boxed{\frac{LSB^2}{12} = \sigma^2}$$

Zato: $SNR = 10 \cdot \log \frac{U_{SRMS}^2}{U_{NRMS}^2} = 10 \log \frac{A^2 \cdot 12}{2 \cdot LSB^2} = 10 \log \frac{2^{2B} \cdot 12 \cdot LSB^2}{2 \cdot LSB^2 \cdot 4} =$

B-bitni pretvornik: $2^B \times LSB = \text{obseg}$

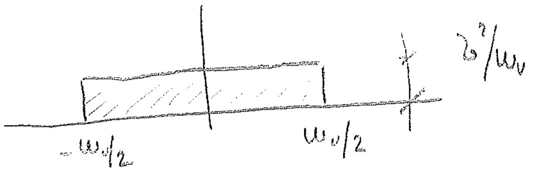
$A = \frac{2^B \times LSB}{2}$

$$= 10 \log \frac{3}{2} + 10 \cdot 2B \cdot \log 2 = \boxed{1.76\,dB + 6.02\,B\,dB}$$

B	SNR
8	50dB
16	98dB
24	146dB

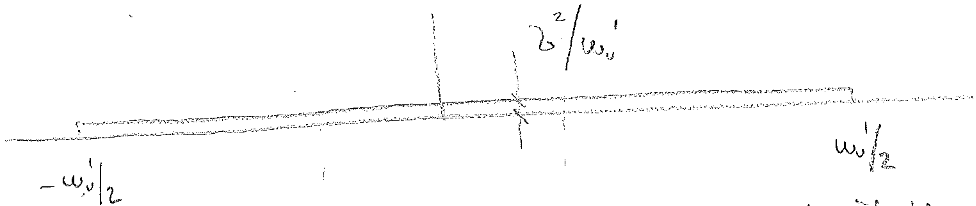
} malo bitov = majhen SNR = veliko šuma!

na srečo je šum enakomerno porazdeljen po kompletnem spektru



⇓

vzorči z veliko frekvenco → isti šum porazdeli po širšem območju!



malo število bitov, hitrejše vzorčenje

⇓

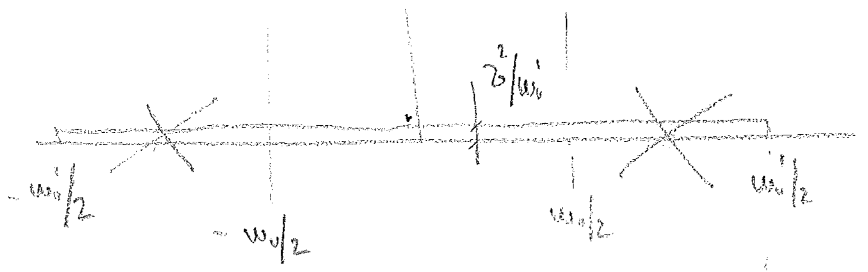
manj šuma v območju $-\omega_v/2$ do $\omega_v/2$

↓

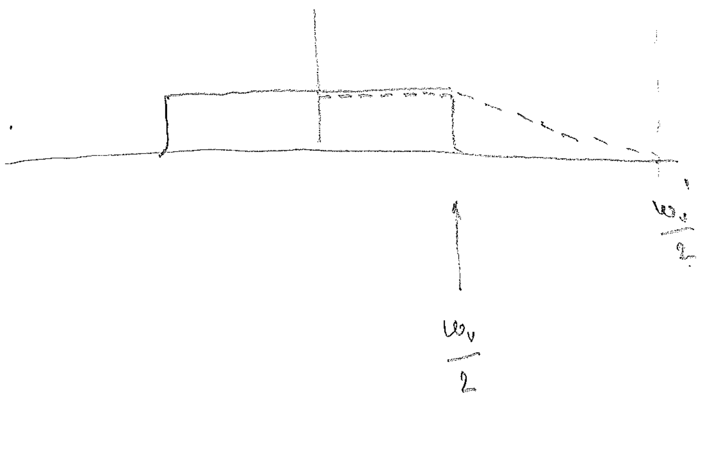
torej

↓

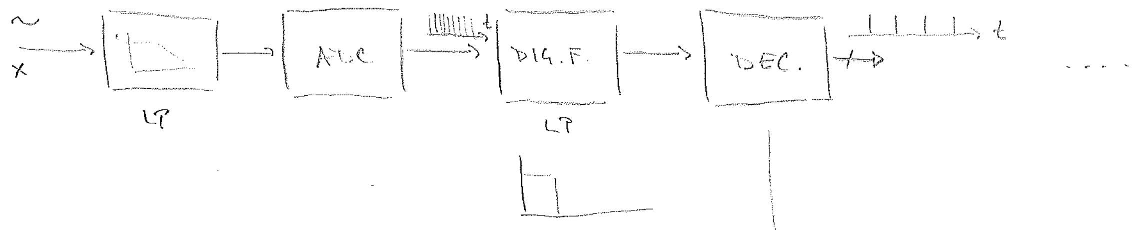
vzorči hitro & digitalno filtriraj!



oversampling!

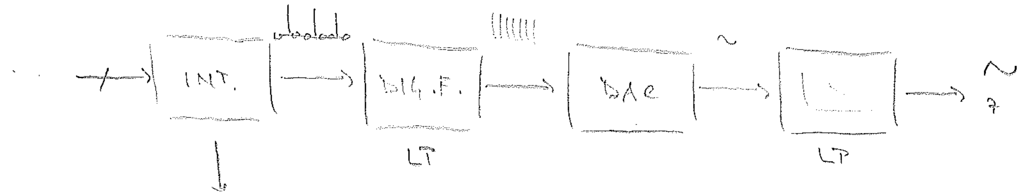


veriga zajemanja:

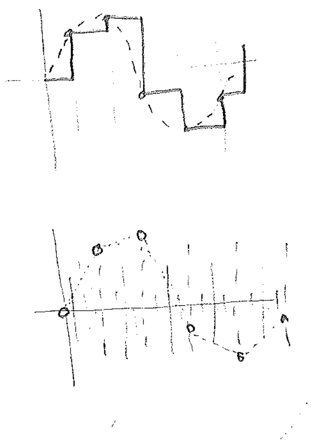


decimacija

veriga generiranja, nadaljevanje



interpolacija



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decimacija → izpuščanje vzorcev → zgled

Gsim(100, 0.04, 1, 0) mag(DFT( ))

Decimate(W1, #, 1) " –

do kod gre? : pokaži na spektru

demo

---

interpolacija → vrivanje vzorcev → zgled

↓

lin ...

ničle! → kaj se zgodi? → Gsim(50, 0.04, 1, 0)

vstavinicle(W1, #)

demo

oversampling: dobre!