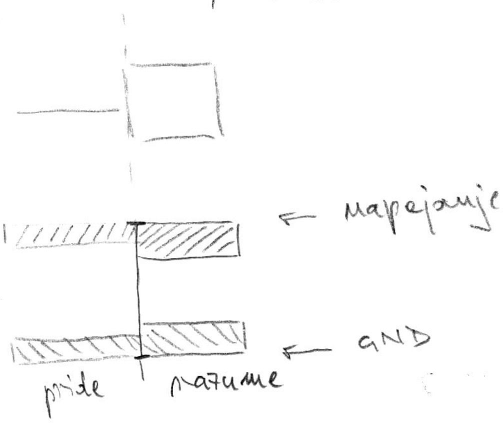
digitalna elektronika - uvod

- zakaj digitalno: natančnost, matematično čisto (točno)
- računanje: potrebujemo predznake, postopke
- definicija signalov: 0/1, dve vrednosti

  več vrednosti: zapis z več ciframi
  binarni zapis
  hex zapis
  pretvarjanje
  obseg zapisanih števil

- signali v realnosti

  $0/1 \rightarrow \underbrace{x \text{ voltov} / y \text{ voltov}}_{\text{različno!}}$

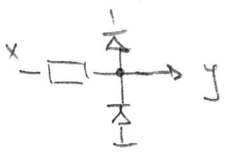


izhodni tok
vhodni tok

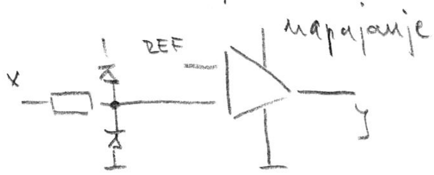
Vodilo: več posamičnih signalov, vsak za eno cifro

- digitalni signali ≡ šibki, omejeni po velikosti in toku

  a) prevelik → omejen

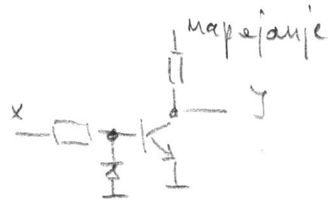


  b) premajhno → pravilno

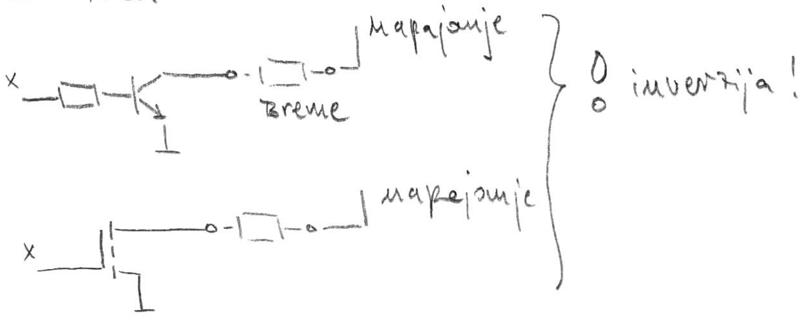


morda še histereza

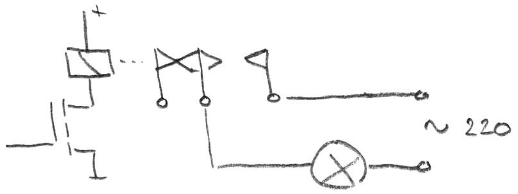
enostavneje

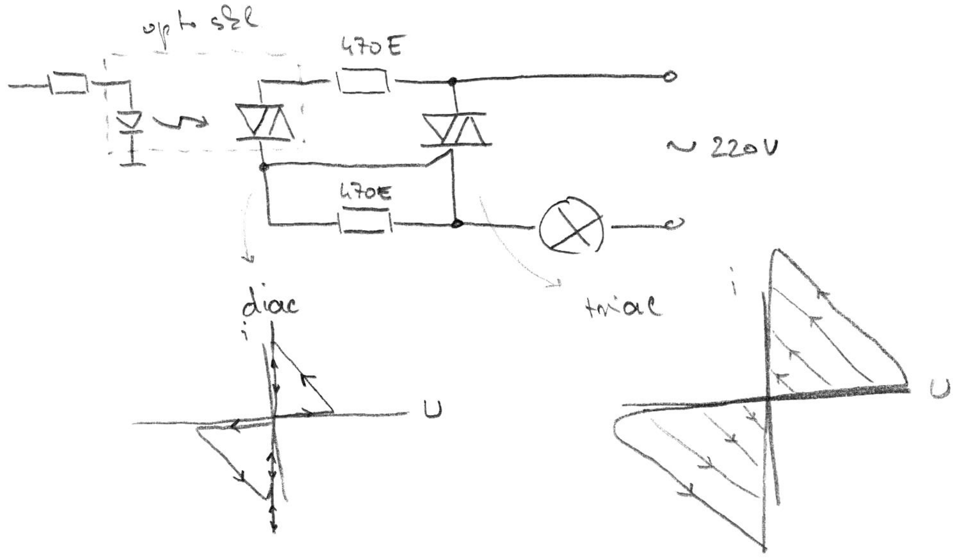


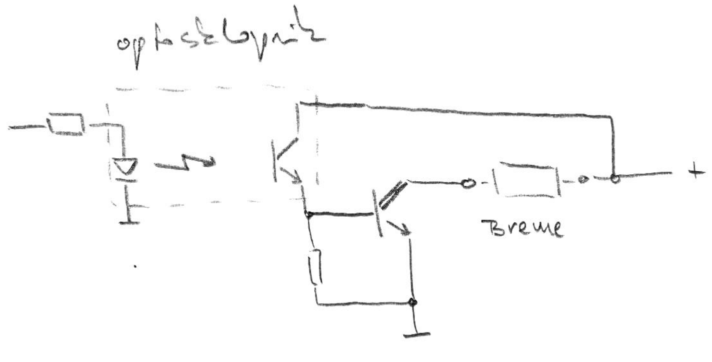
c) več toka



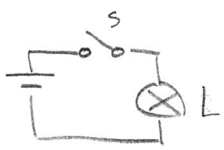
d) več toka & galvanska ločitev



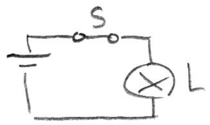




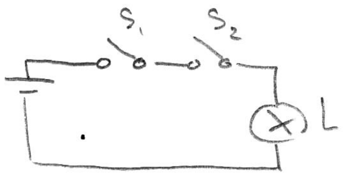
osnovna log. vezja ≡ pomembne operacije



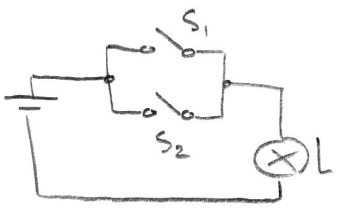
$L = S$



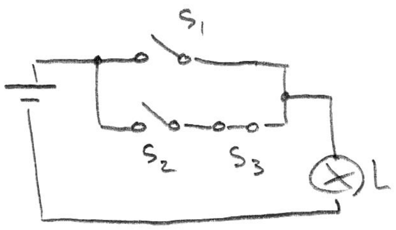
$L = \overline{S}$



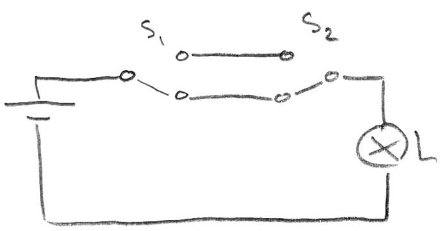
$L = S_1$ in S_2 $= S_1 \cdot S_2$



$L = S_1$ ali S_2 $= S_1 + S_2$



$L = S_1$ ali (S_2 in ne S_3) $= S_1 + (S_2 \cdot \overline{S_3})$



$L =$ (S_1 in S_2) ali (ne S_1 in ne S_2)

$= S_1 \cdot S_2 + \overline{S_1} \cdot \overline{S_2}$

osnovni gradniki

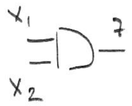
kot smo začeli s stikali želimo začeti z gradniki, ki ne zahtevajo mehanskih posegov ⇒ logična vrata

a) negacija : NOT ≡ $\overline{}$

x	z
0	1
1	0

$z = \bar{x}$

b) konjunkcija : AND ≡ ·



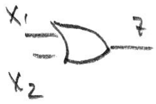
x_2	x_1	z
0	0	0
0	1	0
1	0	0
1	1	1

$z = x_2 \cdot x_1$

lahko z več vh. spr.

$\boxed{\text{vsi } x \equiv 1 \Rightarrow z = 1}$

c) disjunkcija : OR ≡ +



x_2	x_1	z
0	0	0
0	1	1
1	0	1
1	1	1

$z = x_2 + x_1$

lahko z več vh. spr.

$\boxed{\text{vsaj en } x \equiv 1 \Rightarrow z \equiv 1}$

dodatno : negirana konjunkcija , negirana disjunkcija

NAND

x_2	x_1	z
0	0	1
0	1	1
1	0	1
1	1	0

NOR

x_2	x_1	z
0	0	1
0	1	0
1	0	0
1	1	0

pakiranje v čipe

operacije : seštevanje

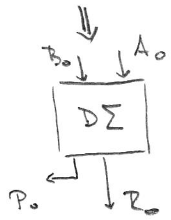
decimalno

$$\begin{array}{r} 1234 \\ +\ 5678 \\ \hline 6912 \end{array}$$

binarno

$$\begin{array}{rcccc} & A_3 & A_2 & A_1 & A_0 \\ + & B_3 & B_2 & B_1 & B_0 \\ \hline & R_3 & R_2 & R_1 & R_0 \end{array}$$

$\Downarrow$



delni seštevalnik :

možnosti vh. signalov

$$\begin{array}{c} 0 \\ 0 \\ \hline 0\,0 \end{array} \quad \begin{array}{c} 0 \\ 1 \\ \hline 0\,1 \end{array} \quad \begin{array}{c} 1 \\ 0 \\ \hline 0\,1 \end{array} \quad \begin{array}{c} 1 \\ 1 \\ \hline 1\,0 \end{array} \quad \Rightarrow$$

B_0	A_0	P_0	R_0
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

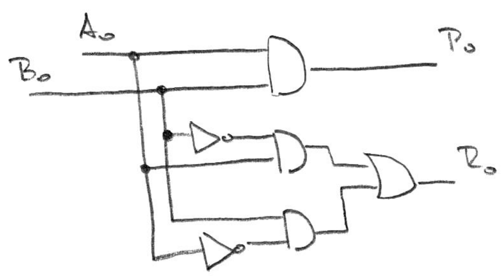
dve tabeli : P_0 prepoznam
R_0 izpišem

$$P_0 = B_0 \cdot A_0$$

$$R_0 = \overline{B_0} \cdot A_0 + B_0 \cdot \overline{A_0}$$

$\Downarrow$

realizacija : prevajanje v mreže

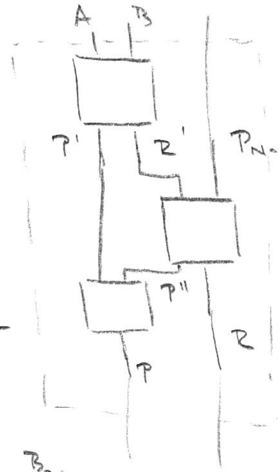


$\equiv$

veba delnega seštevalnika

- seštevanje 3 bite : A, B, P_{N-1} ?
  - dva seštevalnika ?

poln seštevalnik

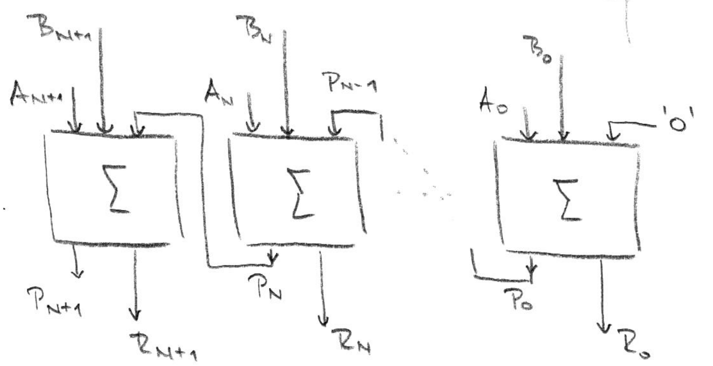


možnosti 1

A	B	P'	R'
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

možnosti 2

R'	P_{N-1}	P''	R
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0



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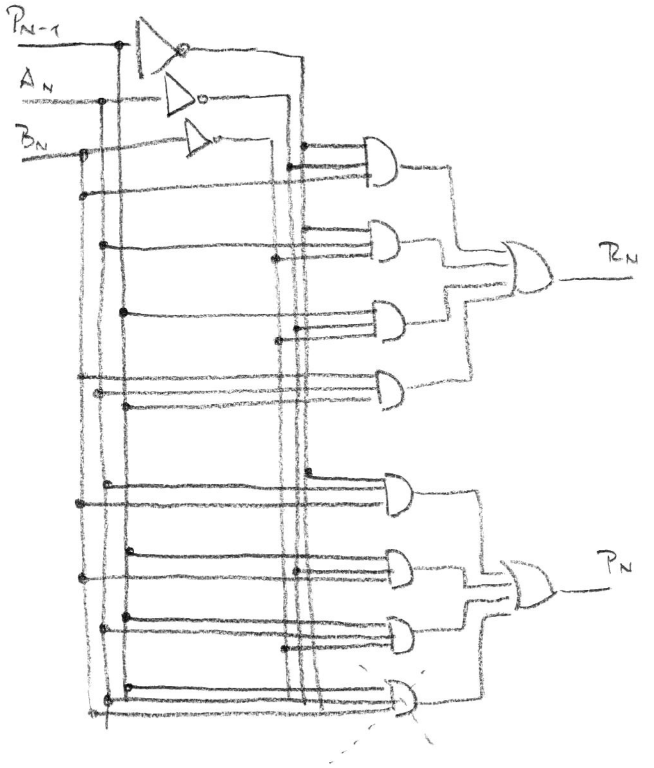
vaje v ti sepi

P_{N-1}	A_N	B_N	P_N	R_N
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

dve enačbi

$$R_N = \overline{P_{N-1}} \cdot \overline{A_N} \cdot B_N + \overline{P_{N-1}} \cdot A_N \cdot \overline{B_N} + P_{N-1} \cdot \overline{A_N} \cdot \overline{B_N} + P_{N-1} \cdot A_N \cdot B_N ;$$

$$P_N = \overline{P_{N-1}} \cdot A_N \cdot B_N + P_{N-1} \cdot \overline{A_N} \cdot B_N + P_{N-1} \cdot A_N \cdot \overline{B_N} + P_{N-1} \cdot A_N \cdot B_N ;$$



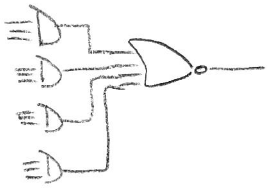
PKNO

ali gre tudi enostavneje?

izpišemo lahko ničle

$$\overline{P_N} = \overline{P_{N-1}}\cdot\overline{A_N}\cdot\overline{B_N} + \overline{P_{N-1}}\cdot\overline{A_N}\cdot B_N +$$
$$+\,\overline{P_{N-1}}\cdot A_N\cdot\overline{B_N} + \overline{P_{N-1}}\cdot A_N\cdot B_N\,;$$

realizacija



poenostavljanje

zgled: log. f. $\quad F = \overline{A}\,\overline{B} + \overline{A}B$

$F = 1$ kadar $\underbrace{\begin{matrix}\overline{A} \text{ in } \overline{B} \text{ ali} \\ \overline{A} \text{ in } B\end{matrix}}_{}$

torej B ni važen!

lahko bi bilo:
$F = \overline{A}$

nekaj pravil

združljivost: $A \cdot B \cdot C = (A \cdot B) \cdot C = A \cdot (B \cdot C)$

$A + B + C = (A + B) + C = A + (B + C)$

zamenljivost: $A \cdot B = B \cdot A$

$A + B = B + A$

prioriteta: najprej $\overline{}$, potem $\cdot$, za konec $+$

izpostavljanje: $AB + AC = A(B + C)$

$(A + B) \cdot (C + A) = A + (B \cdot C)$

$AB+AC$	$A(B+C)$	A B C
0	0	0 0 0
0	0	0 1 0
0	0	1 0 0
1	1	1 1 0
0	0	0 0 1
0	0	0 1 1
1	1	1 0 1
1	1	1 1 1

pravila AND: $A \cdot A = A$ $\quad A \cdot 0 = 0$

$A \cdot 1 = A$ $\quad A \cdot \overline{A} = 0$

pravila OR: $A + A = A$ $\quad A + 0 = A$

$A + 1 = 1$ $\quad A + \overline{A} = 1$

pravilo kompl. $\overline{\overline{A}} = A$

A	$\overline{A}$	$\overline{\overline{A}}$
0	1	0
1	0	1

De Morgan $\overline{A + B} = \overline{A} \cdot \overline{B}$

$\overline{A \cdot B} = \overline{A} + \overline{B}$

A B	$\overline{A + B}$	$\overline{A} \cdot \overline{B}$
0 0	1	1
0 1	0	0
1 0	0	0
1 1	0	0

$$P_N = A_N B_N \left(\overline{P_{N-1}} + P_{N-1}\right) + P_{N-1}\left(\overline{A_N}\cdot B_N + A_N\cdot\overline{B_N}\right) =$$

$$= A_N B_N + P_{N-1}\underbrace{\left(\overline{A_N}\cdot B_N + A_N\cdot\overline{B_N}\right)}$$

se pogosto pojavlja → ima svoj čip

$$y = \underbrace{x_1\cdot\overline{x_2} + \overline{x_1}\cdot x_2}$$

$y = 1$ kadar sta
x različna!

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

$$y = x_1 \oplus x_2$$ XOR

$$P_N = A_N B_N + P_{N-1}\left(A_N \oplus B_N\right);$$

$$R_N = \overline{P_{N-1}}\left(\overline{A_N}\cdot B_N + A_N\cdot\overline{B_N}\right) + P_{N-1}\left(\overline{\overline{\overline{A_N}\cdot\overline{B_N} + A_N\cdot B_N}}\right) =$$

$$= \overline{P_{N-1}}\left(A_N\oplus B_N\right) + P_{N-1}\cdot\left(\overline{\overline{\overline{A_N}\cdot\overline{B_N}}\cdot\overline{A_N\cdot B_N}}\right) =$$

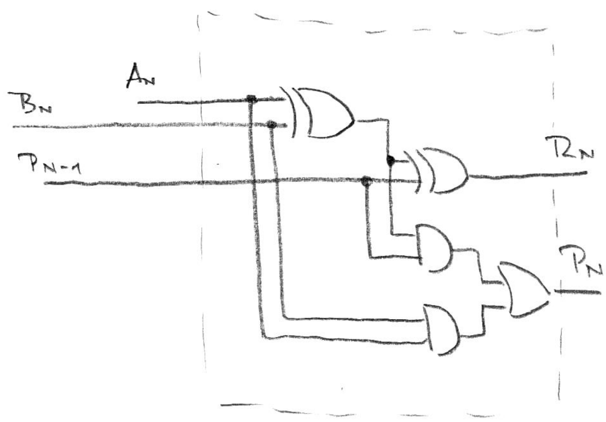
$$= \overline{P_{N-1}}\left(A_N\oplus B_N\right) + P_{N-1}\cdot\overline{\left(\overline{\overline{A_N}} + \overline{\overline{B_N}}\right)\cdot\left(\overline{A_N} + \overline{B_N}\right)} =$$

$$= -''- + P_{N-1}\overline{\left(\underbrace{A_N\overline{A_N}}_{0} + B_N\overline{A_N} + A_N\overline{B_N} + \underbrace{B_N\overline{B_N}}_{0}\right)} =$$

$$= -''- + P_{N-1}\cdot\overline{B_N\overline{A_N} + A_N\overline{B_N}} =$$

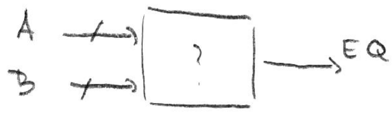
$$= -''- + P_{N-1}\cdot\overline{B_N\oplus A_N} =$$

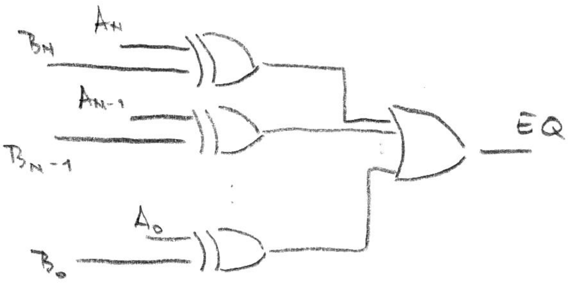
$$= P_{N-1}\oplus B_N\oplus A_N$$



polni seštevalnik

pristop: dodatna raba XOR vrat





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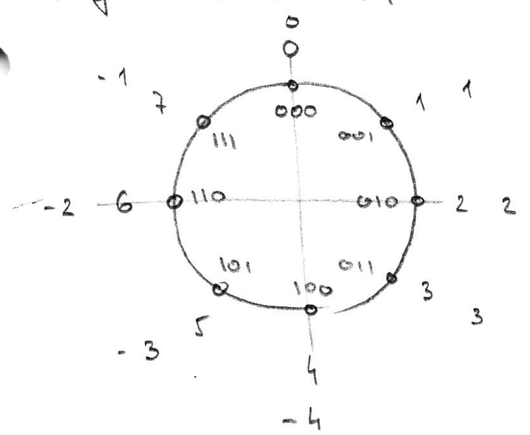
negativna števila



št. premica?

nesmisel!

za poz. cela števila $0 \div 2^N - 1$

za poz/neg cela št. $-2^{N-1} \div 2^{N-1} - 1$

N = št. mest bin. zapisa

seštevanje: koraki v desno $1 + 2 = 3 \Rightarrow$ 2 koraka v desno

odštevanje: koraki v levo $3 - 2 = 1 \Rightarrow$ -"- v levo

$$R = A - B =$$
$$= A + (2^N - B) = A + (8 - B) =$$
$$= A + (7 - B + 1) =$$

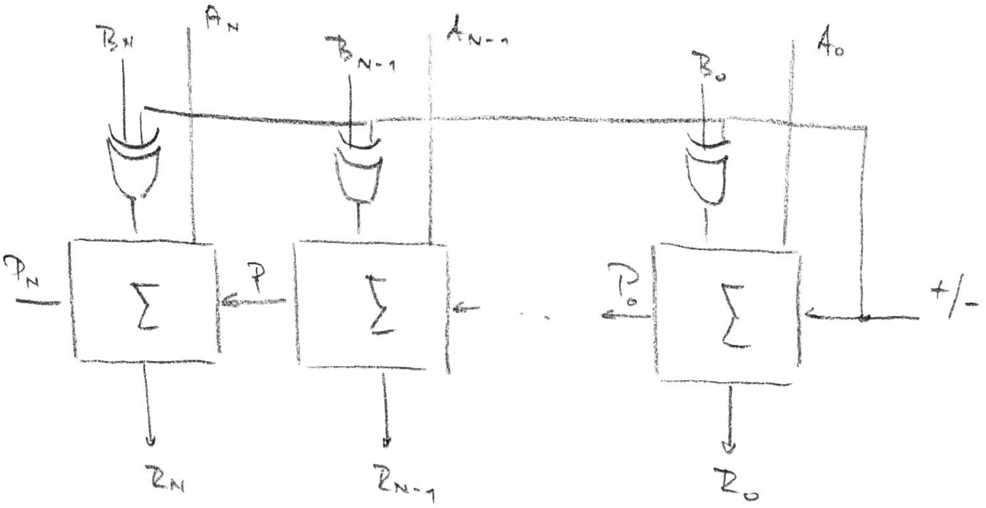
B	7-B
000	111
001	110
010	101
011	100
⋮	⋮

torej ←

$7 - B \equiv \overline{B}$!

$$= A + (\overline{B} + 1)$$

odštevanje tako, da prištejemo komplement, nato pa še ena!



Karnaugh diagrami → Matjaž na vajah

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nadomestek stikala / preklopnika

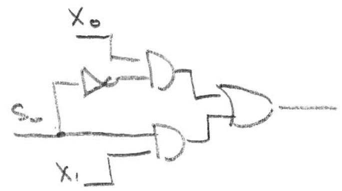
velja: $S_0 = 0 \Rightarrow Z = X_0$

$S_0 = 1 \Rightarrow Z = X_1$

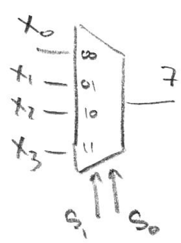
v tabeli:

S_0	Z
0	X_0
1	X_1

$\equiv \quad Z = \bar{S}_0 X_0 + S_0 X_1$



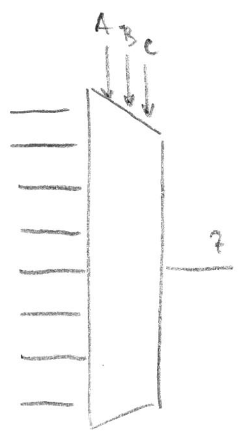
večji mux



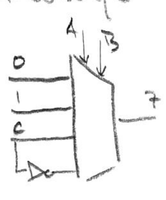
$\Rightarrow Z = \bar{S}_1 \bar{S}_0 X_0 + \bar{S}_1 S_0 X_1 + S_1 \bar{S}_0 X_2 + S_1 S_0 X_3$;

realizacija log. funkcij

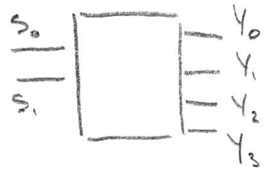
ABC	Z
000	0
001	0
010	1
011	1
100	0
101	1
110	1
111	0



enostavneje



selektor



S_1	S_0	Y_3	Y_2	Y_1	Y_0
0	0	1	1	1	0
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	1	1	1

Nekaterih ima še E (enable) vhod, takrat jih lahko vežemo več skupaj v večji selektor

enable

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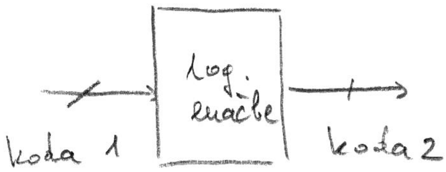
prekoder



za zgled Binarni zapis v Grey zapis

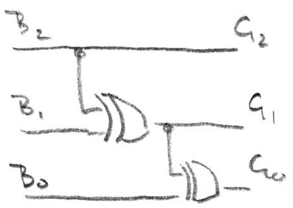
B_2	B_1	B_0	G_2	G_1	G_0
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	1
0	1	1	0	1	0
1	0	0	1	1	0
1	0	1	1	1	1
1	1	0	1	0	1
1	1	1	1	0	0

$$G_2 = B_2$$

$$G_1 = \overline{B_2} \cdot B_1 + B_2 \cdot \overline{B_1} = B_1 \oplus B_2$$

$$G_0 = \overline{B_2} \cdot (B_1 \oplus B_0) + B_2 \cdot \overline{(B_1 \oplus B_0)} = B_2 \oplus B_1 \oplus B_0$$

torej



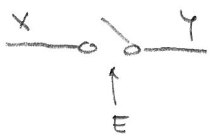
obratno:

$$B_2 = G_2$$

$$B_1 = G_2 \oplus G_1$$

$$B_0 = \overline{G_2} \cdot (G_1 \oplus G_0) + G_2 \cdot \overline{(G_1 \oplus G_0)}$$

3-stanjsko vezje

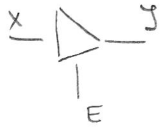


velja

$y = x$ za $E = 1$

$y =$ nedefiniran za $E = 0$

stikalo razklenjeno!



3-state buffer

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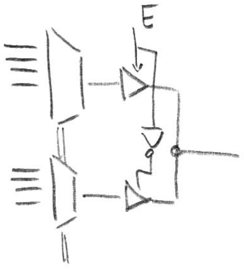
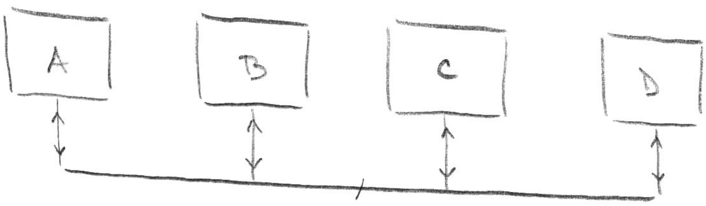
večji MUX

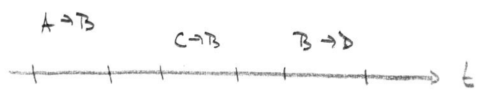


Vodilo : želimo

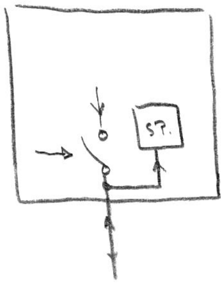


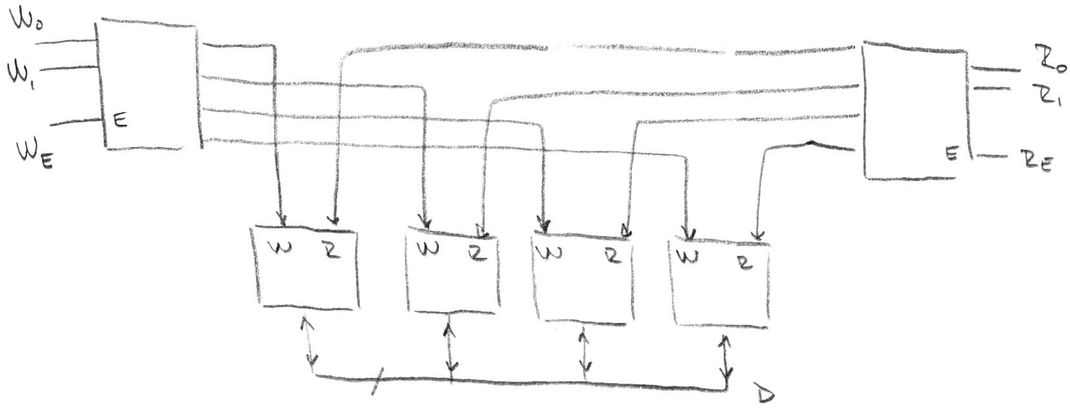
opis želja

prenos podatkov



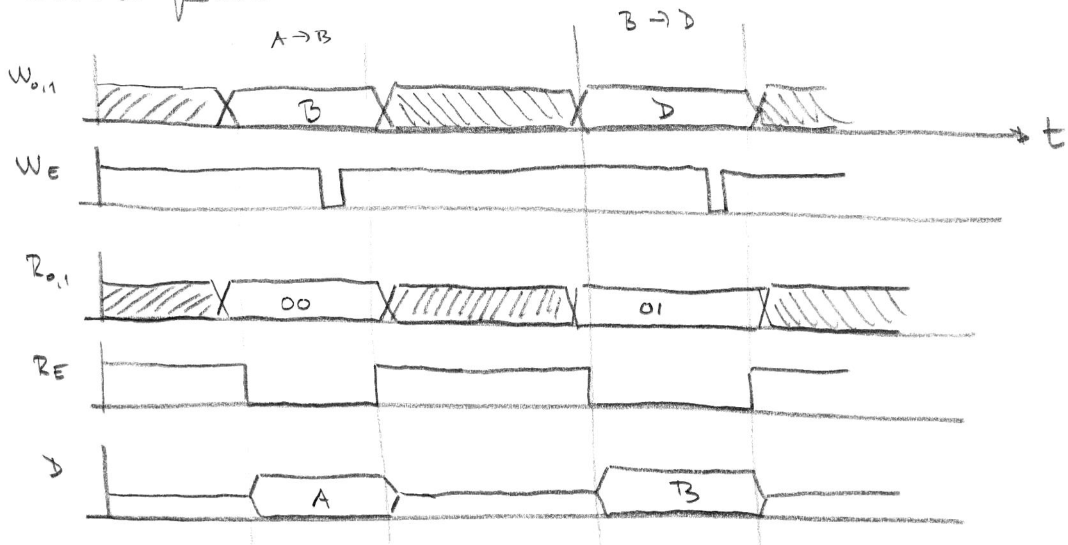
rabimo :





komentar

časovni potek

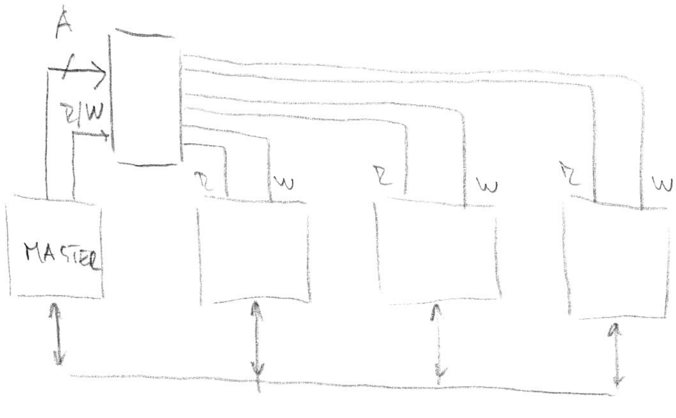


alternativa: do enot gredo signali $W_{0,1}$ in $R_{0,1}$ ter W_E in R_E, dekodiranje je izvedeno lokalno v enotah

↓

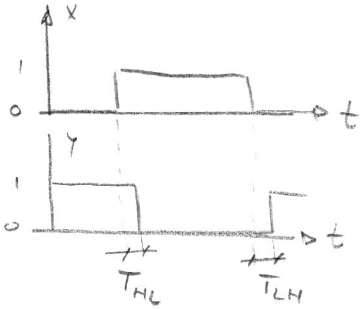
naslovno vodilo, RD, WR

kadar porazdeljeni prenos: vedno sodeluje A vodilo gre le do ostalih enot



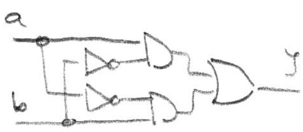
Zamujanje

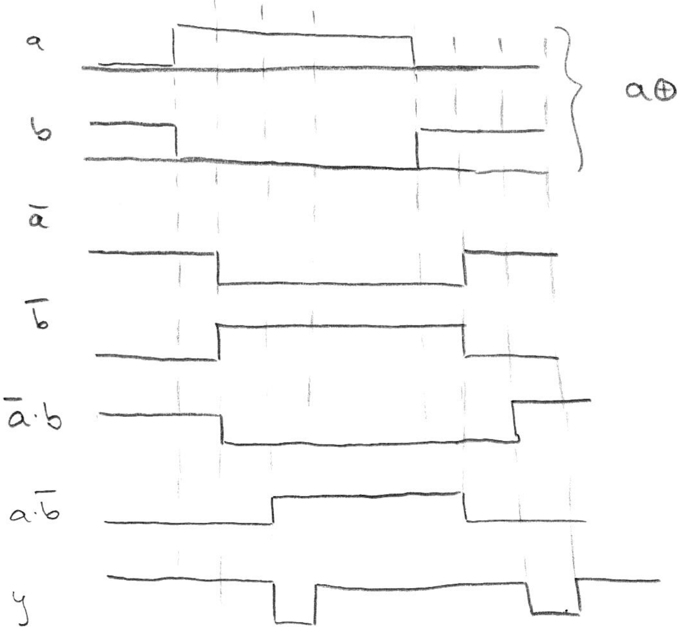
$s_v \xrightarrow{\text{traja}} s_i$



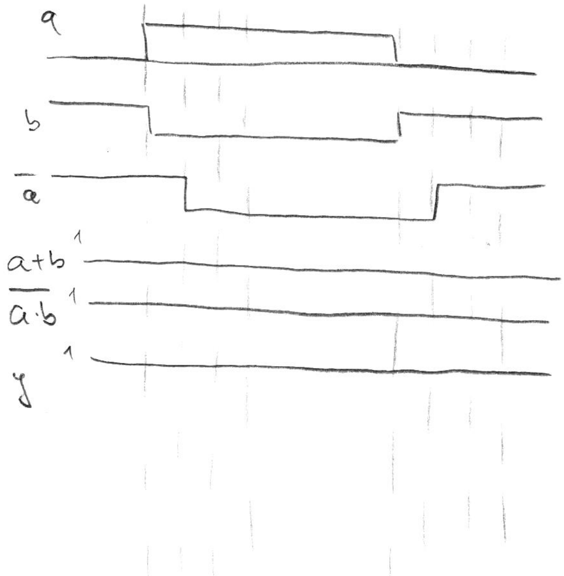
zgled: XOR

$y = \bar{a} \cdot b + a \cdot \bar{b}$

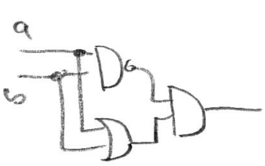




$a \oplus b = 1$



$$\bar{a}b + a\bar{b} = \bar{a}b + a\bar{b} + a\bar{a} + b\bar{b} =$$
$$= \bar{a}(a+b) + \bar{b}\,(a+b) =$$
$$= (a+b)(\bar{a}+\bar{b}) = (a+b)\overline{ab}$$

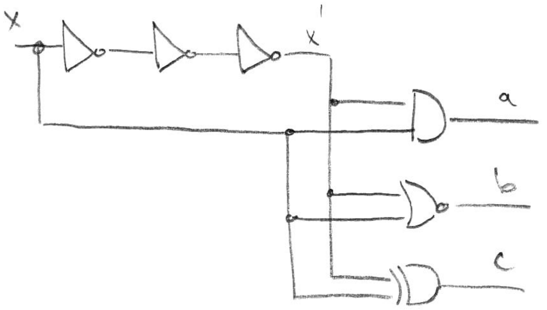


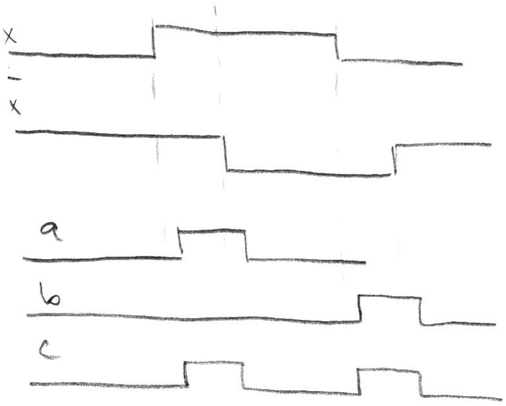
različno dolge poti razširjanja signalov

⇓

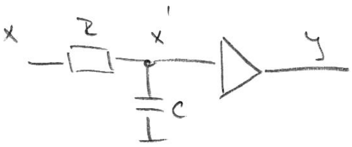
težave!

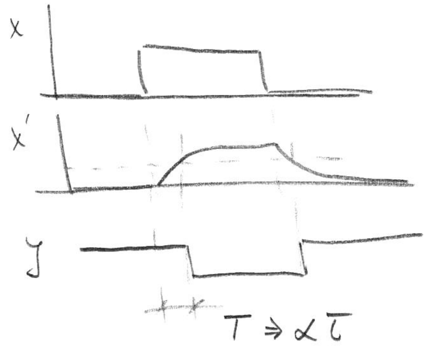
lahko je tudi koristno: detekcija preskokov signala





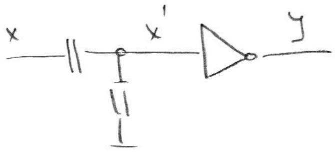
kadar potrebujemo zakasnjene signale, lahko uporabimo RC mrežo (slabo)

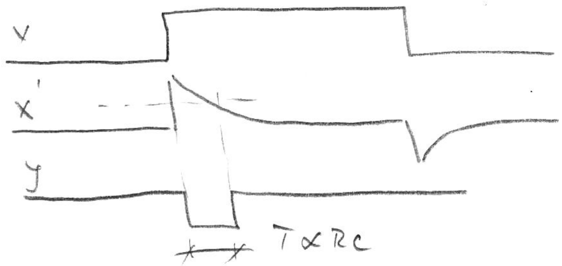




slabo zaradi tega, ker:
- je kasnitev odvisna od log. nivojev, ki so za različna vezja različna
- uporabljamo analogne elemente (toleranca)

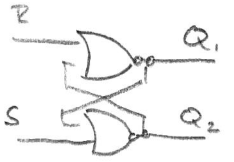
formiranje sunkov z RC





Spominske celice

RS



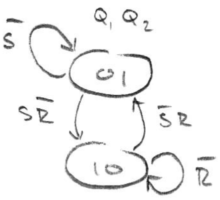
$Q_1 = \overline{R + Q_2} = \overline{R} \cdot \overline{Q_2} = \overline{R} \cdot (S + Q_1)$?

$Q_2 = \overline{S + Q_1}$ enačba ?

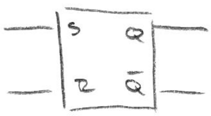
po korakih: postavi: $Q_1 = 0 \Rightarrow Q_2 = 1$ in $Q_1 =$ res 0
$Q_2 = 0 \Rightarrow Q_1 = 1$ in $Q_2 =$ res 0 } dve možni stanji

preklapljanje iz enega stanja v drugo

diagram prehodov med stanji



prepovedano stanje "!"



D

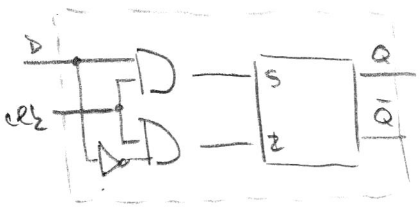
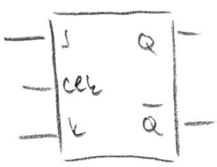


diagram prehodov: le ob clk

čim krajše

detektor prehodov

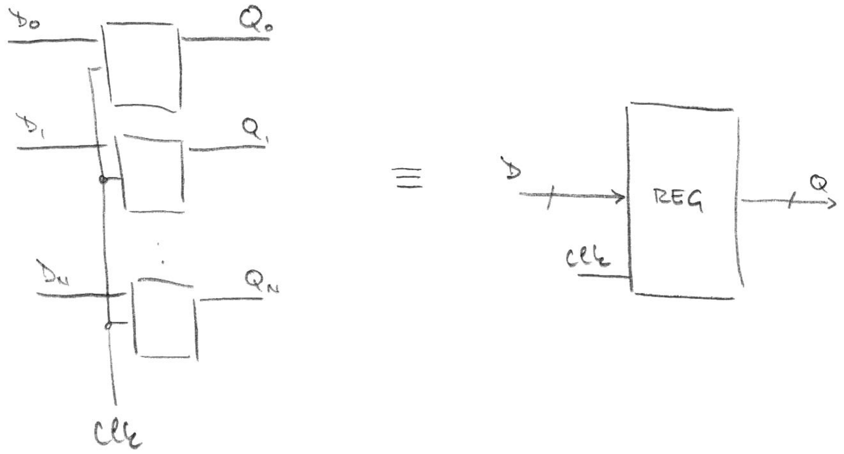
JK



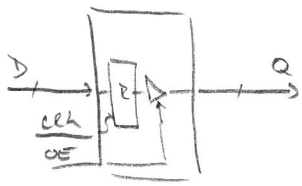
J	K	Q
0	0	Q
0	1	0
1	0	1
1	1	$\overline{Q}$

strukture s FF

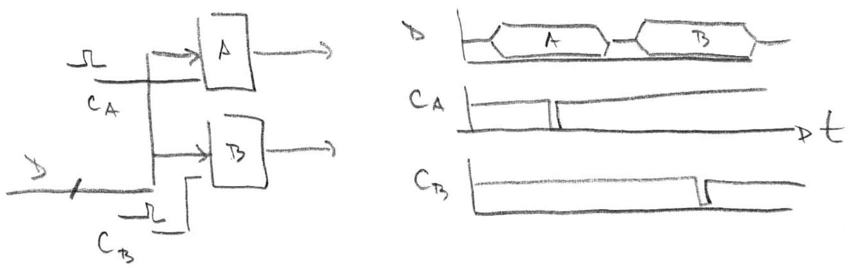
- register : več D-FF, skupna ura



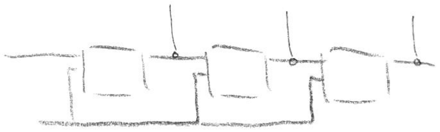
- register z OE

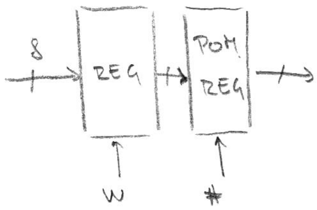


- raba registrov : vpisovanje z vodila

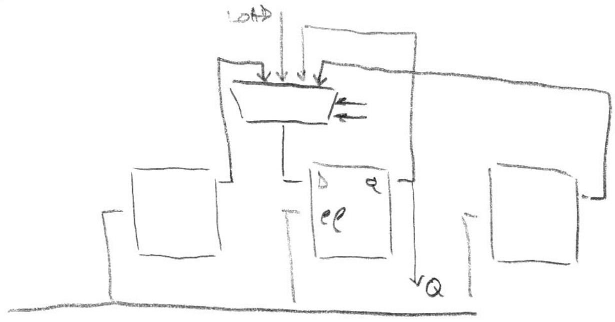


- posmični registar



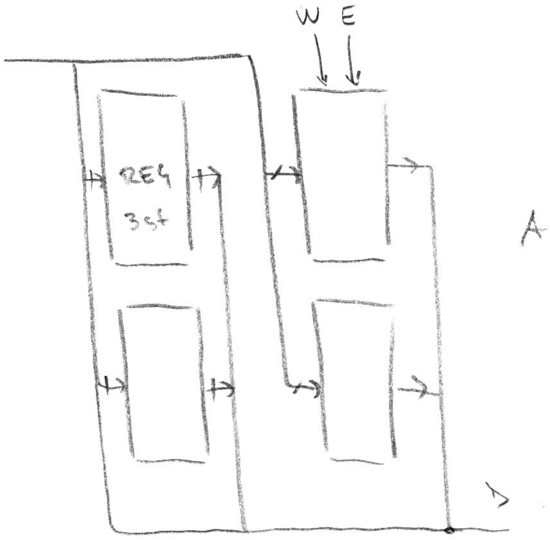


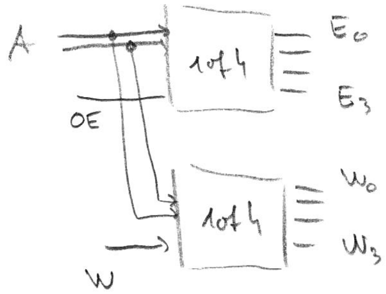
posmak u desno = ÷ 2

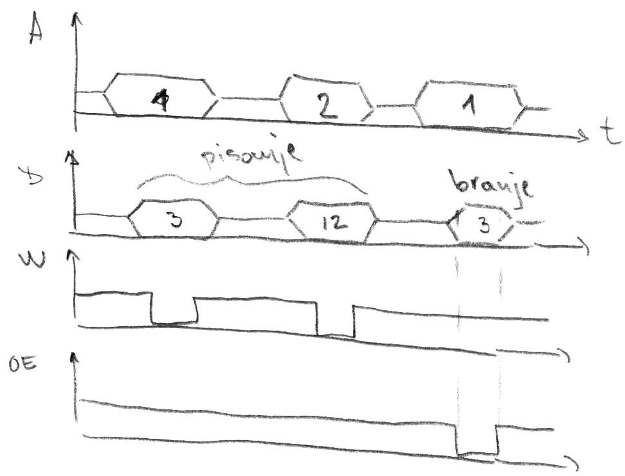


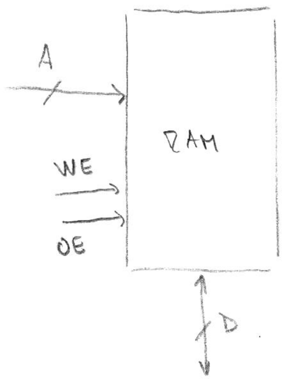
LOAD | LEFT | RIGHT | HOLD

- RAM

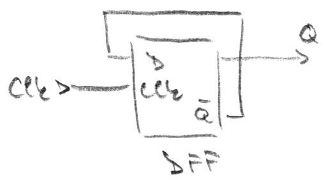


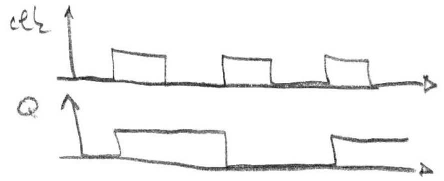




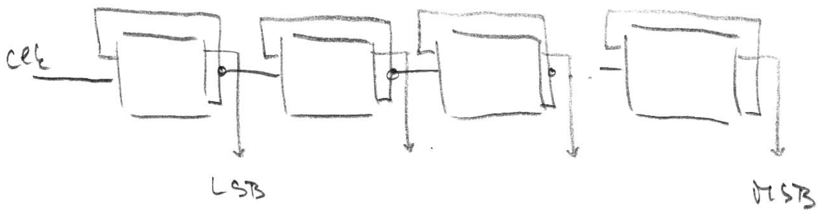


števniki : asinhroni



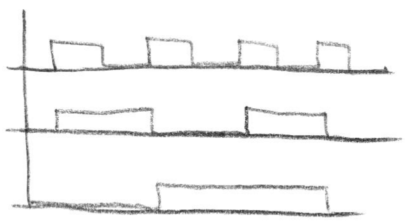


šteje 0,1,0,1



šteje do $2^N - 1$

$N =$ št. FF



obstajajo verzije : load

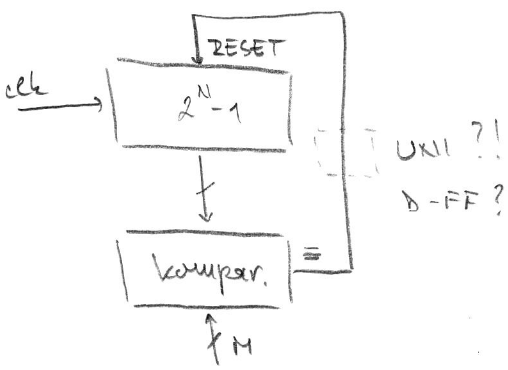
UP/DOWN

lastnosti : hitro

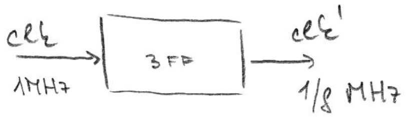
nedefinirano stanje po clk

šteje binarno

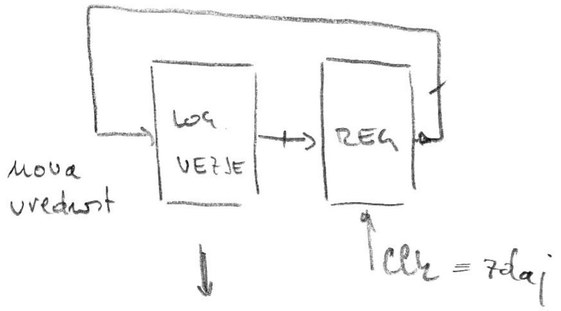
šteje do $M \neq 2^N - 1$ $\Rightarrow$



delilnik frekvence



primeri

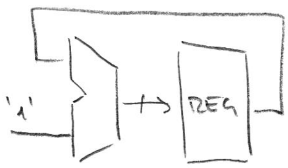


naprimer: prištej 1 ⇒

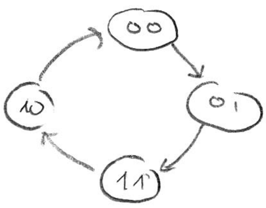
lastnosti: malo počasneje

izhodi se menjajo sočasno

stanje lahko tudi na druge načine!



: generalno:



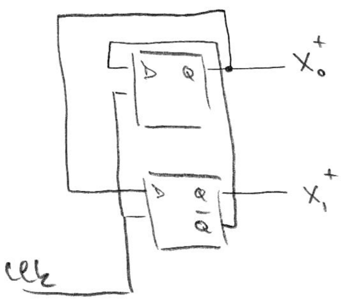
A) diagram prehodov

⇓

x_1	x_0	x_1^+	x_0^+
0	0	0	1
0	1	1	1
1	1	1	0
1	0	0	0

B) tabela prehodov

⇓

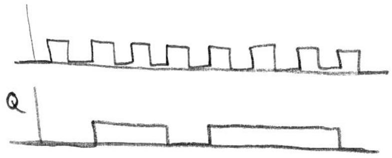


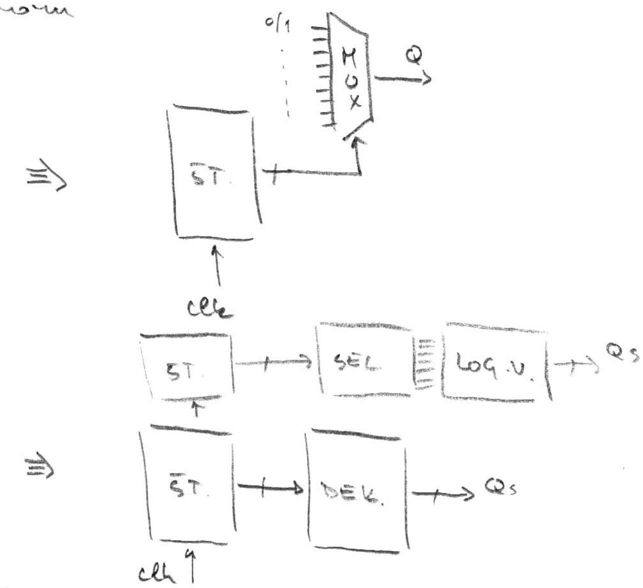
⇐

$$x_1^+ = x_0$$
$$x_0^+ = \overline{x_1}$$

C) enačbe

generiranje signalov s števcem

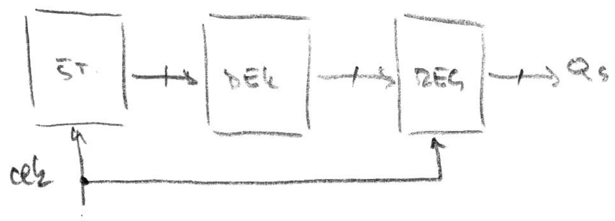




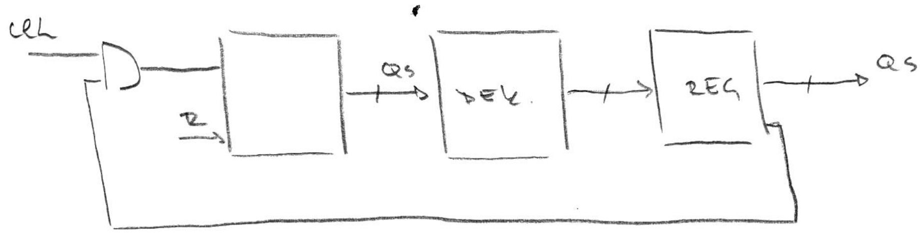
dekodiranje vreče časnitve: nevarnost kratkih simbolov

↓ rešitev

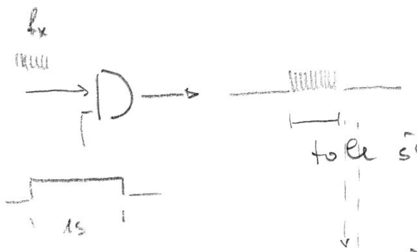
sinhronizacija



enkratni niz



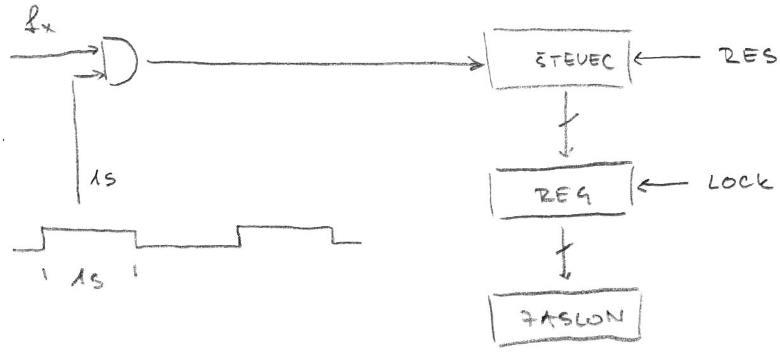
merilnik frekvence

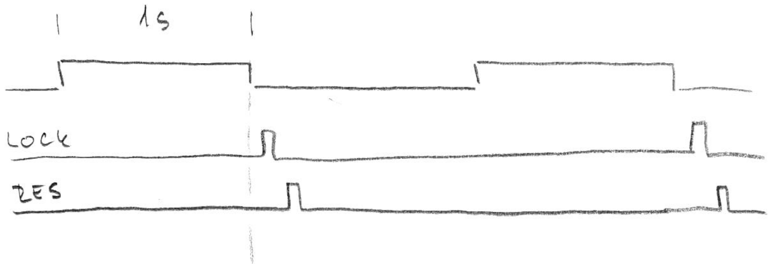


tole štejemo

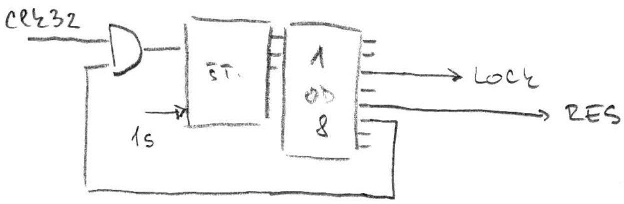
prepišemo iz števca!

reset števca!

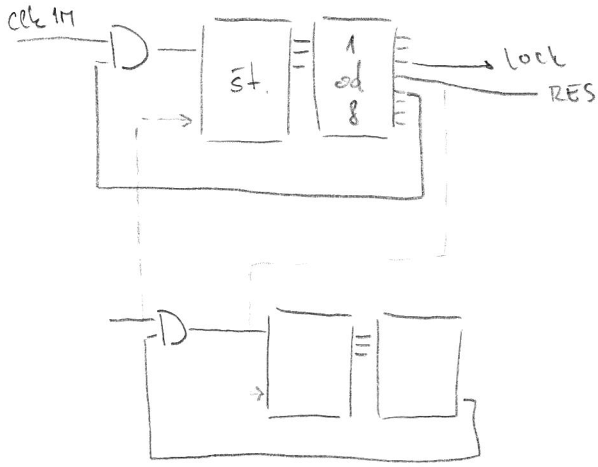




1x po štetju

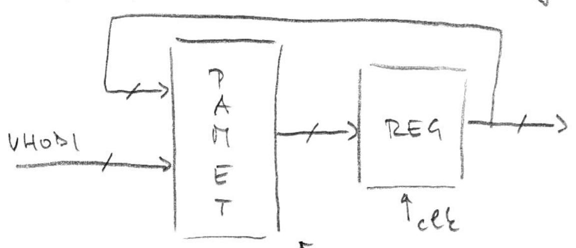


ali pa



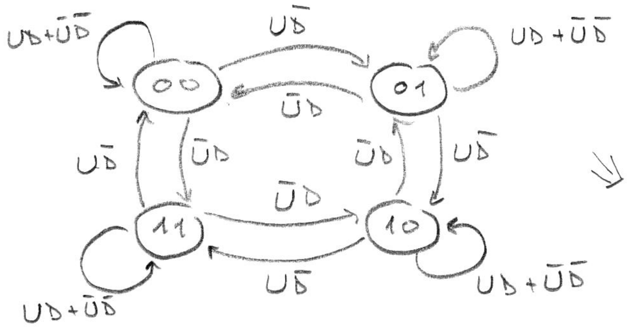
sinhroni avtomati

vezja za generiranje zaporedij stanj
reagirajo na trenutno stanje in vhodne signale



pamet: se odloča o naslednjem stanju na podlagi trenutnega stanja in vhodov

zgled: števec ↑ ↓



U	D	X_1	X_0	X_1^+	X_0^+	J_1	K_1	J_0	K_0
0	0	0	0	0	0	0	X	0	X
1	1	0	0	0	0	0	X	0	X
1	0	0	0	0	1	0	X	1	X
0	1	0	0	1	1	1	X	1	X
0	0	0	1	0	1	0	X	X	0
1	1	0	1	0	1	0	X	X	0
1	0	0	1	1	0	1	X	X	1
0	1	0	1	0	0	0	X	X	1
0	0	1	0	1	0	X	0	0	X
1	1	1	0	1	0	X	0	0	X
1	0	1	0	1	1	X	0	1	X
0	1	1	0	0	1	X	1	1	X
0	0	1	1	1	1	X	0	X	0
1	1	1	1	1	1	X	0	X	0
1	0	1	1	0	0	X	1	X	1
0	1	1	1	1	0	X	0	X	1

J	K	Q^+
0	0	Q
0	1	0
1	0	1
1	1	$\bar{Q}$

	J	K
0→0	0	X
0→1	1	X
1→0	X	1
1→1	X	0

X_1X_0 \ UD	00	01	11	10
00		1		
01				1
11	1	1	1	
10	1		1	1

X_1X_0 \ UD	00	01	11	10
00		1		1
01	1		1	
11	1		1	
10		1		1

$(U \oplus D) \cdot \bar{X}_0 + \overline{(U \oplus D)} \cdot X_0$

$= U \oplus D \oplus X_0$

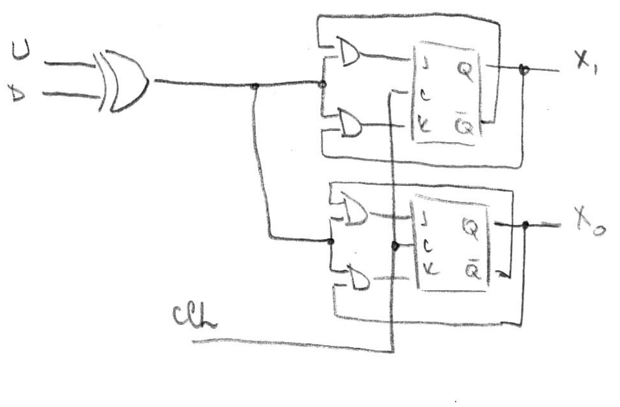
$J_1 = (U \oplus D) \cdot \bar{X}_1$

$K_1 = (U \oplus D) \cdot X_1$

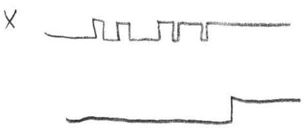
$J_0 = (U \oplus D) \cdot \bar{X}_0$

$K_0 = (U \oplus D) \cdot X_0$

ok!



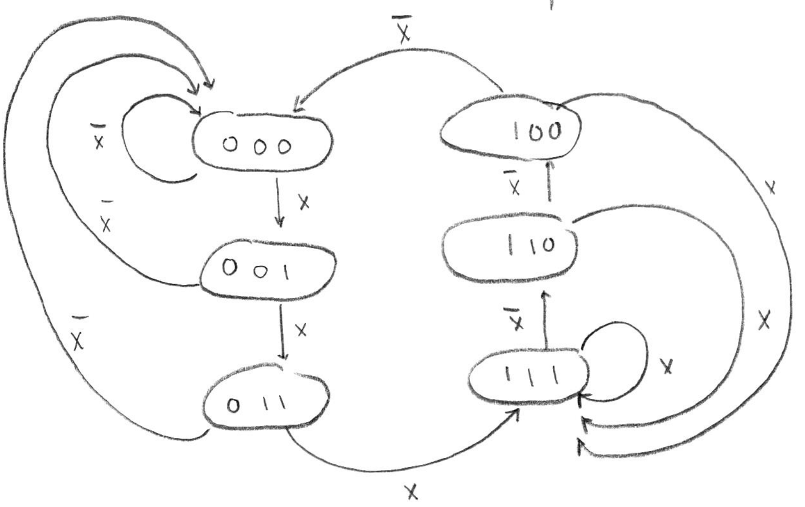
DEBOUNCE



običajno stikalo daje tole

rabimo tole

pamet: vzorči; ko 3x zapored vzorčiš 1 (0)
je rezultat 1 (0)
naraj grede isto



x	$A_2 A_1 A_0$	A_2^+	A_1^+	A_0^+
0	000	0	0	0
1	000	0	0	1
0	001	0	0	0
1	001	0	1	1
0	011	0	0	0
1	011	1	1	1
1	111	1	1	1
0	111	1	1	0
1	110	1	1	1
0	110	1	0	0
1	100	1	1	1
0	100	0	0	0
x	010	0	0	0
x	101	0	0	0

x \ $A_2 A_1 A_0$	000	001	011	010	110	111	101	100
0	0	0	0	0	1	1	0	0
1	0	0	1	0	1	1	0	1

$A_2^+ = A_2 \cdot A_1 + x \bar{A}_2 A_1 A_0 + x A_2 \bar{A}_1 \bar{A}_0$

0	0	0	0	0	0	1	0	0
1	0	1	1	0	1	1	0	1

$A_1^+ = \bar{A}_2 \cdot A_0 \cdot x + A_2 A_1 x +$
$+ A_2 \bar{A}_1 \bar{A}_0 x + A_2 A_1 A_0 \bar{x}$

0	0	0	0	0	0	0	0	0
1	1	1	1	0	1	1	0	1

$A_0^+ = \bar{A}_2 A_0 x + A_2 A_1 x +$
$+ \bar{A}_1 \bar{A}_0 x$

ne pozabi: neuporabljena stanja
začetno stanje
definiraj vse možne izhode
iz stanja

x	$A_2 A_1 A_0$	$A_2^+ A_1^+ A_0^+$	$J_2 K_2$	$J_1 K_1$	$J_0 K_0$
0	0 0 0	0 0 0	0 X	0 X	0 X
1	0 0 0	0 0 1	0 X	0 X	1 X
0	0 0 1	0 0 0	0 X	0 X	X 1
1	0 0 1	0 1 1	0 X	1 X	X 0
0	0 1 1	0 0 0	0 X	X 1	X 1
1	0 1 1	1 1 1	1 X	X 0	X 0
1	1 1 1	1 1 1	X 0	X 0	X 0
0	1 1 1	1 1 0	X 0	X 0	X 1
1	1 1 0	1 1 1	X 0	X 0	1 X
0	1 1 0	1 0 0	X 0	X 1	0 X
1	1 0 0	1 1 1	X 0	1 X	1 X
0	1 0 0	0 0 0	X 1	0 X	0 X
X	0 1 0	0 0 0	0 X	X 1	0 X
X	1 0 1	0 0 0	X 1	0 X	X 1

$J_2 = x \cdot A_1 \cdot A_0$;

$K_2 = \bar{x}\,\bar{A_1}\,\bar{A_0} + A_2\,\bar{A_1}\,A_0$;

$J_1 = x\,A_2\,\bar{A_0} + x\,\bar{A_2}\,A_0$;

$K_1 = \bar{x}\,\bar{A_2}\,A_1 A_0 + \bar{x}\,A_2\,A_1\,\bar{A_0} + \bar{A_2}\,A_1\,\bar{A_0}$

$J_0 = x\,\overline{(A_2 \oplus A_1)}\,\bar{A_0}$

$K_0 = \bar{x}\,\bar{A_2}\,A_0 + \bar{x}\,A_2 A_1 A_0 + A_2\,\bar{A_1}\,A_0$

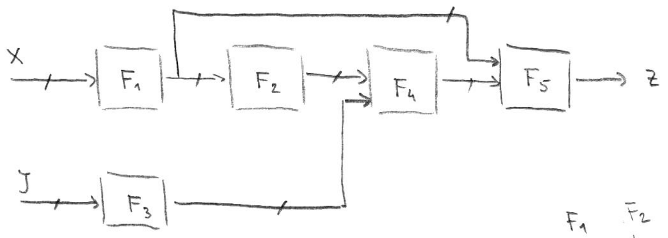
slebše

zaporedje operacij

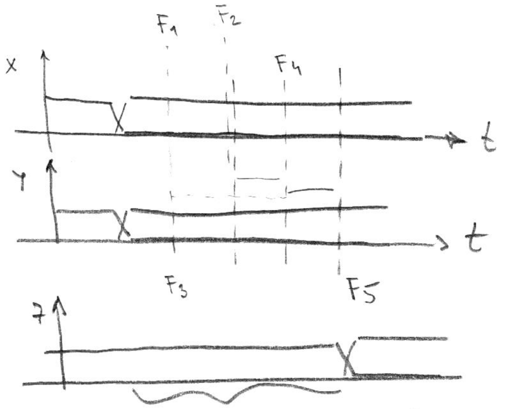
- a) veliko vezje → hitrost

  b) malo vezje → več časa

a)



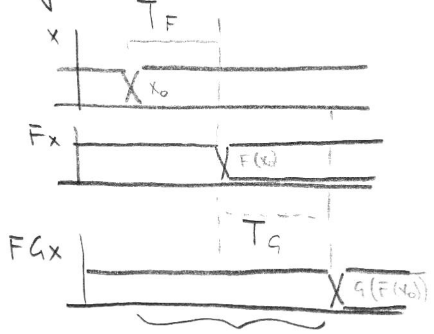
časovni potek



čas računanja = vsota časov !

med računanjem morajo biti vhodi konstantni !

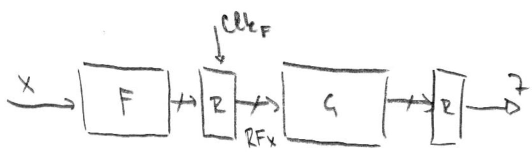
naredi pospeši z vstavljanjem registrov

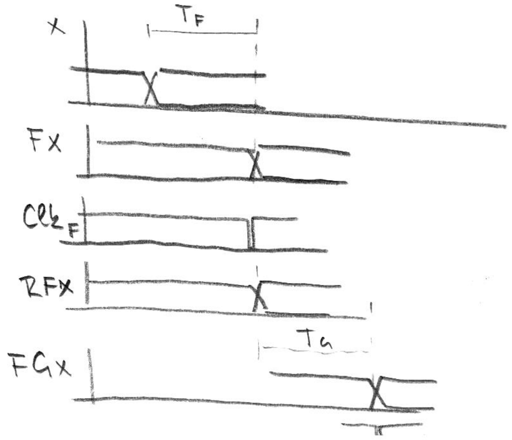


X ≡ konst

perioda več = $\frac{1}{T_F + T_G}$

⇓



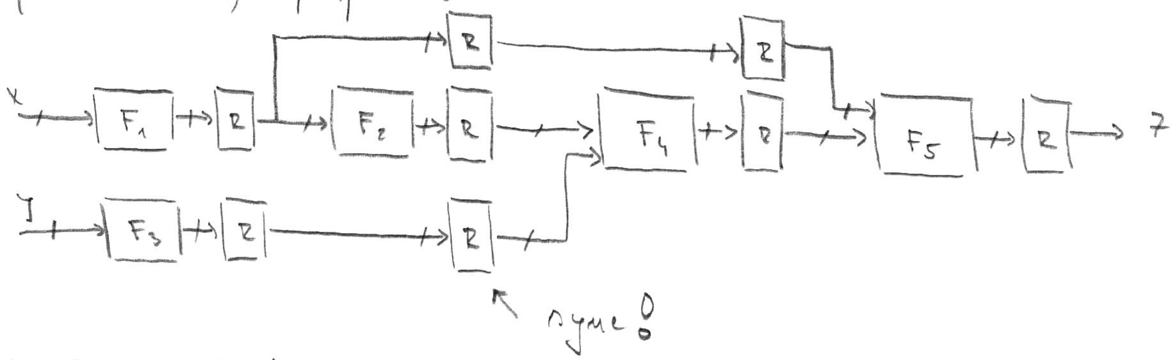


od prega x do prega z mine enako časa, vendar lahko x spremenimo že po T_F !

perioda več = $\frac{1}{T_F}$ ali $\frac{1}{T_G}$

kateri je manj

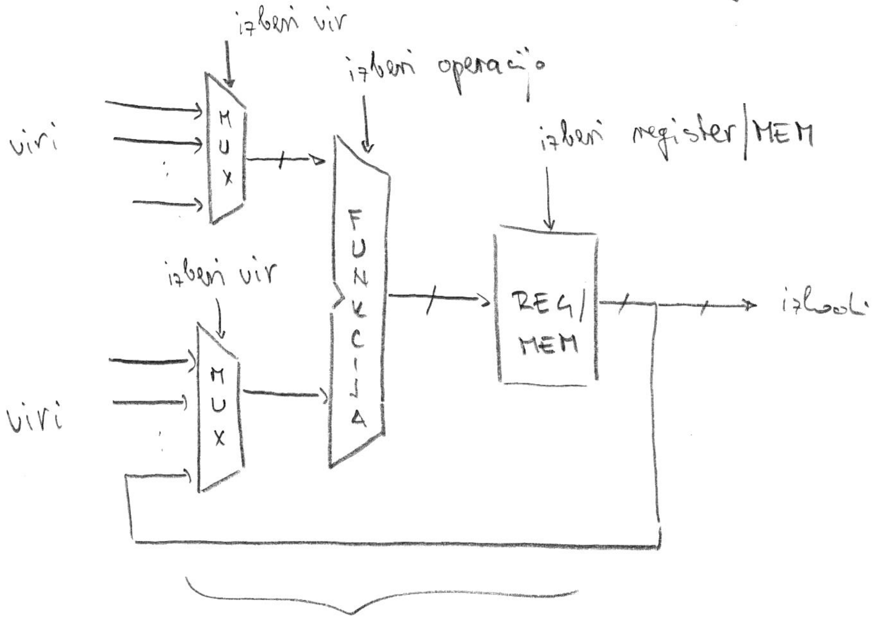
torej lahko a) pospešimo



pipeline tehnika : hitrost računanja je odvisna od najpočasnejšega člena ⇒ razdeli računanje na enako počasne kose

največ je vedno prilagojeno računanju ene same funkcije hitro, a ne fleksibilno

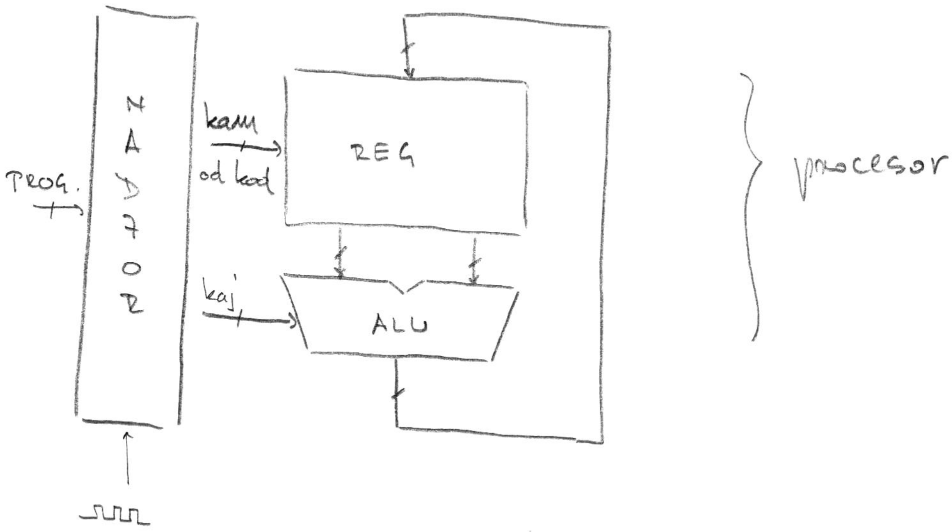
b) porabi ne čas, pomembna je fleksibilnost



izberi: vir, operacijo, register

zadeve tečejo po nadzornem delu + programu

⇓


NADZOR
PROG.
kam
od kod
REG
kaj
ALU
procesor

$\boxed{M \times N}$

$$0011 \times 0110 = \begin{array}{rl} & 0 \times 00011 \\ + & 1 \times 00110 \\ + & 1 \times 01100 \\ + & 0 \times 11000 \\ \hline & 10010_2 = 18_{10} \end{array}$$

$3 \times 6 = 18$

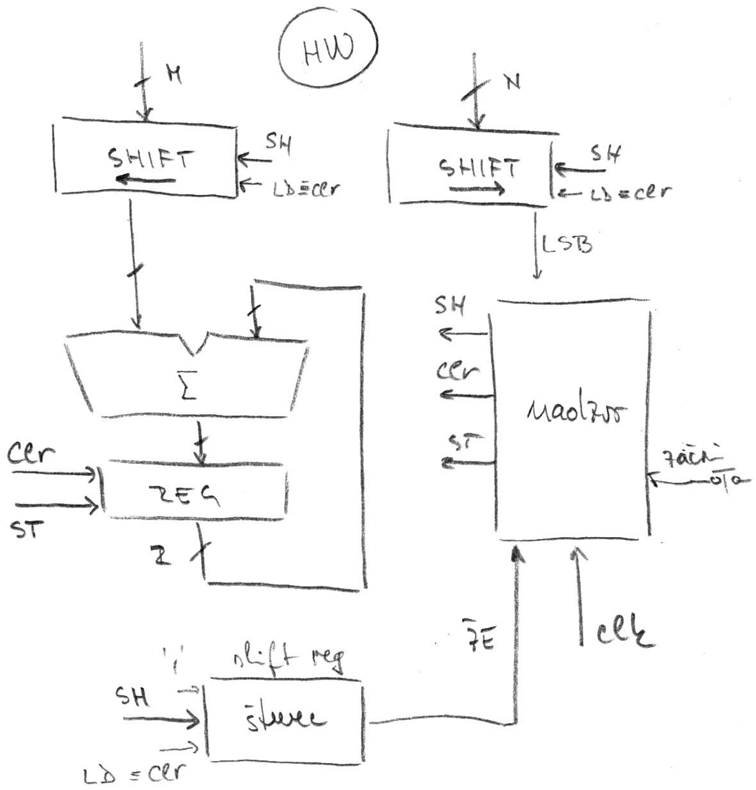
⓪ zbriši rezultat

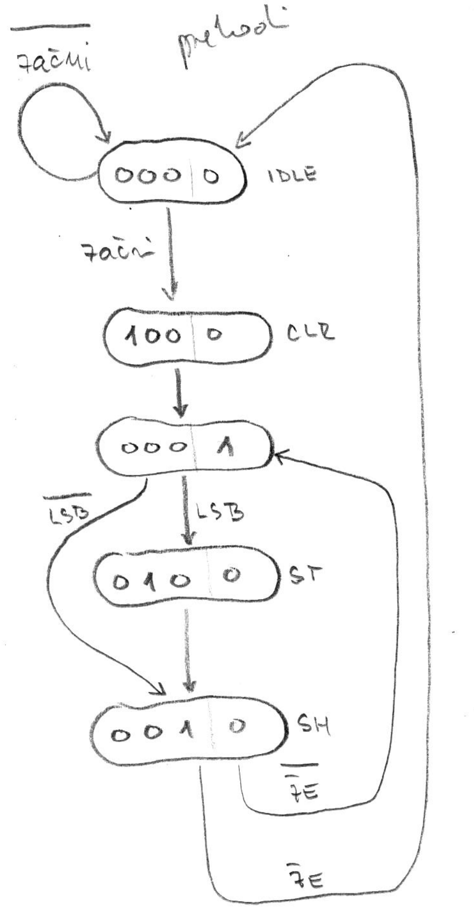
algoritem: ① če je LSB od N = 1 ⇒ prištej rezultatu M

② pomakni M v levo za eno mesto
pomakni N v desno za eno mesto

③ si že dovolj ponovljal zanko? — je kateri od ... v shift-reg == 0 ?

④ konec za DA, skoči na ① za NE

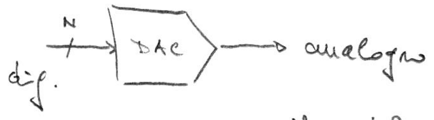




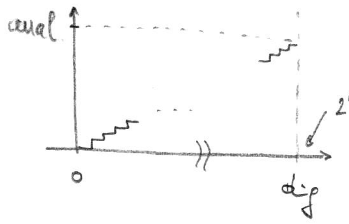
akcija: ↑

clr, ST, SH, SPARE

DAC



majhen tok



2^N-1 ; N ≡ št. bitov } obseg izh. vr.

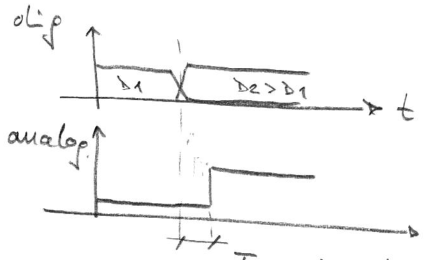
(0 ÷ 5V)

(0 ÷ 2mA)

obseg vh. vr.

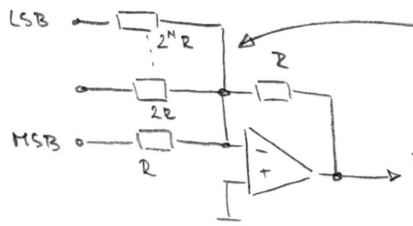
8-bitni : 0 ÷ 256

12-bitni : 0 ÷ 4095



T_c, čas konverzije [ns ÷ ms]

kako to narediti?



N uporov, binarno utežene vrednosti

$$z = k\left(MSB + \frac{1}{2} MSB_{-1} + \frac{1}{4} MSB_{-2} + \frac{1}{8} MSB_{-3} + \cdots \frac{1}{2^N} LSB\right]$$

slabost: težko naredimo točne upore $R \cdots 2^N R$

zgled: 8 bitni DAC: pri LSB $\Rightarrow R_{LSB} = 2^8 R = 256R$

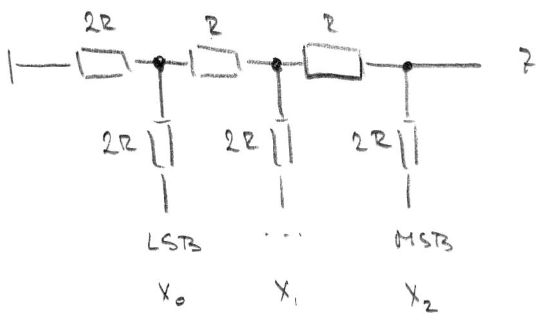
če želimo ta bit pravilno upoštevati, mora biti napaka pri upoštevanju MSB manjša od vrednosti LSB

$$\Downarrow$$

$$\Delta MSB \leq \frac{1}{2^8} = \frac{1}{256} \doteq 0.4\%$$

drago!

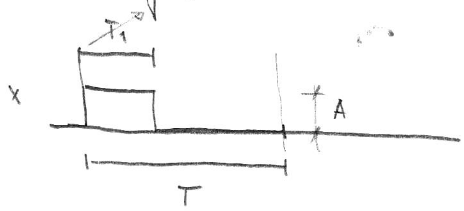
nemogoče zaradi velikega razpona vrednosti

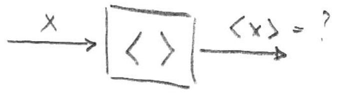


⇒ reši s Theveninom

$$z = \frac{X_2}{2} + \frac{X_1}{4} + \frac{X_0}{8}$$

DAC drugače:

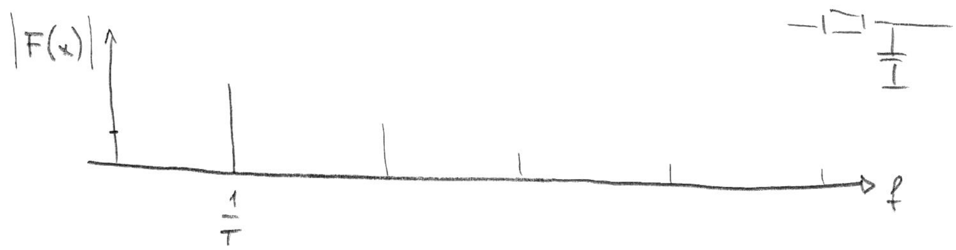




$$\langle x \rangle = A \cdot \frac{T_1}{T}$$

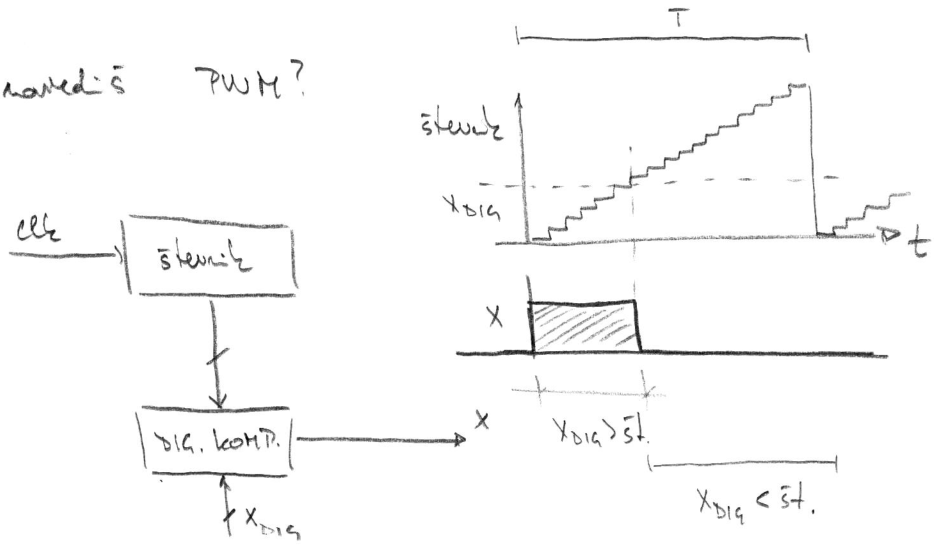
⇓

izberi dovolj dolno vezje za povprečenje



dimenzioniraj tole tako, da obdržiš le DC komponento

kako narediš PWM?



Zgled: clk = 10MHz, števnik ≡ 10 bitni $\Rightarrow T = \frac{2^{10}}{10 \cdot 10^6} \doteq 0{,}1$ ms

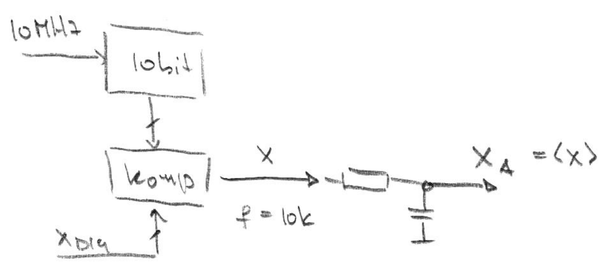
⇓

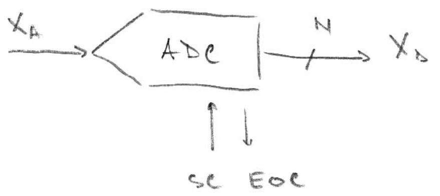
$f = 10$ kHz, $\omega = 2\pi f \doteq 63$ kHz

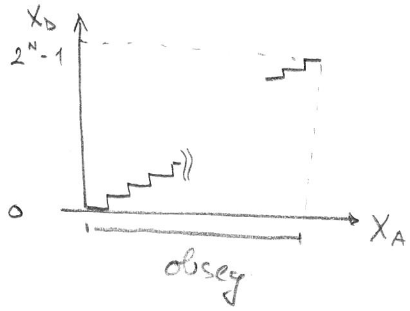
če želim <x>, mora se zadušiti pri 63k na vrednost, ki je manjše od 1LSB = $\frac{1}{1000}$

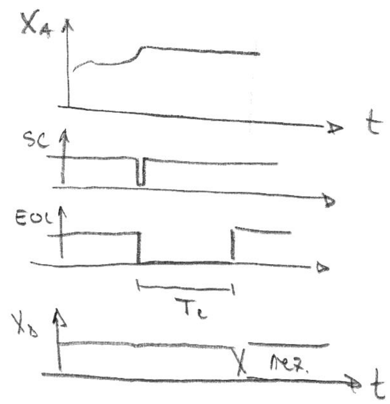
torej mora RC začeti z dušenjem tri dekade pred 63kHz

to pa je pri 63Hz = $\omega_p = \frac{1}{RC} \Rightarrow$ 100kΩ in 0,15μF

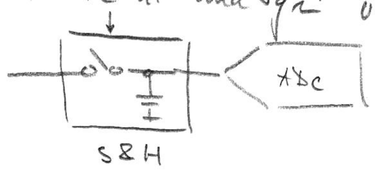




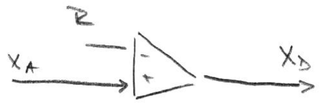




- ločljivost : koliko razl. predelckov
- obseg : analogni
  digitalni $0 \div 2^N-1$ ⇒ zapisi: kodiranje : lin
  dvojiški kompl.
- čas konverzije T_c : od ns do s, odvisno od ločljivosti in tehnike
- vh. signal mora med tem ostati stalen ⇒ vzemi analogni vzorec



tehnike : (A) komparator

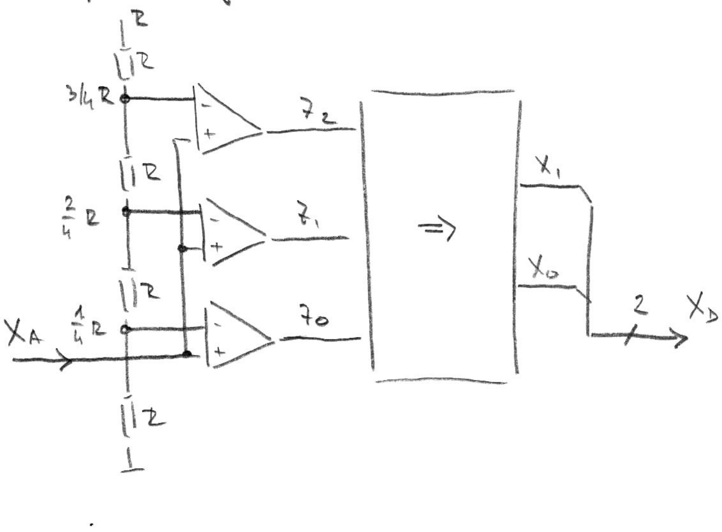


$$X_D \Rightarrow \begin{cases} 0 & \text{za } X_A < R \\ 1 & \text{za } X_A > R \end{cases}$$

↓

ločljivost 1 bit

(A') več komparatorjev ≡ "flash"



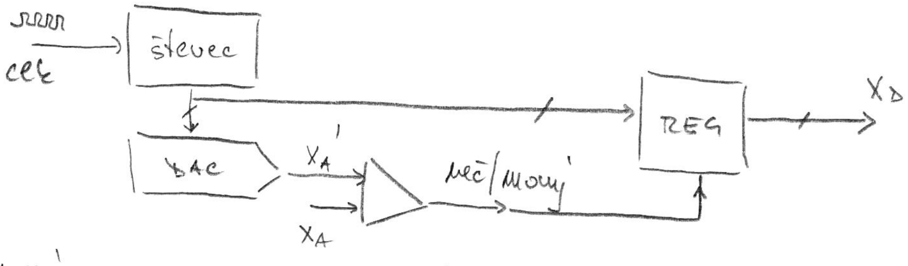
X_A	z_2	z_1	z_0	X_D
$X<\frac{1}{4}R$	0	0	0	00
$\frac{1}{4}R<X<\frac{2}{4}R$	0	0	1	01
$\frac{2}{4}R<X<\frac{3}{4}R$	0	1	1	10
$\frac{3}{4}R<X$	1	1	1	11

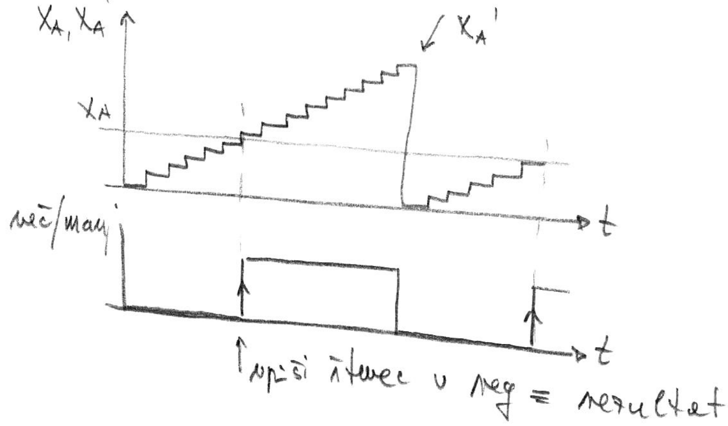
prekoder

⇒

flesh : ločljivost : do 9 bitov ⇒ težave: ofseti komparatorjev
hitrost : ~ ns!
točnost : Ref, komp.

(B) zaporedna primerjava

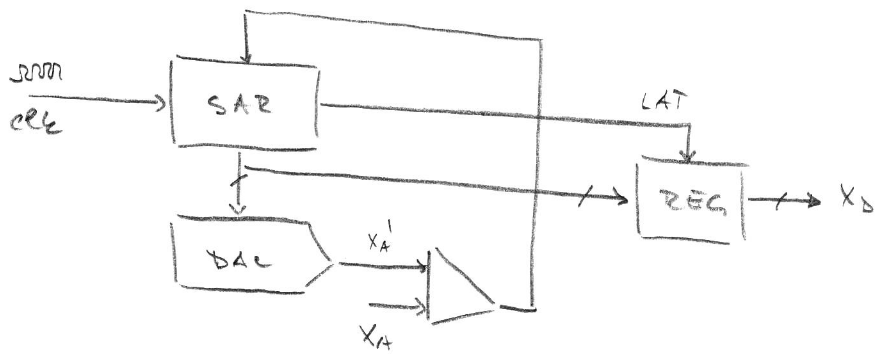


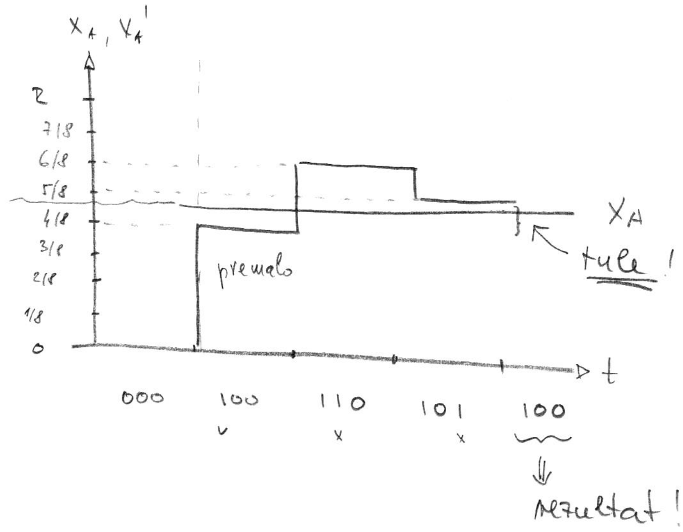


ločljivost : velika!
hitrost : slabo, razlogi
točnost : točnost DAC ≡ ref komp.

variante : števec gor/dol ⇒ hitreje na cilju
analogni integrator namesto DAC ⇒ cenejše

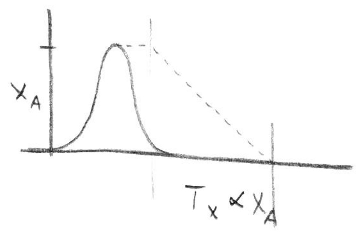
(C) Bisekcija & primerjava

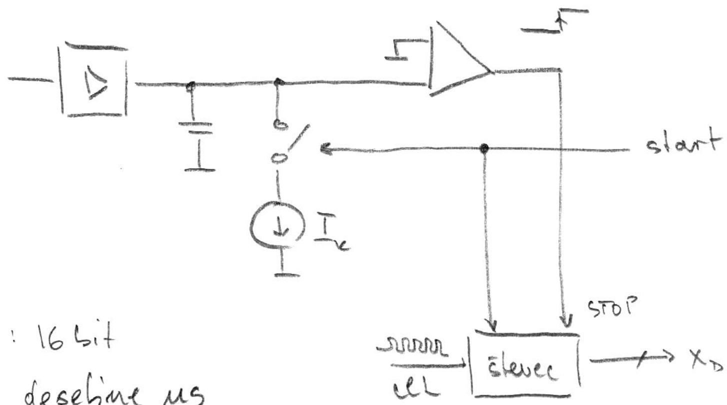




ločljivost: DAC, SAR logika: 16 bit .. 20 bit
hitrost: N-bitov ≡ N primerjav: desetine ns do desetin μs
točnost: Ref, DAC, komparator

Ⓓ Wilkinson





ločljivost: T_C, f_{clk}: 16 bit
hitrost: ločljivost, desetine μs
točnost: Ref I_k, C, komparator, f_{clk}

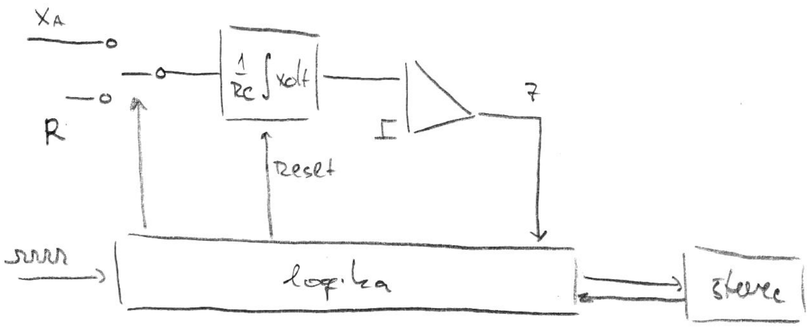
$$Q = C \cdot X_A = I_k \cdot T_X \Rightarrow T_X = \frac{C}{I_k} X_A$$

$$\underbrace{X_D}_{8192} = T_X \cdot f_{clk} = \frac{\overset{10^{-9}\,F}{C} \cdot f_{clk}}{\underbrace{I_k}_{10^{-3}\,A}} \cdot \underbrace{X_A}_{10\,V}$$

$$f_{clk} = \frac{8192 \cdot 10^{-3}}{10 \cdot 10^{-9}} = 819{,}2 \cdot 10^{6} \doteq \underline{\underline{800\ \text{MHz}}}$$

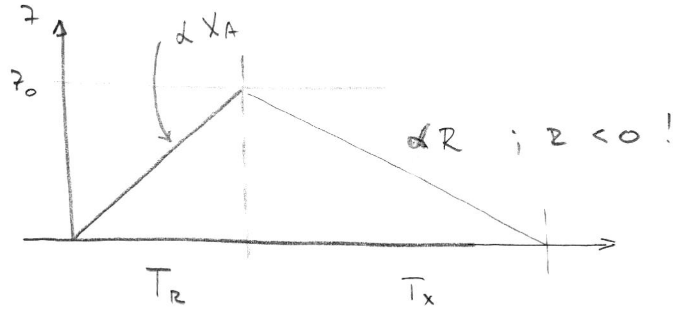
$$T_X = \frac{X_D}{f_{clk}} = \frac{8000}{800 \cdot 10^{6}} \doteq 10\,\mu s$$

(E) dvojna strmina



(1) integriraj X_A znan čas

(2) od-integriraj R dokler ni integratorjev izhod $\varnothing$



$$z_0 = \frac{1}{RC}\int_0^{T_R} X_A\,dt = \frac{X_A T_R}{RC}$$

$$z_0 = \frac{1}{RC}\int_0^{T_x} R\,dt = \frac{R\,T_x}{RC}$$

$$X_A T_R = R\,T_x \Rightarrow T_x = T_R \frac{X_A}{R}$$

$T_x \downarrow N_x \cdot f_{clk}$, $T_R \downarrow N_R \cdot f_{clk}$

$$\underbrace{\qquad}\quad \boxed{N_x = N_R \frac{X_A}{R}}$$

ločljivost: dlje meriš, boljše je ⇒ 24bit!

hitrost: ne, nekaj milisekunde

točnost: samo časovna stabilnost komponent med eno meritvijo (regulirljivo) in $\underline{R}$

tipično: T_R = mnogokratnik $1/f_{omrežja}$ ⇒ ni vpliva

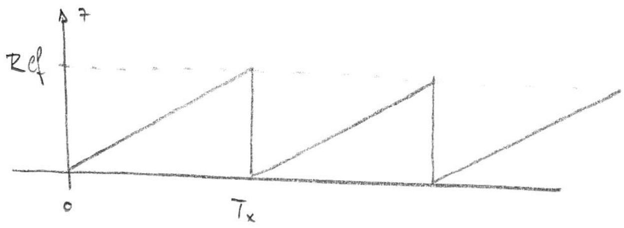
$1/50$, $1/60$ ⇒ 300ms!

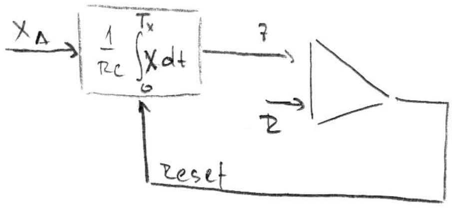
N_R = 2000, 4000, 20000

↓

f_{clk} = 6666, 13333, 66666

V → f : kostoven prenos informacij





$$z = \frac{1}{RC}\int_0^{T_x} X_A\,dt = \underbrace{\frac{X_A}{RC}\,T_x = Ref}$$

$$\boxed{T_x = RC\,\frac{Ref}{X_A}}$$

$$\boxed{f_x = \frac{1}{T_x} = X_A\,\frac{1}{Ref\cdot RC}}$$

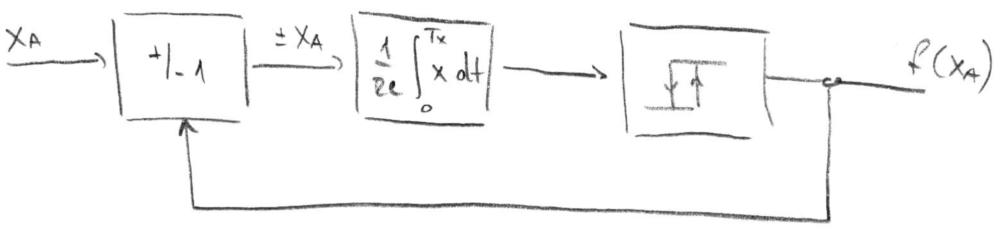
težava z Reset

mora biti končno dolg

⇓

$$T_p = T_x + T_R \Rightarrow f_x' = \frac{1}{T_R + RC\,\frac{Ref}{X_A}} \neq lin.\ f.\ X_A$$

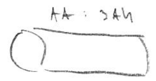
zato raje



charge balanced

Napajanja

kakšne baterije ?



≡ 1,5V

različne velikosti

0.3Ah

≡ 9V

koliko voltov v resnici ? → kemija : diagram AOF

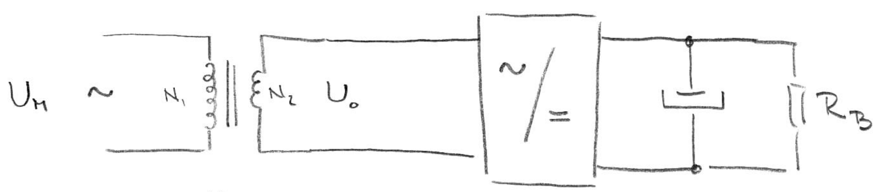
diagrami Duracell

koliko energije v resnici ? : diagrami Duracell

cena ? ~ $3Ah \times 1.2V \,\Big|_{0.1W} = \underbrace{3.6\,Wh \equiv 1€}_{\text{DRAGO}}$

akumulator : napetost ~ 1.25V ; energija } manj € + polnenje

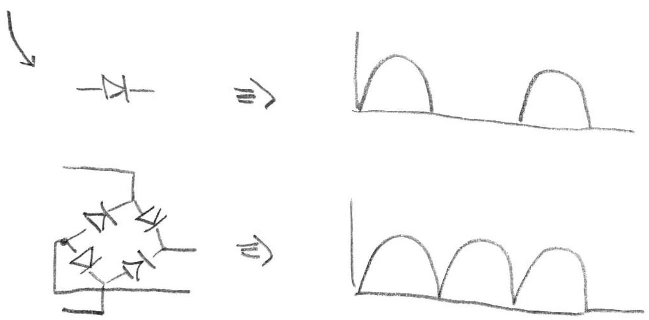
napajanje iz omrežja



$$U_o = \frac{N_2}{N_1} U_M$$

$$I_o = I_M \frac{N_1}{N_2}$$

$$P_o = P_M$$

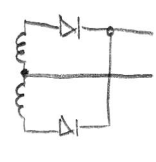


ϕ žice : manj A/mm^2

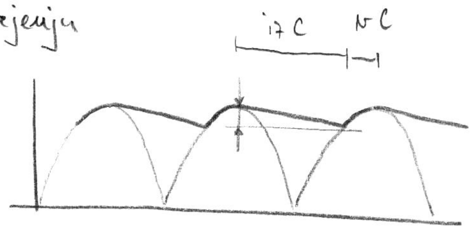
ϕ jedra : $\frac{P}{45}$ [cm^2] ?

ovoji : manj /V

alt:



po glajenju



$$Q_{17} = Q_v$$

$$Q_{17} = \underbrace{C\cdot\Delta U = I_{17}\cdot T_{17}}$$

$$\Delta U = I_{17}\,\frac{T_{17}}{C} \doteq \frac{I_B}{C}\cdot\underbrace{0{,}01}_{1/100\ s}$$

za primer: $\underbrace{I_B = 1A,\ C = 1000\mu}$

$$\Delta U = \frac{1\cdot 0{,}01}{1000\cdot 10^{-6}} = \frac{10^{+4}}{10^{3}} = 10V \ !$$

napak: rabiš velik C! 10000μ ni nenavadno!

posledica: isti velik C moraš napolniti v krajšem času

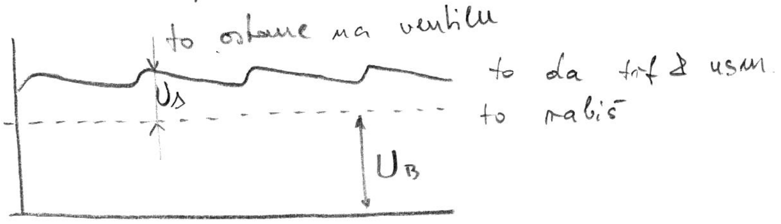
⇓

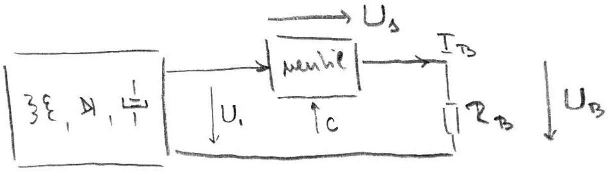
1ms? ⇒ tok polnenja je

$$\underbrace{I_v = I_{17}\cdot\frac{T_{17}}{T_v} \doteq 10\div100\ I_{17}}\ !$$

močne diode
debela žica
preboj v sunkih !

rešitev: stabilizacija → mečeš stran moč





jasno: na ventilu se troši moč $P_v = U_s\cdot I_B = (U_1 - U_B)\cdot I_B$

$$\langle W_B\rangle = \frac{1}{T}\int_0^T P_v\,dt$$

ventil se greje

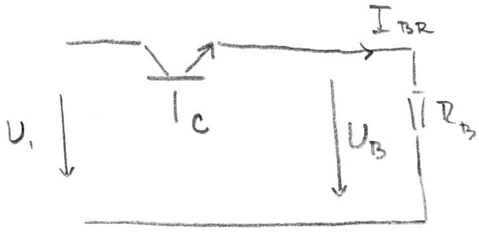
⇓

ventil bo treba hladiti!

za primer: $I_B = 1A$, $U_s = 3V$ ⇒ $W_B = 3W$

hladilnik: °C/W

vezava ventila :



TR = tokov ojač $\Rightarrow$ $I_c = \beta I_B$, $I_E = I_c + I_B = (\beta+1) I_B \doteq \beta I_B$

$$I_B = I_{B0}\left(e^{U_{BE}/U_T} - 1\right)$$

$$= I_{B0}\left(e^{\frac{c-U_B}{U_T}} - 1\right)$$

TR pušča $\Rightarrow$ $\underbrace{I_E \doteq \beta I_{B0}\, e^{\frac{c-U_B}{U_T}} + \overset{\text{funkcija}}{\gamma}(U_1 - U_B)} = I_{BR} = \underbrace{\frac{U_B}{R_B}}$

$$\underbrace{\frac{c-U_B}{U_T} = \ln \frac{\frac{U_B}{R_B} - \gamma(U_1-U_B)}{\beta I_{B0}}}$$

$$\boxed{c = U_B + U_T \ln \frac{U_B - R_B\gamma(U_1-U_B)}{\beta I_{B0} R_B}}$$

če tole damo na c , je U_B znan in konstanten

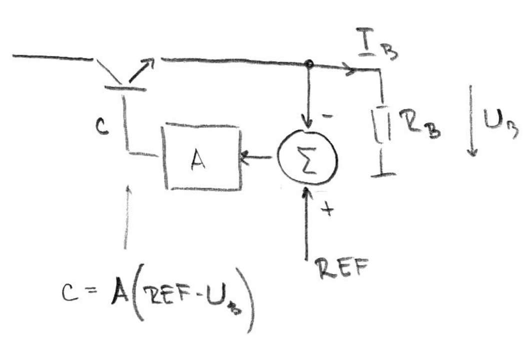
kako tole izračunati v naprej?

⇓

NE GRE !

⇓

trik



$$\underbrace{A(REF-U_B)} = c = \underbrace{U_B + U_T \ln \frac{\cdots}{\cdots}}$$

kako narediti $U_B = REF$?

$$\Downarrow$$

$$A \to \infty \quad !$$

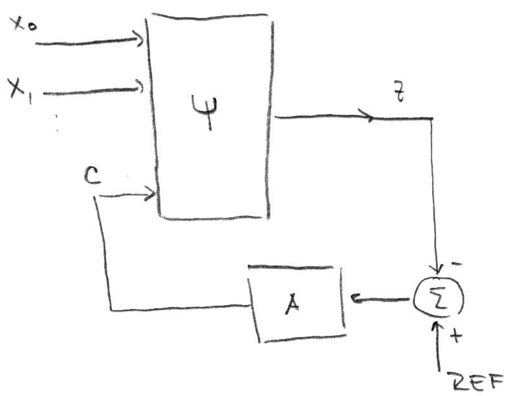
$$\Downarrow$$

$$U_B = REF - \left.\frac{U_B + U_T \ln \cdots}{A}\right|_{A \to \infty} = \underline{REF}$$

rešitev torej: $A \to \infty$ in $\underline{U_B = REF}$

---

tudi splošno



$$z = \Psi(x_0, x_1, \dots, c)$$

$\Downarrow$ nazaj

$$c = \Psi^{-1}(x_0, x_1, \dots, z)$$

$$c = A(REF - z) \longrightarrow A(REF - z) = \Psi^{-1}(\quad)$$

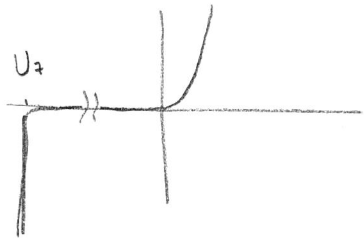
$$\Downarrow$$

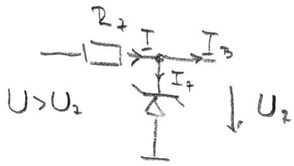
$$z = REF - \frac{\Psi^{-1}(\quad)}{A}$$

za $A \to \infty \Rightarrow \underline{z = REF}$

manjka še referenca REF

- elektronika : Zenerjeva dioda





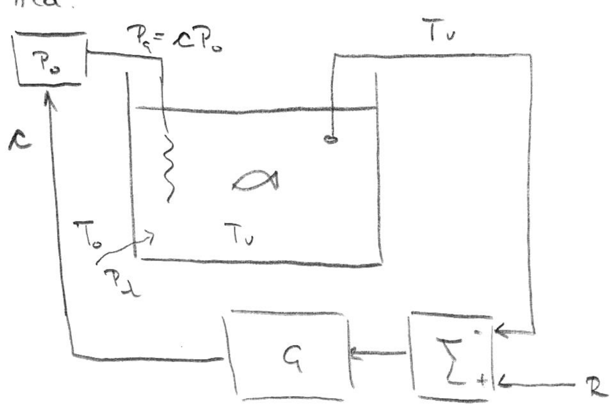
pogoj : $I_Z > 0$; $\underbrace{I_Z + I_B} = I = \underbrace{\frac{U-U_Z}{R_Z}}$

$$\frac{U-U_Z}{R_Z} - I_B > 0$$

moč na ZD : $P_Z = U_Z \cdot I_Z$

ZD narejene za omejene moči : 0.25W, 1W, 5W ?

1. red. S10/1



$$P_{IN} = P_q + P_\lambda = cP_o + (T_o - T_v)\lambda$$

$$T_v = P_{IN} \frac{\alpha}{1+\tau_v p}$$

$$T_v = \left[cP_o + (T_o - T_v)\lambda\right] \frac{\alpha}{1+\tau_v p}$$

$$\frac{T_v}{\alpha}(1+\tau_v p) - (T_o - T_v)\lambda = cP_o$$

$$c = \frac{1}{P_o}\left[\frac{T_v}{\alpha}(1+\tau_v p) - (T_o - T_v)\lambda\right]$$

$$c = G(R - T_v)$$

$$G(R - T_v) = \frac{T_v}{P_o \alpha}(1+\tau_v p) - (T_o - T_v)\frac{\lambda}{P_o}$$

$$T_v\left[\frac{1+\tau_v p}{P_o \alpha} + \frac{\lambda}{P_o} + G\right] = GR + T_o \frac{\lambda}{P_o} \quad \Big/ \cdot \alpha P_o$$

$$T_v\left[(1+\tau_v p) + \alpha\lambda + \alpha P_o G\right] = \alpha P_o G R + T_o \alpha \lambda$$

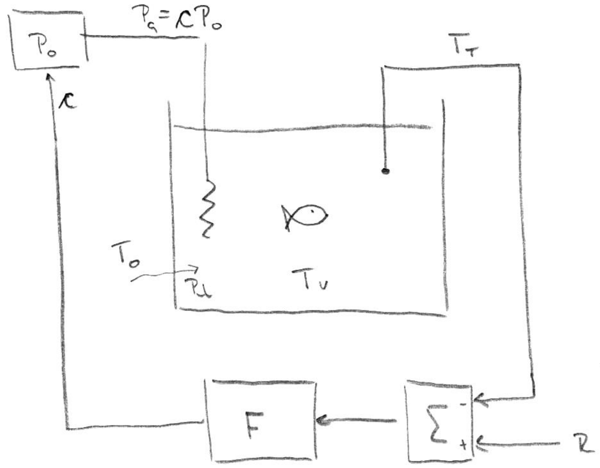
$$T_v = \frac{\alpha P_o G R + T_o \alpha \lambda}{\tau_v p + 1 + \alpha\lambda + \alpha P_o G}$$

$$= \frac{\alpha P_o G R + T_o \alpha \lambda}{1 + \alpha\lambda + \alpha P_o G} \cdot \frac{1}{1 + \underbrace{\frac{\tau_v}{1+\alpha\lambda+\alpha P_o G}} p}$$

τ_v je navidezno manjši

stac. stanje: $G \to \infty$

$$T_v = \left.\frac{\alpha P_o G R + T_o \alpha \lambda}{\alpha P_o G + \alpha\lambda + 1}\right|_{G \to \infty} = \underline{\underline{R}} \checkmark$$



$$c = F\left(R - T_T\right)$$

$$c = F\left(R - T_v \frac{1}{1+\tau_T p}\right)$$

$$T_T = T_v \frac{1}{1+\tau_T p}$$

$$P_{IN} = P_G + P_\lambda = c P_0 + \left(T_0 - T_v\right)\lambda$$

$$T_v = P_{IN} \frac{\alpha}{1+\tau_v p}$$

$$T_v = \left[c P_0 + (T_0 - T_v)\lambda\right] \cdot \frac{\alpha}{1+\tau_v p}$$

$$\frac{T_v}{\alpha}\left(1+\tau_v p\right) - \left(T_0 - T_v\right)\lambda = c P_0$$

$$\boxed{F\left(R - T_v \frac{1}{1+\tau_T p}\right) = \frac{T_v}{P_0 \alpha}\left(1+\tau_v p\right) - \left(T_0 - T_v\right)\frac{\lambda}{P_0}}$$

① $F = G \Rightarrow P_0 \alpha G R - \dfrac{\alpha G T_v P_0}{1+\tau_T p} = T_v\left(1+\tau_v p\right) - \left(T_0 - T_v\right)\alpha\lambda$

$$T_v\left[\frac{\alpha G P_0}{1+\tau_T p} + \left(1+\tau_v p\right) + \alpha\lambda\right] = P_0 \alpha G R + T_0 \alpha \lambda$$

$$T_v \frac{\alpha G P_0 + \left(1+\alpha\lambda+\tau_v p\right)\left(1+\tau_T p\right)}{1+\tau_T p} = P_0 \alpha G R + T_0 \alpha \lambda$$

$$T_v = \frac{P_0 \alpha G R + T_0 \alpha \lambda}{\alpha G P_0 + 1 + \tau_T p + \alpha\lambda + \alpha\lambda\tau_T p + \tau_v p + \tau_v \tau_T p^2} \cdot \left(1+\tau_T p\right)$$

$$= \frac{\left(P_0 G R \alpha + T_0 \alpha \lambda\right)\left(1+\tau_T p\right)}{\tau_v \tau_T p^2 + \left[\tau_T\left(1+\alpha\lambda\right) + \tau_v\right] p + \alpha\left(G P_0 + \lambda\right) + 1}$$

ideja od prej : $G \to \infty$

stac. stanje : odvodi $\equiv 0$!

$$T_v = \left.\frac{P_0 G R \alpha + T_0 \alpha \lambda}{\alpha G P_0 + \alpha\lambda + 1}\right|_{G\to\infty} = R \; ! \quad ✓✓$$

dinamika: oblika $T_v = \dfrac{?}{Ap^2 + Bp + C} \Rightarrow$ D.E. $\Rightarrow A\ddot{T}_v + B\dot{T}_v + CT_v = ?$

rešitve iščemo v obliki $e^{\beta t} \Rightarrow T_v = T_{v0} e^{\beta t}$

$$\dot{T}_v = T_{v0}\, \beta e^{\beta t}$$

$$\ddot{T}_v = T_{v0}\, \beta^2 e^{\beta t}$$

torej: $\underbrace{T_{v0}\, e^{\beta t}}_{\neq 0} \underbrace{\left[A\beta^2 + B\beta + C\right]}_{=0} = 0$ homogeni del

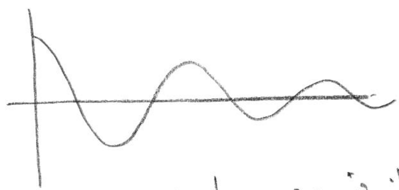
ničle: $\beta_{1,2} = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

zato so rešitve: $T_v = T_{v0}\, e^{\beta_{1,2} t}$

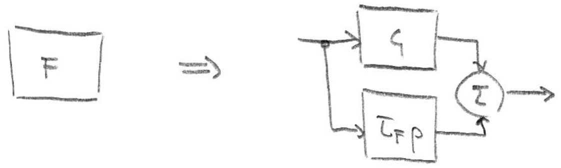
kakšni so β? $\longrightarrow$ za $G \to \infty$ gre $C \to \infty$

torej sta $\beta_{1,2}$ konj. kompl.

rešitve: nihanje okoli končne vr.!



<u>vedno se izniha</u>, saj sta A in B > 0

(2) $F = G + \tau_F p \Rightarrow$ 

$\Downarrow$

$$GR + \tau_F R p - \frac{G T_v}{1+\tau_T p} - \frac{\tau_F p T_v}{1+\tau_T p} = \frac{T_v}{\alpha P_0}(1+\tau_v p) - (T_0 - T_v)\frac{\lambda}{P_0}$$

$$T_v\left[\frac{(1+\tau_v p)}{\alpha P_0} + \frac{\lambda}{P_0} + \frac{G}{1+\tau_T p} + \frac{\tau_F p}{1+\tau_T p}\right] = GR + R\tau_F p + \frac{T_0\lambda}{P_0} \Big/ \cdot \alpha P_0(1+\tau_T p)$$

$$T_v\left[(1+\tau_v p)(1+\tau_T p) + \alpha\lambda(1+\tau_T p) + \alpha P_0 G + \alpha P_0 \tau_F p\right] = \alpha \cancel{P_0}(1+\tau_T p)\frac{GRP_0 + RP_0\tau_F p + T_0\lambda}{\cancel{P_0}}$$

$\downarrow$

$$T_v = \frac{\alpha G R P_0 + \alpha R P_0 \tau_F p + T_0\lambda\alpha}{1+(\tau_v+\tau_T)p + \tau_v\tau_T p^2 + \alpha\lambda(1+\tau_T p) + \alpha P_0 G + \alpha P_0 \tau_F p}(1+\tau_T p)$$

$\downarrow$

$$= \frac{P_0 G R\alpha + \alpha R P_0 \tau_F p + T_0 \alpha\lambda}{\tau_v\tau_T p^2 + \left[\tau_T(1+\alpha\lambda) + \tau_v + \alpha P_0 \tau_F\right]p + \alpha(P_0 G + \lambda) + 1}$$

$\uparrow$

tole dodatno!

$\Downarrow$

$A_{zdaj} = A_{prej}$

$C_{zdaj} = C_{prej}$, $B_{zdaj} \geqslant B_{prej}$

stac. stanje: $G \to \infty$ $\quad T_v = \left.\frac{P_0 G R\alpha + T_0\alpha\lambda}{\alpha P_0 G + \alpha\lambda + 1}\right|_{G\to\infty} = \underline{\underline{R}}$ ✓ ok

dinamika: $B_{zdaj} > B_{prej}$, lahko ga elektronsko izberemo?
izberemo pravo časovno konstanto τ_F
najboljša je tista, pri kateri ni
nihanja

v imenovalcu nastopa še $\tau_F p R \Rightarrow$ odvod reference
če to moti: integriraj

kako je z močjo?

(3) $F = \frac{1}{\tau_1 p}$; $\tau_1 \gg \tau_T$?

$$\frac{R}{\tau_1 p} - \frac{T_v}{\tau_1 p (1+\tau_T p)} = \frac{T_v}{P_0 \alpha}(1+\tau_v p) - (T_0 - T_v)\frac{\lambda}{P_0}$$

↓

zanemari! $\tau_1 \gg \tau_T$

$$T_v\left[\frac{1+\tau_v p}{P_0 \alpha} + \frac{\lambda}{P_0} + \frac{1}{\tau_1 p}\right] = \frac{R}{\tau_1 p} + T_0 \frac{\lambda}{P_0} \quad / \cdot \alpha P_0 \tau_1 p$$

$$T_v\left[(1+\tau_v p)\tau_1 p + \alpha \lambda \tau_1 p + \alpha P_0\right] = \alpha R P_0 + T_0 \alpha \lambda \tau_1 p$$

$$T_v = \frac{\alpha R P_0 + T_0 \alpha \lambda \tau_1 p}{\tau_1 p + \tau_v \tau_1 p^2 + \alpha \lambda \tau_1 p + \alpha P_0}$$

$$T_v = \frac{\alpha R P_0 + T_0 \alpha \lambda \tau_1 p}{\tau_v \tau_1 p^2 + (1+\alpha\lambda)\tau_1 p + \alpha P_0}$$

stac. stanje: $T_v = \frac{\alpha R P_0}{\alpha P_0} = \underline{\underline{R}}$? w ok

dinamika: dovolj velik τ_1 zagotavlja, da bo $\underline{\underline{B^2 \geq 4AC}}$

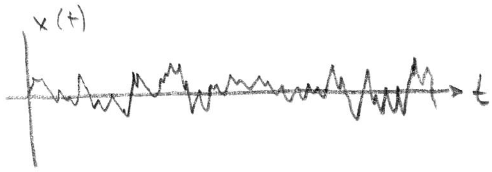
Šumi : naključni signal

šum ≠ motnja

vir je znan, predvidljiv : se da izločiti, omiliti

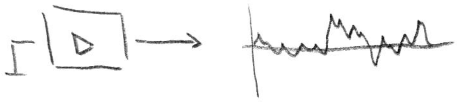
nepredvidljivo : ne znamo določiti frekvence, faze, trenutne vrednosti

šum :



šum moti, ker definira sp. mejo občutljivosti merjenja !

Vsa el. vezja generirajo šum !



gibanje elektronov !

---

opisovanje šuma :

$\langle x(t) \rangle = 0$!

ne pomaga, če merimo sr. vrednost

—"— , če merimo amplitudo (max odmik) le poredko, neperiodično

—"— , če merimo prehode skozi ϕ

—"— , če vzorčimo in dobimo $x(nT)$

---

matematika

$$- \langle x(t) \rangle = \frac{1}{T} \int_{-T/2}^{T/2} x(t)\,dt \implies \langle x(t) \rangle = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)\,dt = 0 \; !$$

$- \langle x^2(t) \rangle \neq \phi$! : seveda, če naključno napetost pripeljemo na upornik, se bo ta še vedno grel !

---

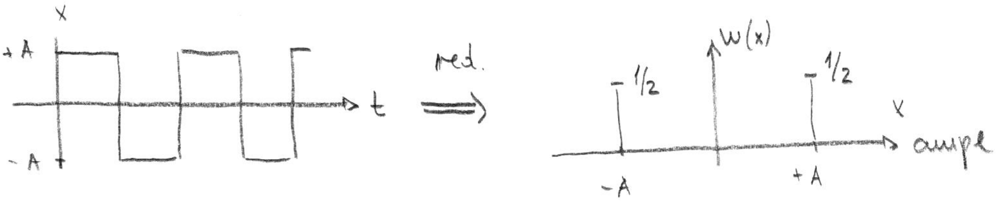
prej : porazdelitve

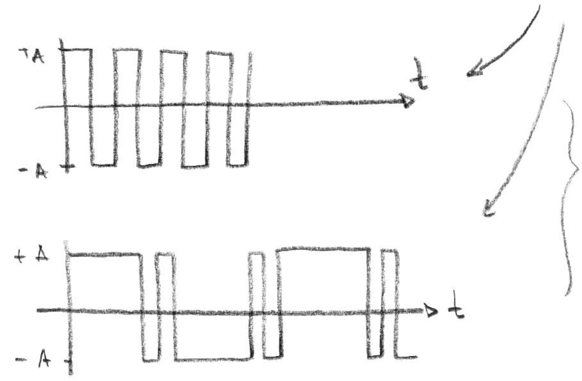
predstavimo tako, da reduciramo podatke

 $\Rightarrow$ povemo A, f, φ

reduciramo: po velikosti
po času
} mi mislimo, da je iz reduciranih podatkov mogoče rekonstruirati original!

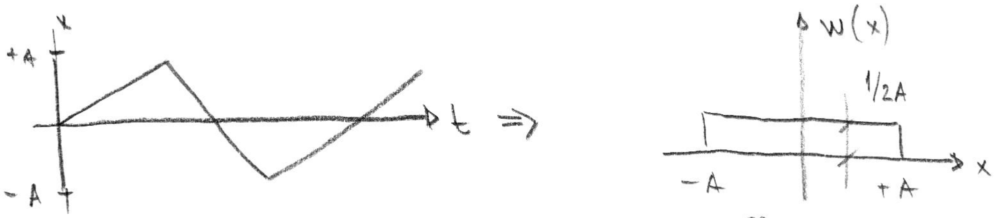
zgled:





kateri je pravi?

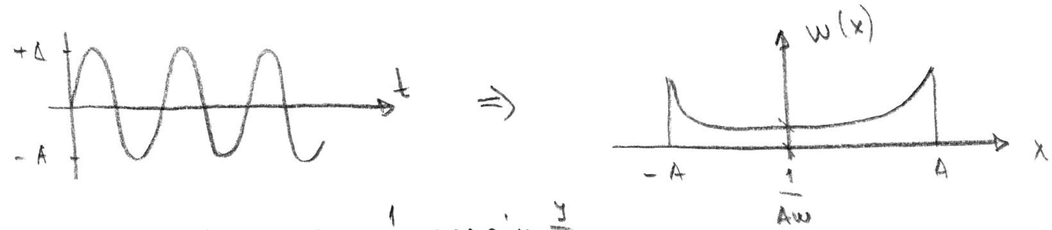
zgled:



vedeti moramo: $\int_{-\infty}^{\infty} w(x)\,dx = 1$

$\Downarrow$

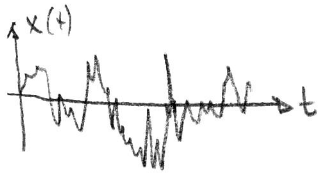
zgled:



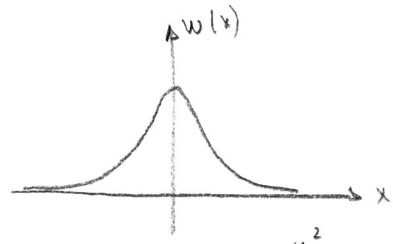
$y = A \sin \omega t \Rightarrow t = \frac{1}{\omega} \arcsin \frac{y}{A}$

$$\frac{dt}{dy} = \frac{1}{A\omega} \frac{1}{\sqrt{1-\left(\frac{y}{A}\right)^2}} = w(x)$$

šum ?



⇒ ?



$$w(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

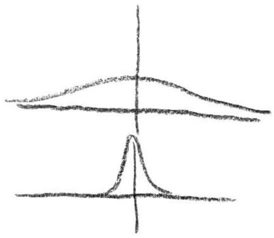
gaussova porazdelitev

$w(x)$ = gostota verjetnosti

$$\int_{-\infty}^{\infty} w(x)\,dx = 1$$

veliki šum

majhen šum



ploščini ste enaki : 1

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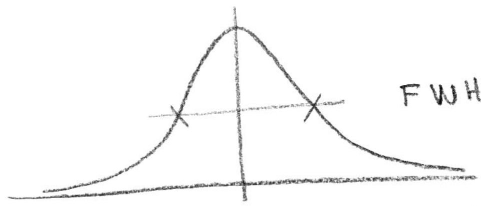
parameter, ki določa raztresenost $= \sigma$: kako ga določiti?

- iz diagrama porazdelitve ocenimo



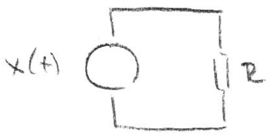
FWHM = širina zvona na ½ višine

$$\text{FWHM} = 2.35 \cdot \sigma$$

- iz zajetih vzorcev, predelanih v diagram porazdelitve, izračunamo

$$\sigma^2 = \int_{-\infty}^{\infty} (x - \langle x \rangle)^2\, w(x)\,dx$$

iz fizike vrednotimo šum



trenutna moč : $P(t) = \dfrac{x^2(t)}{R}$

povp. moč : = $\langle P(t)\rangle = \dfrac{\langle x^2(t)\rangle}{R}$; $R=1$ za referenco $\langle P(t)\rangle = \langle x^2(t)\rangle$

a) po matematiki :

$$\langle x^2(t)\rangle = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x^2(t)\,dt = \underbrace{\int_{-\infty}^{\infty} w(x)\,x^2\,dx}_{\text{statistika}}$$ (drugi moment)

, poljubna porazdelitev

vemo tudi: za Gaussa!

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\langle x\rangle)^2\,w(x)\,dx \; ; \quad \text{za nas } \langle x\rangle = 0$$

↓

$$\sigma^2 = \int_{-\infty}^{\infty} x^2\,w(x)\,dx \Rightarrow \sigma^2 = \langle x^2(t)\rangle$$

σ = merilo za moč

$$\sigma = \sqrt{\langle x^2(t)\rangle} = \sqrt{\langle P\rangle}\Big|_{R=1}$$

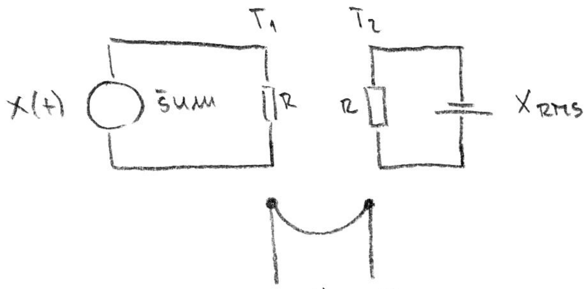
b) po fiziki iščemo ekvivalentno napetost, ki enako ogreje upornik

$$\langle P(t)\rangle = \underbrace{\frac{\langle x^2(t)\rangle}{R} \equiv \frac{X_{EF}^2}{R}}$$

$$X_{EF} = X_{RMS} = \sqrt{\langle x^2(t)\rangle}$$

Root Mean Square

Merjenje RMS vrednosti : primerjava



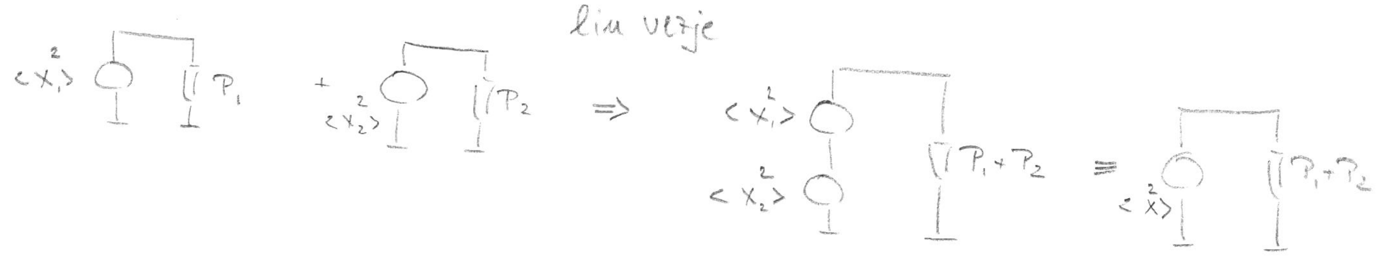
$\Delta U = k\,\Delta T = 0$

↓

$$T_1 = T_2 \Rightarrow P_1 = P_2 \Rightarrow P_š = \frac{\langle x^2(t)\rangle}{R} = P_{DC} = \frac{X_{RMS}^2}{R}$$

seštevanje virov

lin vezje



$$P_1 + P_2 = \frac{\langle x_1^2\rangle}{R} + \frac{\langle x_2^2\rangle}{R} = \frac{\langle x^2\rangle}{R} \Rightarrow \boxed{\langle x^2\rangle = \langle x_1^2\rangle + \langle x_2^2\rangle}$$

---

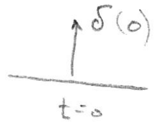
kaj imamo povedati o šumih po frekvenci?

- poznamo $x(t)$
- povezava prostora t in f: Fourier, razvoj v vrsto za period. sig.
  integral za neper. sig.

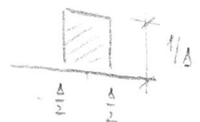
$$F(x(t)) = F(i\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-i\omega t}\, dt$$

$$x(t) = F^{-1}(F(i\omega)) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(i\omega)\, e^{i\omega t}\, d\omega$$

zgled:



$$F(i\omega) = \int_{-\infty}^{\infty} \delta(0)\, e^{-i\omega t}\, dt =$$



$$= \lim_{\Delta\to 0} \frac{1}{\Delta}\int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} e^{-i\omega t}\, dt = \lim_{\Delta\to 0} \frac{1}{\Delta}\frac{1}{-i\omega} e^{-i\omega t}\Big|_{-\Delta/2}^{\Delta/2} =$$

$$= \lim_{\Delta\to 0} \frac{1}{\Delta}\frac{i}{\omega}\left[e^{-i\omega\frac{\Delta}{2}} - e^{i\omega\frac{\Delta}{2}}\right] = \ldots \left[\cancel{\cos\omega\frac{\Delta}{2}} - i\sin\omega\frac{\Delta}{2} - \cancel{\cos\omega\frac{\Delta}{2}} - i\sin\omega\frac{\Delta}{2}\right]$$

$$= \lim_{\Delta\to 0} \frac{-i^2\cdot 2}{\Delta\omega}\sin\omega\frac{\Delta}{2} = \lim_{\Delta\to 0}\frac{\sin\frac{\omega\Delta}{2}}{\frac{\omega\Delta}{2}} = 1$$

poglejmo integral

$$\underbrace{\int_{-\infty}^{\infty} x^2(t)\,dt}_{\text{energija}\cdot R \text{ za čas } T\to\pm\infty} = \int_{-\infty}^{\infty} x(t)\cdot x(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(t)\cdot\left[\int_{-\infty}^{\infty} F(i\omega)e^{i\omega t}\,d\omega\right]dt =$$

zamenjamo vrstni red int.

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F(i\omega)\cdot\left[\int_{-\infty}^{\infty} x(t)\,e^{i\omega t}\,dt\right]d\omega =$$

nova spr. $\Omega = -\omega$, $\omega = -\Omega$, $d\omega = -d\Omega$

$$= \frac{-1}{2\pi}\int_{\infty}^{-\infty} F(-i\Omega)\underbrace{\left[\int_{-\infty}^{\infty} x(t)\,e^{-i\Omega t}\,dt\right]}_{F(i\Omega)}d\Omega =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F(-i\Omega)\cdot F(i\Omega)\,d\Omega = \frac{-1}{2\pi}\int_{\infty}^{-\infty} F(i\omega)\cdot F(-i\omega)\,d\omega =$$

$$= \boxed{\frac{1}{2\pi}\int_{-\infty}^{\infty}|F(i\omega)|^2\,d\omega = \int_{-\infty}^{\infty} x^2(t)\,dt}$$

parsevalov teorem

energija $\equiv$ energija

ni faze $\rightarrow$ ni pomembna za energijo

---

težava: energija $\xrightarrow[t\to\infty]{}$ ∞ !, zato raje računamo pov. moč

$$\frac{x_{ef}^2}{R} = \langle P\rangle = \frac{1}{R}\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x^2(t)\,dt = \frac{1}{R}\lim_{T\to\infty}\frac{1}{2T}\frac{1}{2\pi}\int_{-\infty}^{\infty}|F(i\omega)|^2\,d\omega = \quad ; \ \omega = 2\pi\nu$$

$$\frac{\langle x^2\rangle}{R}$$

$$= \frac{1}{R}\lim_{T\to\infty}\frac{1}{2T}\frac{1}{2\pi}\int_{-\infty}^{\infty}|F(i\omega)|^2\cdot 2\pi\,d\nu =$$

$$= \frac{1}{R}\int_{-\infty}^{\infty}\lim_{T\to\infty}\frac{1}{2T}|F(i\omega)|^2\,d\nu =$$

$$= \frac{1}{R}\int_{0}^{\infty}\underbrace{\lim_{T\to\infty}\frac{1}{T}|F(i\omega)|^2}_{S(\omega)}\,d\nu = \frac{1}{R}\int_{0}^{\infty} S(\omega)\,d\nu\,.$$

spektralna gostota šuma

velja tudi

$$\langle x^2(t)\rangle = x_{ef}^2 = \sigma^2 = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x^2(t)\,dt = \int_0^\infty S(\omega)\,d\nu \quad , \text{ če } S(\omega) = \text{konst}$$

$$\Downarrow$$

$$x_{ef}^2 = S(\omega)\cdot \Delta f$$

---

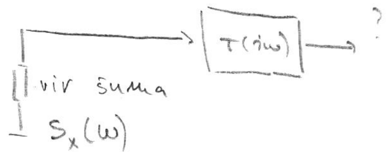
za upornik velja. $S(\omega) = 4kTR$; $k = 1{,}38\cdot 10^{-23}$, $T = 293K$

zgled : $R = 1k\Omega$ $\Delta f = 20kHz$

$$\langle x^2(t)\rangle = x_{ef}^2 = \int_0^{f_M} 4kTR\,d\nu = 4kTR\,\Delta\nu$$

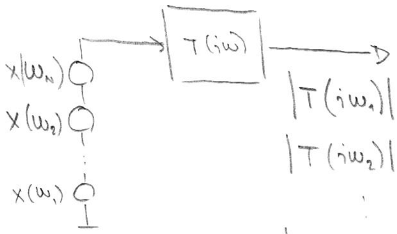
$$x_{ef} = \sqrt{4kTR\,\Delta f} = \sqrt{4\cdot 1{,}38\cdot 10^{-23}\cdot 293\cdot 10^3\cdot 20\cdot 10^3} = 0{,}57\mu V$$

pozor : <u>10 x R šumi $\sqrt{10}$ x bolj !</u>



kako pa tule?

vir šume : spekter, več komponent! → vsaka od njih se razširja skozi vezje kot $x(\omega)\cdot|T(i\omega)|$



$|T(i\omega_1)|\cdot x(\omega_1)$
$|T(i\omega_2)|\cdot x(\omega_2)$
⋮
$|T(i\omega_N)|\cdot x(\omega_N)$

$S_x(\omega)$

$$\boxed{S_y(\omega) = S_x(\omega)\cdot|T(i\omega)|^2}$$

$S_x(\omega)$

$\Downarrow$

$x_{ef}^2 \neq x^2(\omega_1) + x^2(\omega_2) + \cdots x^2(\omega_N)$

nekorelirano!

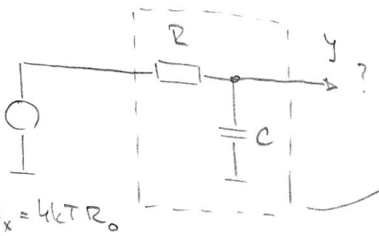
$S_y(\omega)$

$\Downarrow$

$x^2(\omega_1)\cdot|T(i\omega_1)|^2 + \cdots + x^2(\omega_N)\cdot|T(i\omega_N)|^2$

$x_{ef}'^2 \Rightarrow x_{ef}^2\cdot|T(i\omega)|^2$ ← goljufija, ne upoštevamo $x_{ef}(i\omega)$!

pogled za filter



$S_x = 4kTR_0$

$$T(i\omega) = \frac{1}{1+i\omega RC}$$

$$S_y = S_x \cdot |T(i\omega)|^2 = S_x \cdot T(i\omega)\cdot T(-i\omega) = S_x \frac{1}{1+i\omega RC}\cdot\frac{1}{1-i\omega RC} = S_x \frac{1}{1+\omega^2R^2C^2}$$

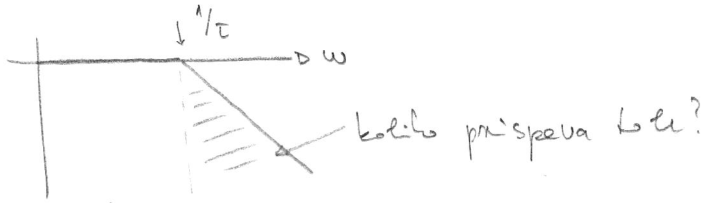
$$x_{ef}^2 = \int_0^\infty S_x \, d\nu = 4kTR_0 \, \Delta f$$

?, saj gre proti ∞!

$$y_{ef}^2 = \int_0^\infty S_y \, d\nu = \int_0^\infty S_x \frac{1}{1+\omega^2R^2C^2} \, d\nu = \frac{S_x}{2\pi RC}\int_0^\infty \frac{d(2\pi\nu RC)}{1+(2\pi\nu)^2R^2C^2} = \frac{S_x}{2\pi RC}\int_0^\infty \frac{dx}{1+x^2}$$

$$= \frac{S_x}{2\pi RC}\,\mathrm{arc\,tg}\,\omega\tau\Big|_0^\infty = \frac{S_x}{2\pi\tau}\left[\frac{\pi}{2}-0\right] = \frac{S_x}{4\tau} = \frac{4kTR_0}{4\tau}$$

zanimivost:



dobimo tole

$$z_{ef}^2 = S_x \cdot \Delta f = 4kTR_0 \frac{1}{2\pi\tau} = \frac{kTR_0}{\frac{\pi}{2}\tau}$$

poglejmo $\frac{z_{ef}}{y_{ef}} = \sqrt{\frac{\cancel{kTR_0}\cdot 2\cdot\cancel{\tau}}{\pi\cancel{\tau}\cdot\cancel{4kTR_0}}} = \sqrt{\frac{2}{\pi}} \doteq 0.8$

$\frac{y_{ef}}{z_{ef}} = \sqrt{\frac{\pi}{2}} = 1.25$ $\Rightarrow$ šrafirani del doda 25%!

vrste šumov – splošno

a) termični šum → posledica temperature, sorazmeren s T

frekvenčni obseg je neomejen → do nekaj 10 GHz

energija za gibanje nosilcev prihaja iz okolice: $T \to 0 \Rightarrow$ ni šuma

$S(\omega) = \text{konst}!$ ⇒ beli šum $\qquad S(\omega) = 4kTR$

b) shot noise = šum zrnatosti: posledica toka

naboj prihaja v kvantih: e_0

$S(\omega) = \text{konst} \Rightarrow$ beli šum

izkustvena formula (izpeljana): $I_{RMS}^2 = 2 e_0 I_{DC} \Delta\nu \Rightarrow$

57 µA pri 1 A, 10 kHz :

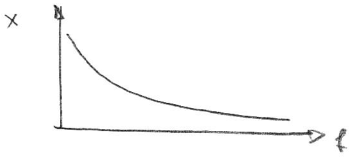
56 pA pri 1 µA, 10 kHz :

$$\frac{\text{signal}}{\text{noise}} = \frac{I_{DC}}{\sqrt{2 e_0 I_{DC} \Delta\nu}} = \sqrt{\frac{I_{DC}}{2 e_0 \Delta\nu}} \Rightarrow \begin{array}{ll} 1\,\text{A}: & 140\,\text{dB} \\ 1\,\mu\text{A}: & 82\,\text{dB} \end{array}$$

izkaže se, da je v tovrstnih težavah šuma še manj

c) nepopolna izdelava elementov povzroče dodaten šum

rožnati šum, 1/f šum, flicker noise



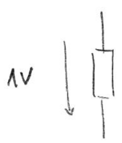
razlog so nepopolna stičišča med materiali.

zgled: upori: žični 0.01 µV do 0.2 µV

metal-film 0.02 µV do 0.2 µV

ogljeno-plastni 0.05 µV do 0.3 µV

ogljeno-mešani 0.1 µV do 3 µV



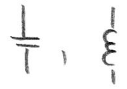
$\Delta\nu = 1$ dekada

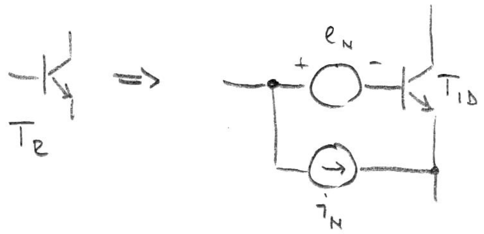
d) res slaba izdelava: naključno izpuščanje in vzpostavljanje stikov

popcorn noise

popisovanje šumov

$$X_{R_R} = \sqrt{4kT R_R \Delta\nu}$$

 ⇒ ne šumi po definiciji : energija = ϕ !

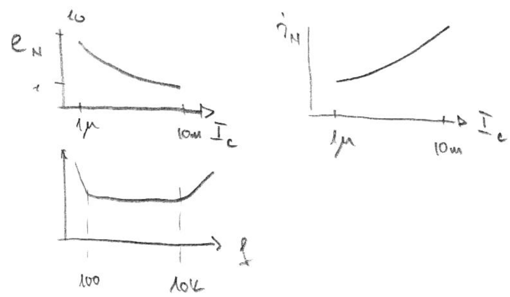


podaja se kot : $e_N(\nu) = \sqrt{S_x(\omega)}$ $[nV/\sqrt{Hz}]$

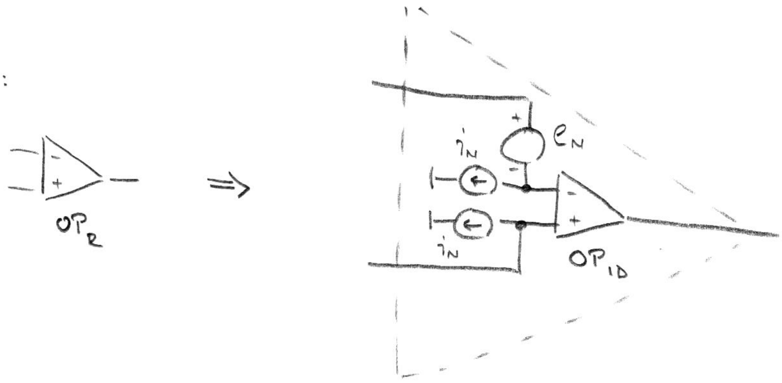
$i_N(\nu) = \sqrt{S_I(\omega)}$ $[pA/\sqrt{Hz}]$

	e_N	i_N
bipolarni	nekaj $nV/\sqrt{Hz}$	$0.2\, pA/\sqrt{Hz}$
FET, MOSFET	$\sim 5\, nV/\sqrt{Hz}$	$0.01\, pA/\sqrt{Hz}$

$$\left.\begin{aligned} X_N^2 &= \int_\nu S(\omega)\, d\nu \\ &= \int_\nu e_N^2(\nu)\, d\nu \end{aligned}\right\} X_N = e_N \sqrt{\Delta\nu}$$

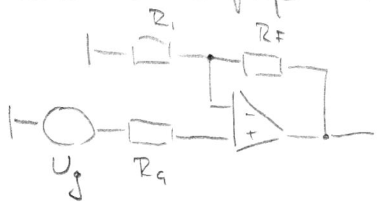


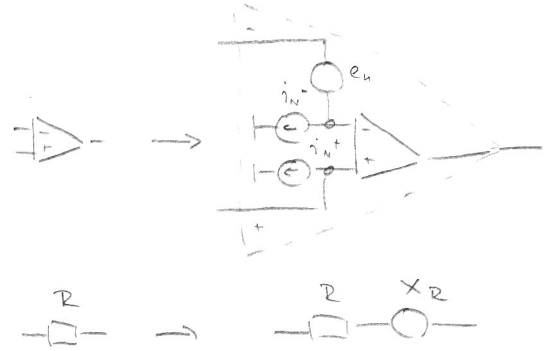
OP:

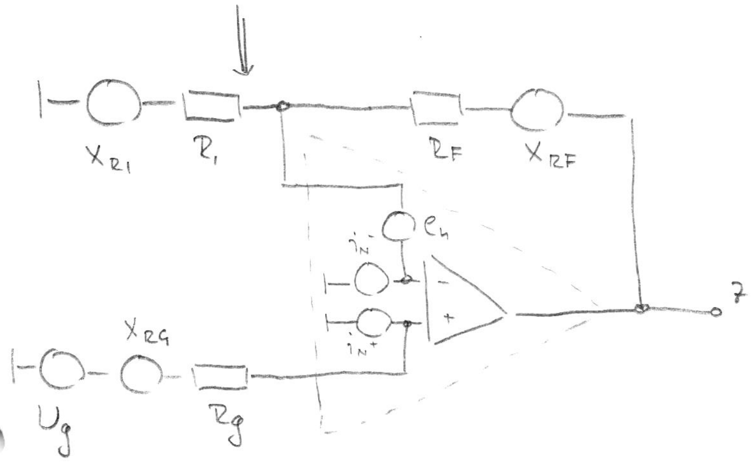


	$e_N\ [nV/\sqrt{Hz}]$	$i_N\ [pA/\sqrt{Hz}]$
LF 356	12	0.01
OP 27	3	0.6
CLC 425	1.05	0.8
LMC 6044	83	0.002

šumi v vezju z OP



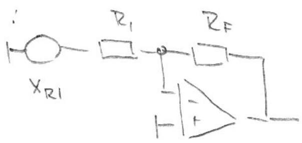




$X_Z \equiv$ geom. vsota vseh prispevkov posameznega vira!

poenostavitev:
- vsi šumi so beli
- vsi se enako razširjajo, ker zanje velja enaka $T(j\omega)$
- vezje je linearno

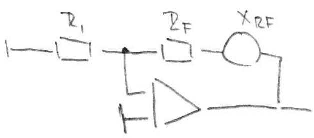
(R_1):



$$Z_{R1} = -X_{R1}\frac{R_F}{R_1} \quad ; \quad X_{R1} = \sqrt{4kTR_1}\cdot\sqrt{\Delta\nu}$$

torej: $$Z_{R1} = \frac{R_F}{R_1}\sqrt{4kTR_1}\cdot\sqrt{\Delta\nu} = \frac{R_F}{R_1}\sqrt{R_1}\sqrt{4kT\Delta\nu} = \frac{R_F}{\sqrt{R_1}}\sqrt{4kT\Delta\nu} = \frac{R_F}{R_1}\sqrt{R_1}\sqrt{4kT\Delta\nu}$$

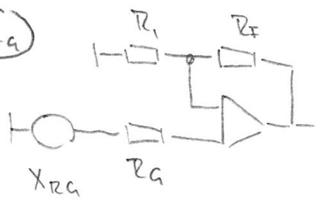
(R_F)



$$Z_{RF} = X_{RF} \quad ; \quad X_{RF} = \sqrt{4kTR_F}\cdot\sqrt{\Delta\nu}$$

torej: $$Z_{RF} = \sqrt{4kTR_F}\cdot\sqrt{\Delta\nu} = \sqrt{R_F}\sqrt{4kT\Delta\nu}$$

(R_G)



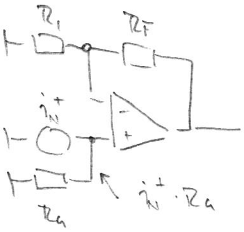
$$Z_{RG} = X_{RG}\left(1+\frac{R_F}{R_1}\right) \quad ; \quad X_{RG} = \sqrt{4kTR_G}\,\sqrt{\Delta\nu}$$

torej: $$Z_{RG} = \left(1+\frac{R_F}{R_1}\right)\sqrt{4kTR_G}\,\sqrt{\Delta\nu} = \left(1+\frac{R_F}{R_1}\right)\sqrt{R_G}\sqrt{4kT\Delta\nu}$$

a) ojačevalnik: torej $R_1 \ll R_F \Rightarrow Z_{R1} > Z_{RF}$: $\sqrt{10}$ pri $A=10$!
pri $|A| > 10$: pomembni R_F!

b) R_F & R_1 izbireš sam, važno je razmerje $\Rightarrow$ majhna R_F & R_1 $\Rightarrow$ pomembni prispevki R_G!

i_N^+



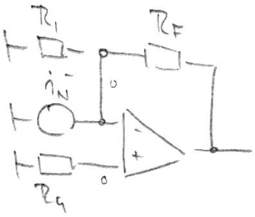
$$z_{iN+} = \left(1+\frac{R_F}{R_1}\right) i_N^+ R_g\sqrt{\Delta\nu} \doteq \frac{R_F}{R_1}\, i_N^+ R_g\sqrt{\Delta\nu}$$

torej: $z_{IN+} = \frac{R_F}{R_1}\, i_N^+ R_g\sqrt{\Delta\nu}$

$i_N^+ = i_N^-$

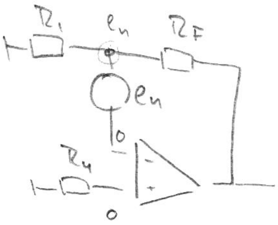
$z_{IN-} \ll z_{IN+}$

i_N^-



$$z_{IN-} = i_N^-\, R_F\cdot\sqrt{\Delta\nu}$$

e_n



$x_{en} = e_N\sqrt{\nu}$ $\qquad \frac{x_{en}}{R_1} + \frac{x_{en}-z_{en}}{R_F} = 0$

$$\frac{e_n\sqrt{\nu}}{R_1} + \frac{\sqrt{\nu}e_n - z_{en}}{R_F} = 0 \Rightarrow \sqrt{\nu}\left(e_nR_F + e_nR_1\right) = z_{en}R_1$$

$$z_{en} = \sqrt{\nu}e_n\left(1+\frac{R_F}{R_1}\right) \doteq z_{en} = e_n\frac{R_F}{R_1}\sqrt{\nu}$$

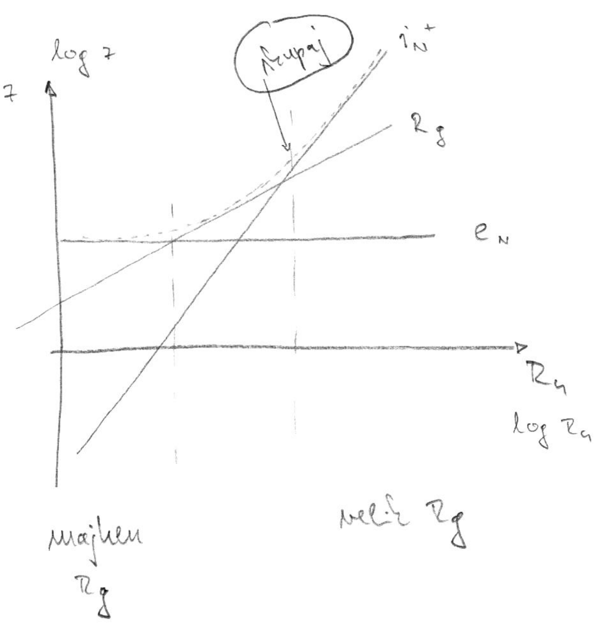
---

ostanejo trije prispevki

- R_g: $\longrightarrow$ $z_{Rg} = \frac{R_F}{R_1}\sqrt{R_g}\sqrt{4kT\Delta\nu}$ ($R_g^{1/2}$)
- i_N^+ $\longrightarrow$ $z_{iN+} = \frac{R_F}{R_1} R_g\sqrt{\Delta\nu}\, i_N^+$
- e_N: $\longrightarrow$ $z_{eN} = \frac{R_F}{R_1} e_N\sqrt{\Delta\nu}$

- ojačanje je za vse prispevke enako!
- $\Delta\nu$ je za vse prispevke enako
- seštej geometrijsko

naš diagram :



; $\sqrt{\Delta\nu}$ = konst

$\frac{R_F}{R_1}$ = — " —

boj proti šumom

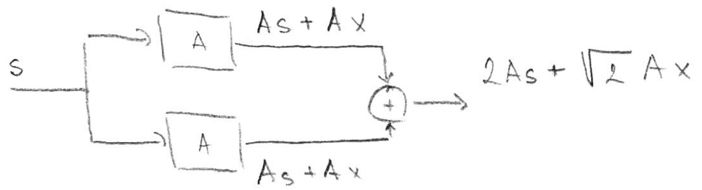
① termični : hladi

② primerna izbira elementov, ki malo šumijo

– " – frekvenčnega pasu

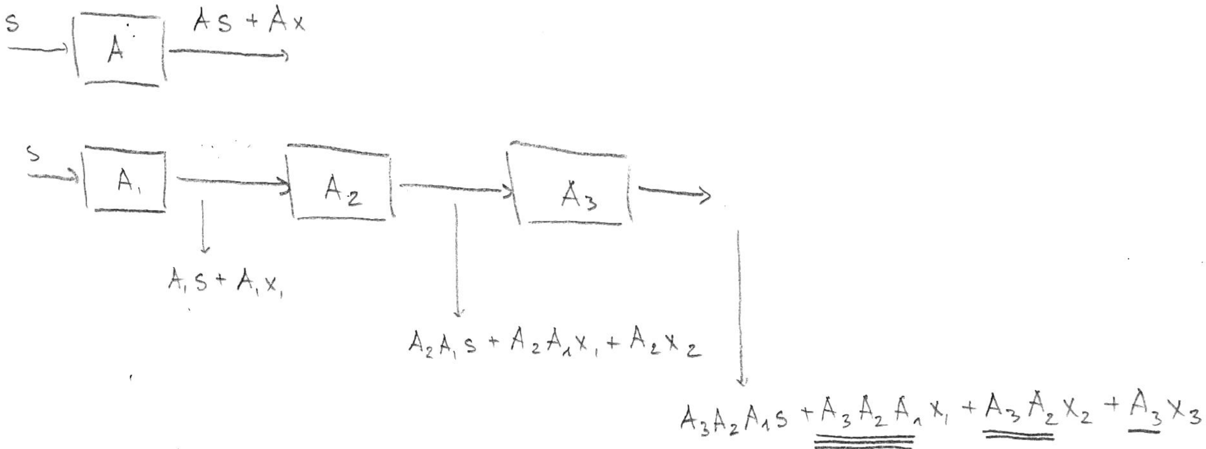
B : filtriranje : ojačuj le tisti del spektra, ki nosi koristno informacijo

D : vzporedne vezave



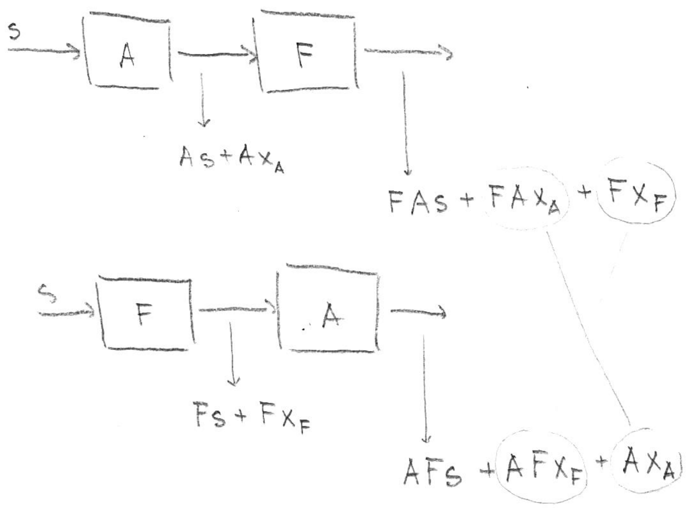
šum se sešteva geometrijsko!

A : optimiraj vh. stopnjo



najbolj upliva šum vhodne stopnje!

C filtriranje



⇐ slabše!

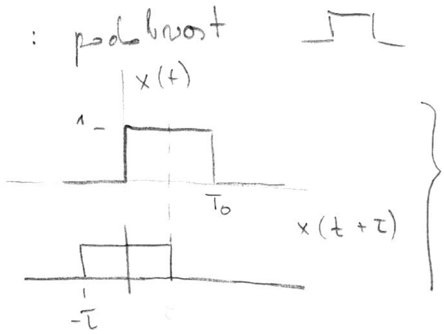
E povprečenje : koherentno : znano vzbujanje, povpreči odziv

nekoherentno : povpreči amplitudo spektra odziva

Avtokorelacija $\equiv$ stopnja podobnosti od premika

def: $$R(\tau) = \langle x(t)\cdot x(t+\tau)\rangle = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)\cdot x(t+\tau)\,dt$$

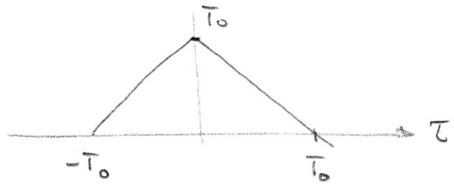
zgled: podobnost



$$\tau > 0 \Rightarrow R(\tau) = \int_{0}^{T_0-\tau} dt = T_0 - \tau$$

$$\tau < 0 \Rightarrow R(\tau) = \int_{-\tau}^{T_0} dt = T_0 + \tau$$

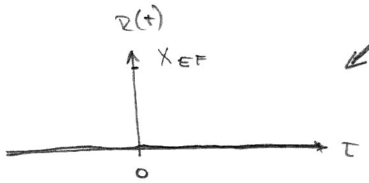
najbolj podobni za $\tau = 0$ $\Leftarrow$



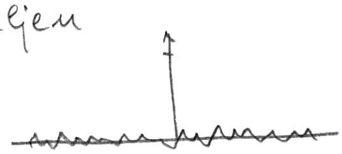
kaj pa za šum?

$R(\tau) = \langle x(t)\cdot x(t+\tau)\rangle$: za $\tau \neq 0$ $R(\tau) = 0$

za $\tau = 0$ $R(\tau) = \langle x^2(t)\rangle = X_{EF}^2$



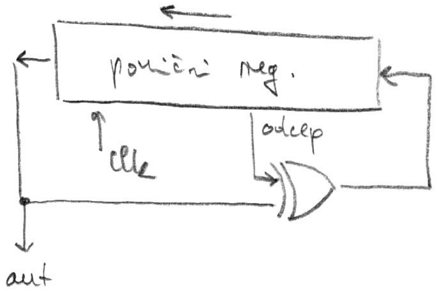
če povprečimo preko manjšega intervala, je del za $\tau \neq 0$ zašumljen



zgled: digitalni šum: 0, 1 ⇒ naključne spremembe

meja: $P(1) = P(0) = 1/2$

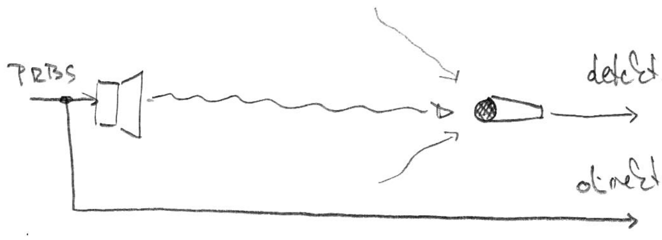
preskoki ob disk. čas. int.



PRBS: pseudo-random bit-sequence

ključ je v pravem odcepu

numerical methods!

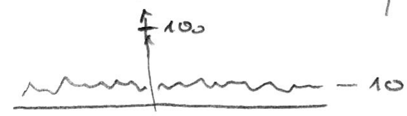


← kasni zaradi poti

načelno: npr 100 clk: ni motenj ⇒ samo kesnitev

$R(\tau = T_k) = 100$ ujemanj

$R(\tau \neq T_k) = \sqrt{100} = \pm 10$ ujemanj



motnje: izgubi se 1/2 signala

???

$$R(\tau = T_k) = 50 \pm \sqrt{50} = 50 \pm 7$$

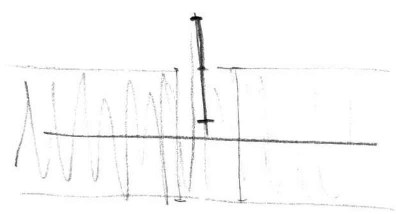
$$R(\tau \neq T_k) = \sqrt{100} = \pm 10$$

odziv ne $\frac{1}{2}N$ ampak $\sqrt{N}$

izgubi se 90% signala

$$R(\tau = T_k) = 10 \pm \sqrt{90} = 10 \pm 9.5$$

$$R(\tau \neq T_k) = \sqrt{100} = \pm 10$$



rešitev: podaljšaj meritev 1000 vzorcev

$$\left.\begin{array}{l} R(\tau = T_k) = 100 \pm \sqrt{900} = 100 \pm 30 \\ R(\tau \neq T_k) = \sqrt{1000} = \pm 33 \end{array}\right\} \text{spet gre}$$

N : št. vzorcev

m : delež dobrih

$$\tau = 0 \qquad m \cdot N + \frac{1-m}{2} N \pm \sqrt{\frac{1-m}{2} N} = \frac{N}{2}\left[2m + 1 - m\right] \pm \sqrt{\frac{N}{2}}\sqrt{1-m}$$

$$\tau \neq 0 \qquad \frac{N}{2} \pm \sqrt{\frac{N}{2}} = \frac{N}{2}\left[m+1\right] \pm \sqrt{\frac{N}{2}}\sqrt{1-m}$$

$N = 10000$, $m = 1$:

$\tau = 0 \Rightarrow$ 10 000 vjemanj

$\tau \neq 0 \Rightarrow 5000 \pm \sqrt{5000} = 5000 \pm 70$

$m = 1/2$:

$\tau = 0 \Rightarrow 5000 + 2500 \pm \sqrt{2500} = 7500 \pm 50$

$\tau \neq 0 \Rightarrow 5000 \pm \sqrt{5000} = 5000 \pm 70$

$m = 1/10$:

$\tau = 0 \Rightarrow 1000 + 4500 \pm \sqrt{4500} = 5500 \pm 67$ ← + 10 % !

$\tau \neq 0 \Rightarrow 5000 \pm \sqrt{5000} = 5000 \pm 70$

$m = 1/100$:

$\tau = 0 \Rightarrow 100 + 4950 \pm \sqrt{4950} = 5050 \pm 70$

$\tau \neq 0 \Rightarrow 5000 \pm \sqrt{5000} = 5000 \pm 70$

$N = 1\,000\,000$, $m = 1/100$:

$\tau = 0 \Rightarrow 10\,000 + 495\,000 \pm \sqrt{495\,000} = 505\,000 \pm 703$!

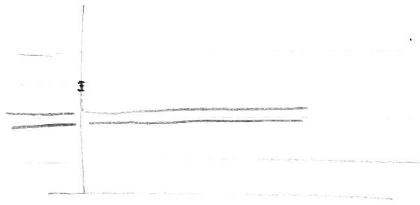
$\tau \neq 0 \Rightarrow 500\,000 \pm \sqrt{500\,000} = 500\,000 \pm 707$

opet se vidi

$$\left.\begin{matrix} N = 100 \\ M = 1 \end{matrix}\right\} \quad \begin{matrix} \tau = 0 & 100 & \\ \tau \neq 0 & 50 \pm \sqrt{50} & = 50 \pm 7 \end{matrix}$$

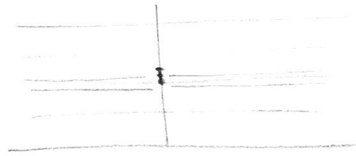
$$\left. M = \tfrac{1}{2} \right\} \quad \tau = 0 \qquad 50 + \frac{50}{2} \pm \sqrt{\frac{50}{2}} = 75 \pm 5$$

$$\tau \neq 0 \qquad 50 \pm 7$$



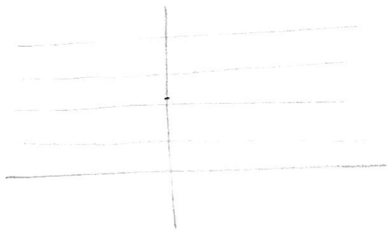
$$\left. M = \tfrac{1}{10} \right\} \quad \tau = 0 \qquad 10 + \frac{90}{2} \pm \sqrt{\frac{90}{2}} = 55 \pm 7.4$$

$$\tau \neq 0 \qquad 50 \pm 7$$



$$\left.\begin{matrix} N = 10000 \\ M = \tfrac{1}{10} \end{matrix}\right\} \quad \tau = 0 \qquad 1000 + \frac{9000}{2} \pm \sqrt{\frac{9000}{2}} = 5500 \pm 67$$

$$\tau \neq 0 \qquad 5000 \pm \sqrt{5000} = 5000 \pm 70$$



$$\left.\begin{matrix} N = 1000 \\ M = \tfrac{1}{10} \end{matrix}\right\} \quad \tau = 0 \qquad 100 + \frac{900}{2} \pm \sqrt{\frac{900}{2}} = 550 \pm 21$$

$$\tau \neq 0 \qquad 500 \pm \sqrt{500} = 500 \pm 22$$