Izračunajmo koeficiente filterskega jedra:

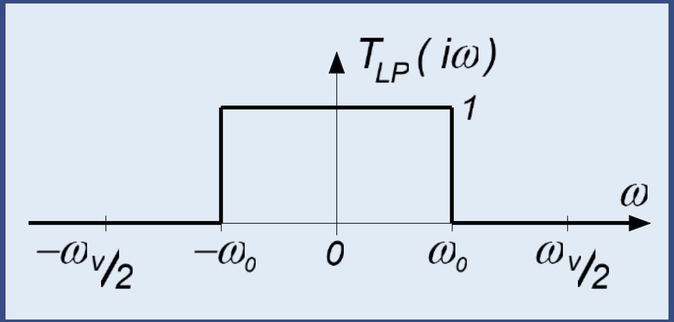
$$h(t) = F^{-1}\big(T_A(i\omega)\big) = \frac{1}{2\pi}\int_{-\infty}^{\infty} T_A(i\omega)e^{i\omega t}\,d\omega$$

Ker je za diskretne funkcije: $t = m \cdot T_v$ in $T_D(i\omega) = \frac{1}{T_v} T_A(i\omega)$,

lahko za diskretno vezijo zapišemo:

$$h(mT_v) = h_m = \frac{1}{2\pi}\int_{-\omega_v/2}^{\omega_v/2} T_v \cdot T_D(i\omega)e^{i\omega m T_v}\,d\omega \quad = \quad \frac{T_v}{2\pi}\int_{-\omega_v/2}^{\omega_v/2} T_D(i\omega)e^{i\omega m T_v}\,d\omega$$

Izračunajmo koeficiente filterskega jedra za nizkoprepustni filter:



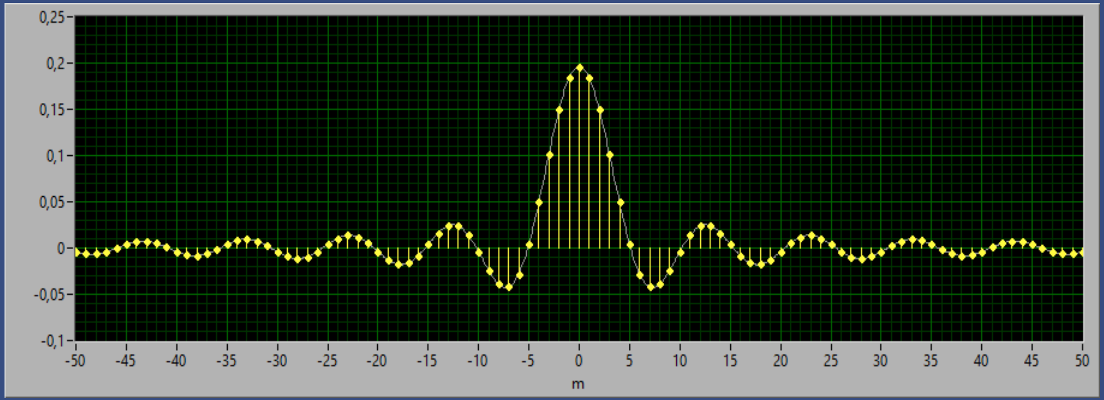
$$T_{LP}(i\omega) = \begin{cases} 1 & za \quad -\omega_0 \leq \omega \leq \omega_0 \\ 0 & drugje \end{cases}$$

$$h(mT_v) = \frac{T_v}{2\pi} \int_{-\omega_v/2}^{\omega_v/2} T_D(i\omega) e^{i\omega m T_v}\, d\omega$$

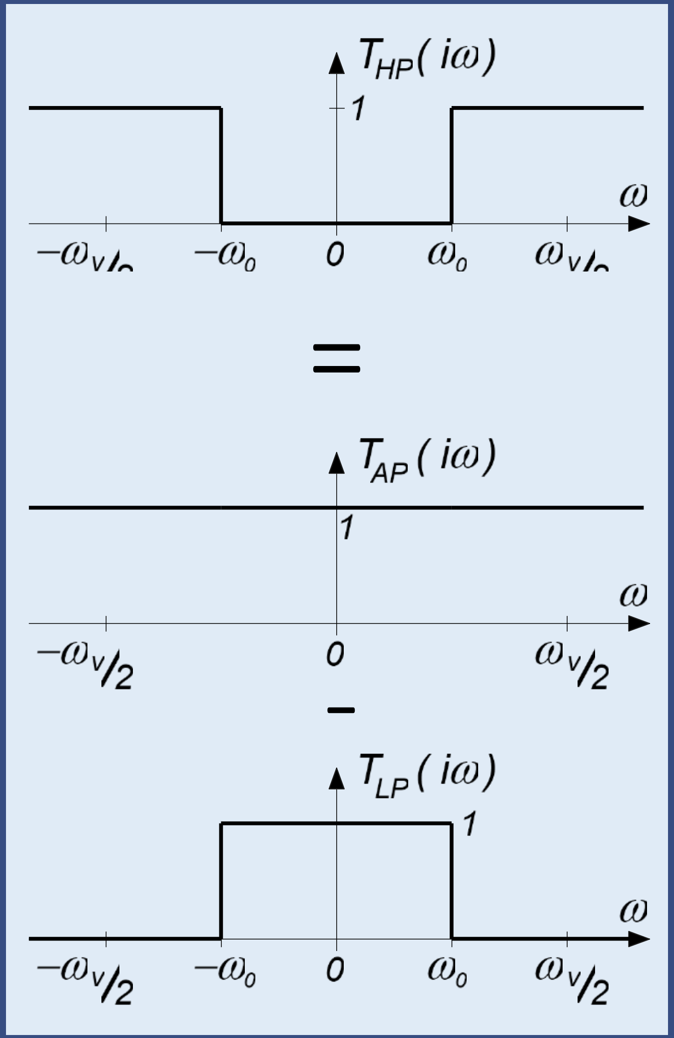
$$h_{LP}(mT_v) = h_{mLP} = \frac{T_v}{2\pi} \int_{-\omega_0}^{\omega_0} e^{i\omega m T_v}\, d\omega = \frac{T_v}{2\pi \cdot imT_v} \cdot e^{i\omega m T_v} \Big|_{-\omega_0}^{\omega_0}$$

$$h_{mLP} = \frac{1}{2\pi \cdot im} \cdot \left[\cos 2\pi m \frac{f_0}{f_v} + i \sin 2\pi m \frac{f_0}{f_v} - \cos 2\pi m \frac{f_0}{f_v} + i \sin 2\pi m \frac{f_0}{f_v} \right] \Rightarrow$$

$$h_{mLP} = 2 \frac{f_0}{f_v} \cdot \frac{\sin 2\pi m \frac{f_0}{f_v}}{\pi m \cdot 2 \frac{f_0}{f_v}}$$



Izračunajmo koeficiente filterskega jedra za visokoprepustni filter:

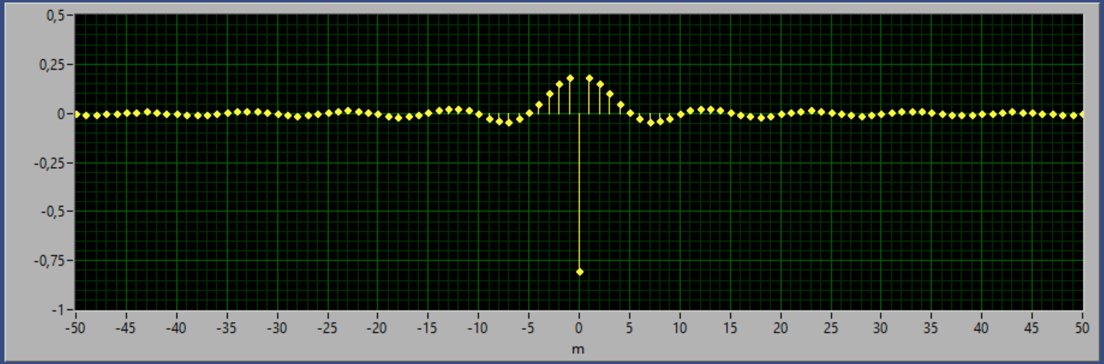


$$T_{HP}(i\omega) = \begin{cases} 0 & za \quad -\omega_0 \leq \omega \leq \omega_0 \\ 1 & \quad drugje \end{cases}$$

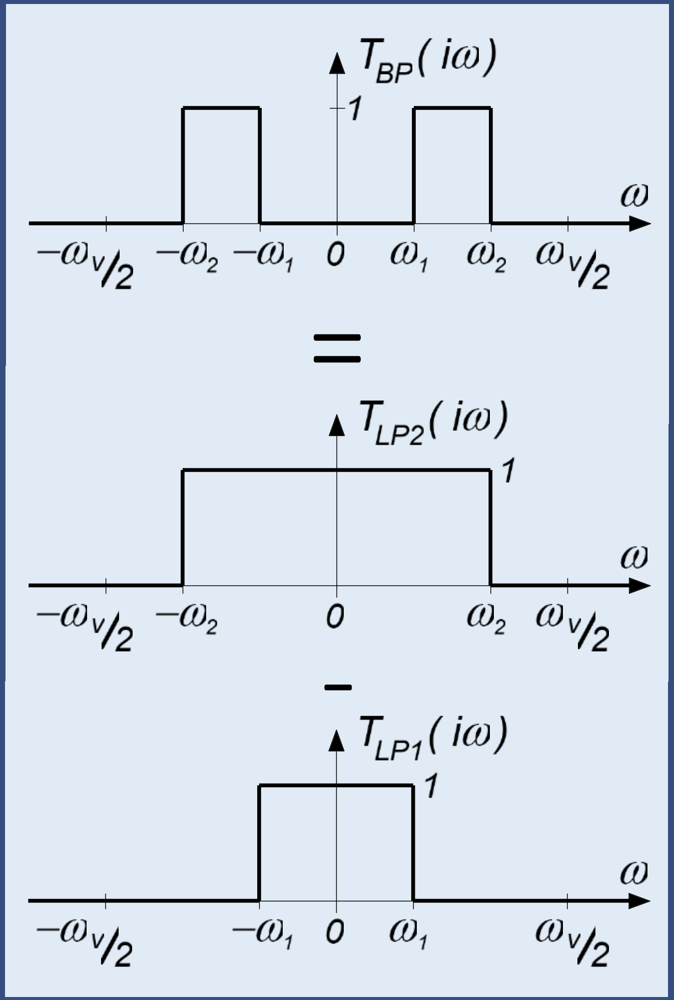
$$h_{mAP} = \begin{cases} 1 & za \quad m = 0 \\ 0 & \quad drugje \end{cases}$$

$$h_{mLP} = 2\frac{f_0}{f_v} \cdot \frac{\sin 2\pi m \frac{f_0}{f_v}}{\pi m \cdot 2\frac{f_0}{f_v}}$$

$$h_{mHP} = \begin{cases} 1 - 2\frac{f_0}{f_v} & za \quad m = 0 \\ -2\frac{f_0}{f_v} \cdot \frac{\sin 2\pi m \frac{f_0}{f_v}}{\pi m \cdot 2\frac{f_0}{f_v}} & za \quad m \neq 0 \end{cases}$$



Izračunajmo koeficiente filterskega jedra za pasovnoprepustni filter:

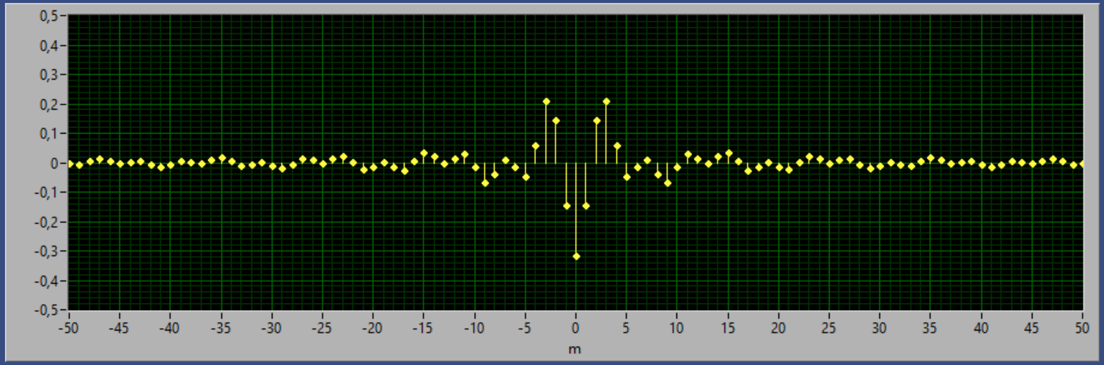


$$T_{BP}(i\omega) = \begin{cases} 1 & za \quad |\omega| \; med \; \omega_1 \; in \; \omega_2 \\ 0 & drugje \end{cases}$$

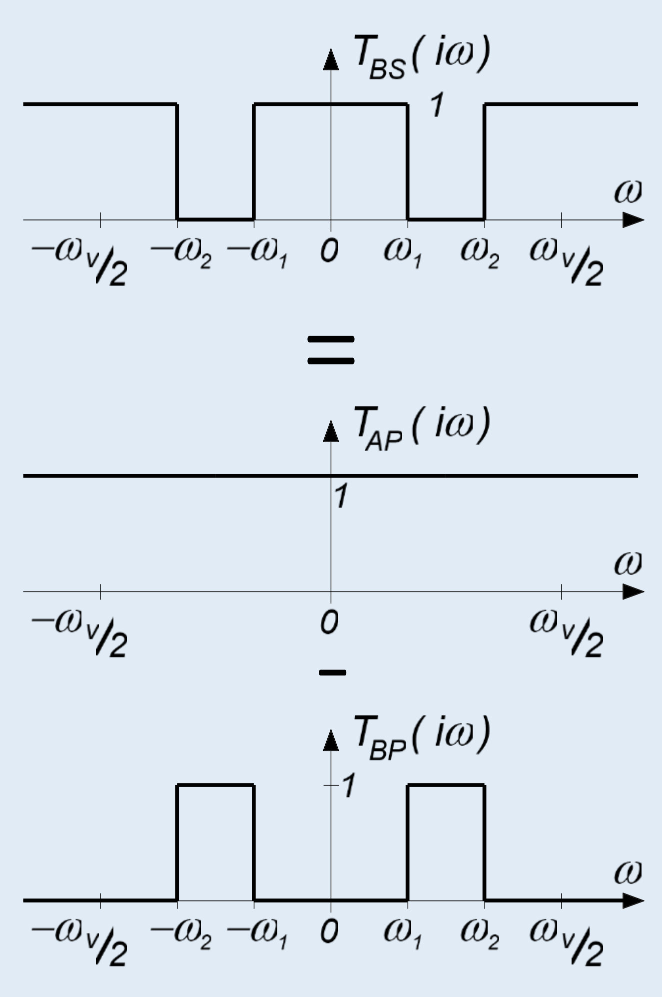
$$h_{mLP2} = 2\frac{f_2}{f_v} \cdot \frac{\sin 2\pi m \frac{f_2}{f_v}}{\pi m \cdot 2\frac{f_2}{f_v}}$$

$$h_{mLP1} = 2\frac{f_1}{f_v} \cdot \frac{\sin 2\pi m \frac{f_1}{f_v}}{\pi m \cdot 2\frac{f_1}{f_v}}$$

$$h_{mBP} = 2\frac{f_2}{f_v} \cdot \frac{\sin 2\pi m \frac{f_2}{f_v}}{\pi m \cdot 2\frac{f_2}{f_v}} - 2\frac{f_1}{f_v} \cdot \frac{\sin 2\pi m \frac{f_1}{f_v}}{\pi m \cdot 2\frac{f_1}{f_v}}$$



Izračunajmo koeficiente filterskega jedra za pasovnozaporni filter:

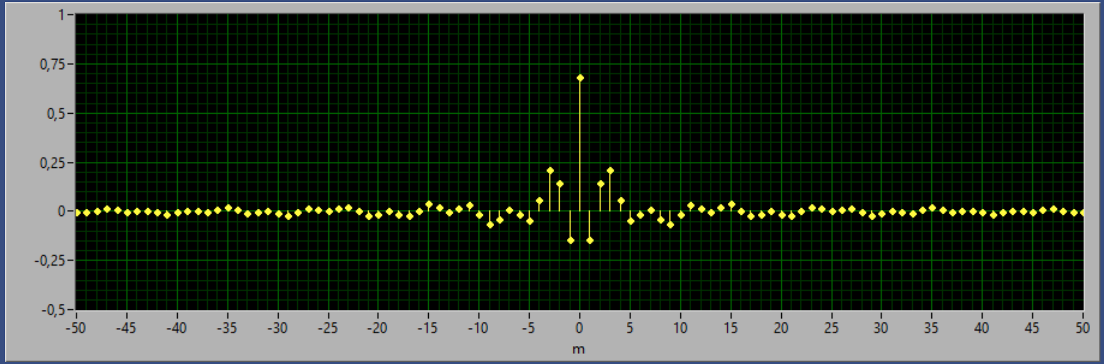


$$T_{BP}(i\omega) = \begin{cases} 0 & za \quad |\omega| \ med\ \omega_1 \ in \ \ \omega_2 \\ 1 & \qquad drugje \end{cases}$$

$$h_{mAP} = \begin{cases} 1 & za \quad m = 0 \\ 0 & \quad drugje \end{cases}$$

$$h_{mHP} = \begin{cases} 1 - 2\dfrac{f_2}{f_v} + 2\dfrac{f_1}{f_v} & za \quad m = 0 \\ 1 - 2\dfrac{f_2}{f_v} \cdot \dfrac{\sin 2\pi m \dfrac{f_2}{f_v}}{\pi m \cdot 2\dfrac{f_2}{f_v}} + 2\dfrac{f_1}{f_v} \cdot \dfrac{\sin 2\pi m \dfrac{f_1}{f_v}}{\pi m \cdot 2\dfrac{f_1}{f_v}} & za \ \ m \neq 0 \end{cases}$$

$$h_{mBP} = 2\frac{f_2}{f_v} \cdot \frac{\sin 2\pi m \frac{f_2}{f_v}}{\pi m \cdot 2\frac{f_2}{f_v}} - 2\frac{f_1}{f_v} \cdot \frac{\sin 2\pi m \frac{f_1}{f_v}}{\pi m \cdot 2\frac{f_1}{f_v}}$$



Hilbertov transform: zasukaj harmonski signal za 90 stopinj

Izračunali smo razvoj v Fourierovo vrsto za splošni harmonski signal:

$$x(t) = A\cos(n\omega t + \varphi) \quad ; \quad \omega = \frac{2\pi}{T},\ n\ je\ naravno\ število \quad \Rightarrow \quad c_{+n} = \frac{A}{2}\,e^{i\varphi} \quad c_{-n} = \frac{A}{2}e^{-i\varphi}$$

Zato:

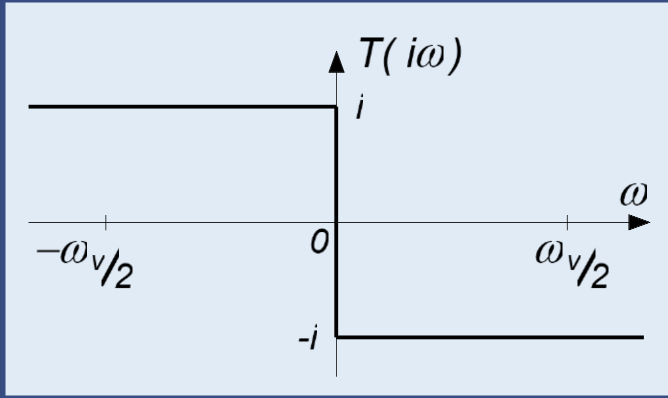
$$x(t) = A\cos n\omega t \quad \Longrightarrow \quad c_{m=+n} = {}^{A}/_{2} \quad , \quad c_{m=-n} = {}^{A}/_{2}$$

$$x(t) = A\sin n\omega t = A\cos\left(n\omega t - \frac{\pi}{2}\right) \quad \Longrightarrow \quad c_{m=+n} = -{}^{iA}/_{2} \quad , \quad c_{m=-n} = {}^{iA}/_{2}$$

Če torej želim kosinusni signal spremeniti v sinusnega, moram:

- koeficiente za $\omega > 0$ pomnožiti z $-i$,
- koeficiente za $\omega < 0$ pa z i.

Izračunajmo koeficiente filterskega jedra za Hilbertov transform:



$$T(i\omega) = \begin{cases} i & za \quad \omega < 0 \\ -i & za \quad \omega > 0 \end{cases} \qquad h(mT_v) = \frac{T_v}{2\pi} \int_{-\omega_v/2}^{\omega_v/2} T_D(i\omega) e^{i\omega m T_v}\, d\omega$$

$$h(mT_v) = h_m = \frac{T_v}{2\pi}\left[\int_{-\omega_v/2}^{0} i \cdot e^{i\omega m T_v}\, d\omega - \int_{0}^{\omega_v/2} i \cdot e^{i\omega m T_v}\, d\omega\right] = \frac{T_v}{2\pi \cdot i m T_v} \cdot \left[i \cdot e^{i\omega m T_v}\Big|_{-\omega_v/2}^{0} - i \cdot e^{i\omega m T_v}\Big|_{0}^{\omega_v/2}\right] =$$

$$h_m = \frac{1}{2\pi m} \cdot \{[\cos 0 + i \sin 0 - \cos \pi m + i \sin \pi m] - [\cos \pi m + i \sin \pi m - \cos 0 + i \sin 0]\} = \frac{2 - 2\cos \pi m}{2\pi m}$$

1 0 1 0

$$h_m = \frac{1 - \cos \pi m}{\pi m} \Longrightarrow h_m = \frac{1 - (-1)^m}{\pi m}$$