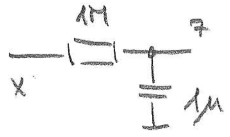
16a



$$T(p) = \frac{1}{1+\tau p} \Rightarrow T(i\omega) = \frac{1}{1+i\omega\tau} \quad ; \tau = RC = 1s$$

iščemo frekvenco, kjer je ojačenje vezja 1/10, torej

$$|T(i\omega)| = \frac{1}{|1+i\omega\tau|} = \frac{1}{10}$$

od tod:

$$10 = \sqrt{1^2 + \omega^2\tau^2}$$

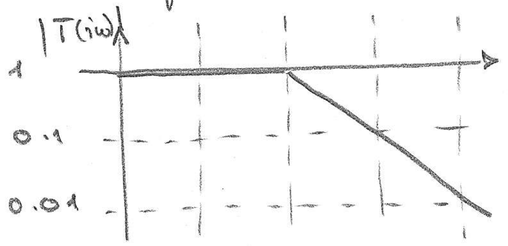
$$100 = 1^2 + (2\pi f)^2 \cdot 1^2$$

$$2\pi f = \sqrt{99} \doteq 10 \Rightarrow f = \frac{10}{2\pi} = \underline{\underline{1{,}59\,Hz}}$$

po bližnici: prelomna frekvenca RC člena

$$\omega_p = \frac{1}{RC} \Rightarrow f_p = \frac{1}{2\pi \cdot 10^6 \cdot 10^{-6}} = 0{,}159\,Hz$$

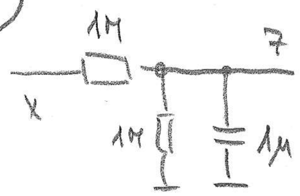
zaradi amplitudne karakteristike RC člena:



ojačenje pade na 1/10 pri frekvenci, ki je 10 kratnik f_p

$$f_x = \underline{\underline{1{,}59\,Hz}}$$

16b



nadomesti vhodna upornika po

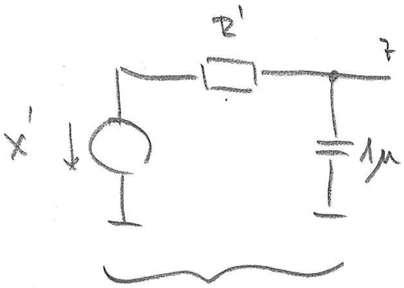
⇓

Thevenin

⇓

$$R' = 1M \parallel 1M = 500k$$

$$x' = x/2$$



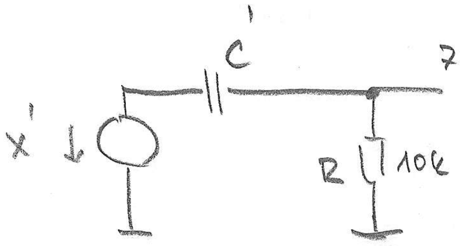
$$T(p) = \frac{z}{x} = \frac{z}{2x'} = 1/10$$

$$\frac{z}{x'} = \frac{2}{10}$$

iščemo frekvenco,

kjer ojačenje tega vezja pade na 2/10

po krajši poti : Thevenin : nadomesti oba kondenzatorja in vir signala



$$X' = \frac{\frac{1}{10n\cdot\cancel{p}}}{\frac{1}{10n\cdot\cancel{p}}+\frac{1}{C_x\cdot\cancel{p}}}X = X\frac{C_x}{C_x+10n}$$

$$C' = C_x \parallel 10n = C_x + 10n$$

$$T(p) = \frac{Z}{X} = \frac{Z\cdot C_x}{X'(C_x+10n)} = \frac{Z}{X'}\cdot\frac{C_x}{C_x+10n} = \frac{\tau' p}{1+\tau' p}\,\frac{C_x}{C_x+10n}$$

$$\downarrow$$

$$T(i\omega) = \frac{i\omega R C'}{1+i\omega R C'}\cdot\frac{C_x}{C_x+10n} = \frac{i\omega R\cancel{(C_x+10n)}}{1+i\omega R(C_x+10n)}\cdot\frac{C_x}{\cancel{C_x+10n}}$$

$$|T(i\omega)| = \frac{1}{\sqrt{2}} = \frac{\omega R C_x}{\sqrt{1+\omega^2R^2(C_x+10n)^2}}$$

$$1+\omega^2R^2\left[C_x^2+2\cdot C_x\cdot 10n+10n^2\right] = 2\omega^2R^2C_x^2$$

$$\underbrace{-\omega^2R^2}_{A}C_x^2+\underbrace{2\omega^2R^2}_{B}C_x\cdot 10n+\underbrace{\omega^2R^2\,10n^2+1}_{C} = 0$$

$$AC_x^2 + BC_x + C = 0$$

$$C_{x\,1,2} = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$= \frac{-2\omega^2R^2\cdot 10n\pm\sqrt{4\omega^4R^4\cdot 10n^2+4\omega^4R^4\,10n^2+\omega^2R^2}}{-2\omega^2R^2}$$

$$\vdots$$

$$= \frac{R\omega\cdot 10n\pm\sqrt{2R^2\omega^2\cdot 10n^2+1}}{\omega R}$$

ker je $C_x > 0 \Rightarrow C_x = 0{,}8\,\mu F$

za R'C člen velja $T(i\omega) = \frac{1}{1+R'C\omega}$

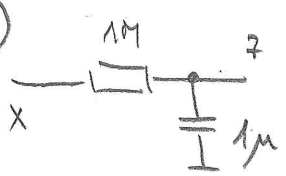
iščem torej $|T(i\omega)| = \frac{1}{|1+R'C\cdot 2\pi f_x|} = \frac{2}{10}$

$$5^2 = 1^2 + (R'C\cdot 2\pi f_x)^2$$

$$\sqrt{24} = R'C\cdot 2\pi f_x$$

$$f_x = \frac{\sqrt{24}}{R'C\cdot 2\pi} = \frac{\sqrt{24}}{500\cdot 10^3\cdot 10^{-6}\cdot 2\pi} = 1{,}559\,Hz$$

16c



$$T(i\omega) = \frac{1}{1+i\omega RC} = \frac{1-i\omega RC}{1^2+\omega^2R^2C^2}$$

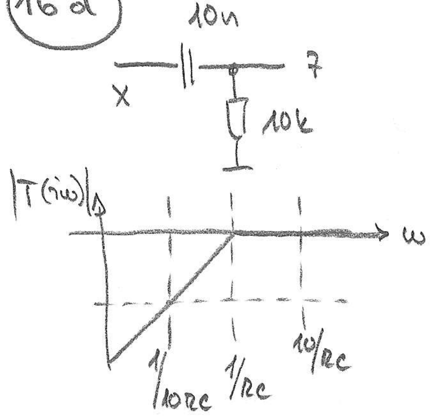
$$tg\,\varphi = \frac{Im(T(i\omega))}{Re(T(i\omega))} = \frac{-\omega RC}{1}$$

$$tg(-45^\circ) = -\omega RC \Rightarrow f_x = \frac{1}{2\pi RC}\,tg\,45^\circ = \underline{0{,}159\,Hz}$$

po bližnjici: ob prelomni frekvenci je ojačenje $1/\sqrt{2}$
in fazni kot 45°

$$f_p = \frac{1}{2\pi RC} = \underline{\underline{0.159\,Hz}}$$

16 d



$$T(p) = \frac{\tau p}{1+\tau p} \Rightarrow T(i\omega) = \frac{i\omega\tau}{1+i\omega\tau}$$

iščemo frekvenco, kjer je $|T(i\omega)| = 1/10$

$$\omega\tau \cdot 10 = |1 + i\omega\tau|$$

$$(10\,\omega\tau)^2 = 1 + \omega^2\tau^2$$

$$99\,\omega^2\tau^2 = 1$$

$$\omega = \frac{1}{\sqrt{99}\,\tau} \Rightarrow f_x = \frac{1}{2\pi\sqrt{99}\,\tau} = \underline{\underline{160\,Hz}}$$

po knjigi:

pri prelomni frekvenci je ojačenje $1/\sqrt{2}$

pri $1/10$ prelomne frekvence je ojačenje $\approx 1/10$

$\Downarrow$

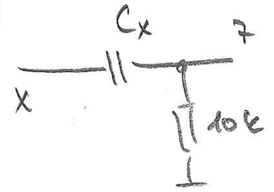
$$f_p = \frac{1}{2\pi RC} = 1{,}592\ kHz$$

$\Downarrow$

ojačenje znaša $1/10$ pri $\underline{\underline{f_x = 159{,}2\ Hz}}$

---

16 e



$f = 20\,Hz$: ojačenje je $1/\sqrt{2}$

$$T(i\omega) = \frac{i\omega\tau}{1+i\omega\tau}$$

zanima me τ, ki ga moram izbrati tako, da bo pri $f = 20\,Hz$ ojačenje $|T(i\omega)| = 1/\sqrt{2}$

$$|T(i\omega)| = \frac{\omega\tau}{|1+i\omega\tau|} = \frac{\omega\tau}{\sqrt{1+\omega^2\tau^2}} = \frac{1}{\sqrt{2}}$$

$$2\omega^2\tau^2 = 1 + \omega^2\tau^2 \Rightarrow \omega^2\tau^2 = 1$$

$$\tau = 1/\omega$$

$$C_x = 1/R \cdot 2\pi \cdot 20$$

$$C_x = 796\,nF \doteq 0{,}8\,\mu F$$

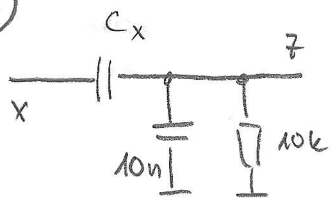
na kratko: ojačenje pade na $1/\sqrt{2}$ pri prelomni frekvenci filtra

$$\Downarrow$$

$$\omega_p = \frac{1}{\tau} \Rightarrow RC_x = \frac{1}{\omega_p} \Rightarrow C_x = \frac{1}{R\cdot 2\pi f_p}$$

---

16 f



rešitev je enostavna zaradi besedice "vsaj"

katerikoli večji od minimalnega kondenzatorja bo dober, torej tudi $C_x = \infty$ F !

ampak poiščimo minimalni C, ki ustreza:

po daljši poti: napiši vozliščno enačbo:

$$\frac{z-x}{\frac{1}{C_x p}} + \frac{z}{\frac{1}{10n\,p}} + \frac{z}{10k} = 0$$

$$(z C_x p - x C_x p + z\,10n\,p)\,10k + z = 0$$

$$z(10k\cdot C_x\cdot p + 10k\cdot 10n\cdot p + 1) = x\cdot 10k\cdot C_x\cdot p$$

$$T(p) = \frac{z}{x} = \frac{10k\cdot C_x\cdot p}{1 + 10k\cdot(C_x+10n)\cdot p}$$

$$\Downarrow$$

$$T(i\omega) = \frac{i\omega\cdot 10k\cdot C_x}{1 + i\omega\cdot 10k\cdot(C_x+10n)\cdot p}$$

$$|T(i\omega)| = \frac{\omega\cdot 10k\cdot C_x}{\sqrt{1+[\omega\cdot 10k\cdot(C_x+10n)]^2}} = \frac{1}{\sqrt{2}}$$

$$2[\omega\cdot 10k\cdot C_x]^2 = 1 + [\omega\cdot 10k\cdot(C_x+10n)]^2$$

$$0 = 2\omega^2\cdot 10k^2\cdot C_x^2 - 1 - \omega^2\cdot 10k^2\cdot C_x^2 - 2\omega^2\cdot 10k^2\cdot C_x\cdot 10n - \omega^2\cdot 10k^2\cdot 10n^2 =$$

reši to kvadratno enačbo ... na C_x

p. še krajši poti s preskušanjem

$$|T(i\omega)| = \frac{\omega \cdot 10k \cdot C_x}{\sqrt{1+\left[\omega \cdot 10k \cdot (C_x + 10n)\right]^2}} \quad ; \quad \omega = 2\pi \cdot 20 \doteq 63\,Hz$$

$$\omega \cdot 10k = 628 \cdot 10^3$$

$$= \frac{628 \cdot 10^3 \cdot C_x}{\sqrt{1 + 395 \cdot 10^9 (C_x + 10^{-8})^2}}$$

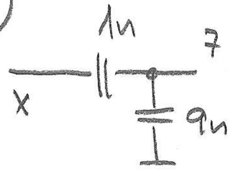
C_x	$\lvert T(i\omega) \rvert$
10n	0,0126
100n	0,1245
1µ	0,7777
0,5µ	0,5290
0,75µ	0,6616
0,85µ	0,7254
0,8µ	0,7045

zaporedni poskusi s kalkulatorjem iščemo $1/\sqrt{2} = 0{,}707$

$\Leftarrow$ dovolj dober rezultat

kapacitivnost C_x mora biti vsaj **0,8µF**

16 g



delilnik napetosti:

$$z = x\,\frac{X_{9n}}{X_{1n}+X_{9n}} = x\,\frac{\frac{1}{9n\cdot \cancel{p}}}{\frac{1}{9n\cdot \cancel{p}}+\frac{1}{1n\cdot \cancel{p}}} =$$

$$z = x\,\frac{1}{10}$$

odgovor: pri vseh frekvencah je amplituda izhodnega signala 1/10 amplitude vhodnega signala

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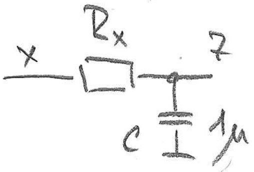
16 h



$$T(i\omega) = \frac{1}{1+i\omega R_x C}$$

formulacija: a) $|T(i\omega)| = A = \underbrace{\frac{1}{\sqrt{1+\omega^2 R_x^2 C^2}} = 1/10}_{\text{izračunaj } R_x}$, $f = 1\,kHz$

b) uporabi izračunani R_x in poišči frekvenco, pri kateri je ojačenje 1/1000

a) $$10 = \sqrt{1+(2\pi\cdot 10^3\cdot R_x\cdot 10^{-6})^2}$$

$$100-1 = 4\pi^2\cdot 10^{-6}\cdot R_x^2 \Rightarrow R_x = \sqrt{\frac{99}{4\pi^2\cdot 10^{-6}}} = \underline{\underline{1{,}58\,k}}$$

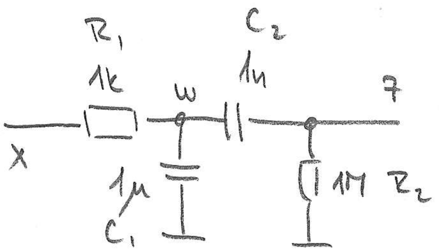
$$|T(i\omega)| = \frac{1}{1000} = \frac{1}{\sqrt{1+(2\pi f_x\cdot R_x\cdot C)^2}}$$

$$1+(2\pi R_x C)^2\cdot f_x^2 = (10^3)^2$$

$$f_x \doteq \frac{10^3}{2\pi R_x C} \doteq 100\,kHz$$

na kratko: RC člen duši na 1/10 pri $10\times f_p$, to je pri 1kHz. Dušenje se poveča za $10\times$ ob podeseterjenju vhodne frekvence, torej pade na 1/1000 pri $f = 100\cdot 1kHz = 100\,kHz$

16 i



$$\tau_1 = 10^3 \cdot 10^{-6} = 10^{-3} = 1\,ms$$
$$\tau_2 = 10^{-9} \cdot 10^{6} = 10^{-3} = 1\,ms$$
$$\Big\}\ \tau_1 = \tau_2 = 1\,ms$$

formalno: vozliščni enačbi za w in z

$$\frac{w-x}{R_1} + \frac{w}{\frac{1}{C_1 p}} + \frac{w-z}{\frac{1}{C_2 p}} = 0 \qquad w = \frac{1+R_2C_2p}{R_2C_2p}\,z$$

$$\frac{z-w}{\frac{1}{C_2 p}} + \frac{z}{R_2} = 0 \implies z = w\,\frac{R_2C_2p}{1+R_2C_2p}$$

$$w - x + wR_1C_1p + wR_1C_2p - zR_1C_2p = 0$$

$$z\,\frac{1+R_2C_2p}{R_2C_2p}\left[1 + R_1C_1p + R_1C_2p\right] - zR_1C_2p = x$$

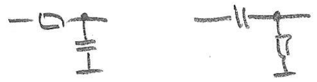
$$xR_2C_2p = z\left[1 + R_1C_1p + \cancel{R_1C_2p} + R_2C_2p + R_1R_2C_1C_2p^2 + R_1R_2C_2^2p^2 - \cancel{R_1C_2p}\right]$$

$$z = x\,\frac{R_2C_2p}{R_1R_2C_2\underbrace{(C_1+C_2)}_{C_1+C_2\approx C_1}p^2 + \underbrace{(R_1C_1+R_2C_2)}_{R_1C_1 = R_2C_2}p + 1}$$

$$\Downarrow$$

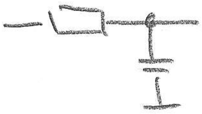
$$T(p) = \frac{RCp}{R^2C^2p^2 + 2RCp + 1} = \frac{RCp}{(RCp+1)^2}$$

$$T(p) = \frac{1}{1+\tau p}\cdot\frac{\tau p}{1+\tau p} \Rightarrow T(i\omega) = \frac{1}{1+i\omega\tau}\cdot\frac{i\omega\tau}{1+i\omega\tau}$$



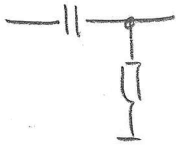
ker so vrednosti smiselno izbrane ($C_2 \ll C_1$) in ($R_2 \gg R_1$) drugi RC ne spreminja lastnosti prvega RC-ja in obratno oba RC-ja imata enako prelomno frekvenco, pri tej prvi suka fazo za $+\varphi_1$, drugi pa za $-\varphi_2$

določimo oba fazna kota



$$\operatorname{tg}\varphi_1 = \frac{Im}{Re} = \frac{-\omega\tau}{1} = -2\pi\cdot 10^{4}\cdot 10^{-3}$$

$$\varphi_1 = \operatorname{arctg}(-2\pi\cdot 10) = -89.1^\circ$$



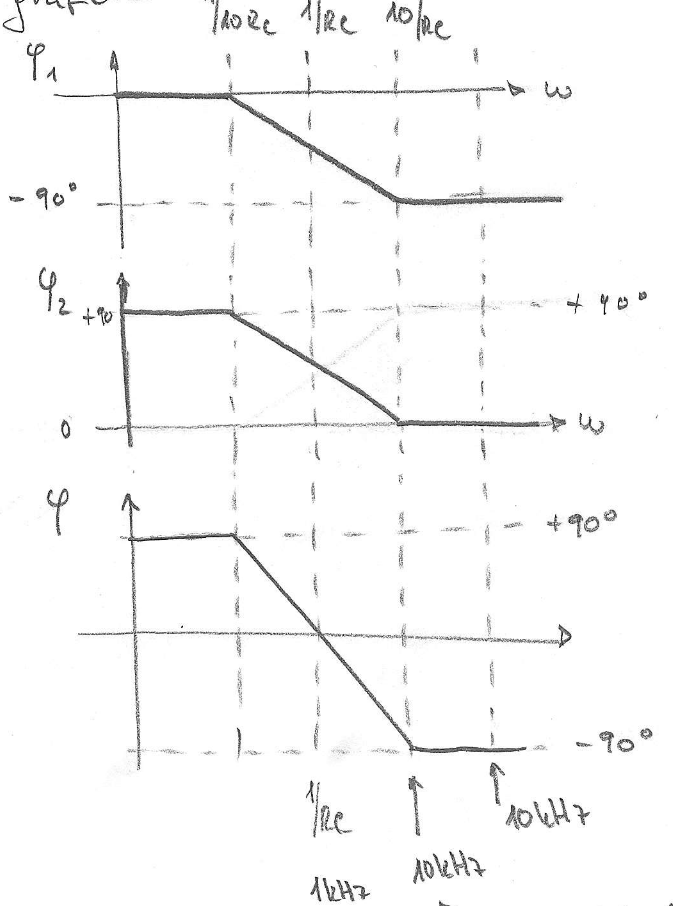
$$T(i\omega) = \frac{i\omega\tau}{1+i\omega\tau} = \frac{i\omega\tau(1-i\omega\tau)}{1+\omega^2\tau^2} = \frac{i\omega\tau+\omega^2\tau^2}{1+\omega^2\tau^2}$$

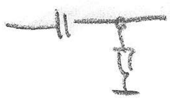
$$\operatorname{tg}\varphi_2 = \frac{Im}{Re} = \frac{\cancel{\omega\tau}}{\omega^2\tau^2} = \frac{1}{\omega\tau}$$

$$\varphi_2 = \operatorname{arc\,tg}\left(1/2\pi\cdot 10^4\cdot 10^{-3}\right) = 0{,}91^\circ$$

skupni fazni kot : $\varphi = \varphi_1 + \varphi_2 = \underline{\underline{-88.19^\circ}}$

grafično



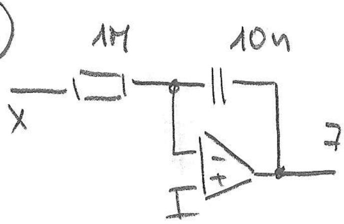


$$\frac{1}{RC} = \frac{1}{1ms} = 1\,kHz$$

$$\omega = 2\pi/RC = 6{,}28\,kHz$$

pri tej frekvenci je fazni kot skoraj -90°

17 a



$$T(p) = -\frac{\frac{1}{10n\cdot p}}{1M} = -\frac{1}{10n\cdot 1M\cdot p} = -\frac{1}{10^{-2}p}$$

$$T(i\omega) = -\frac{1}{i\cdot 10^{-2}\omega}$$

$$|T(i\omega)| = \frac{1}{10^{-2}\cdot 2\pi f} = \frac{1}{10^{-2}\cdot 2\cdot\pi\cdot 10^{3}} \doteq 15.9\ \text{mV}$$

$$\operatorname{tg}\varphi = \frac{10^{-2}\omega}{0} = \infty \Rightarrow \varphi = \underline{\underline{90^\circ}}$$

drugače:

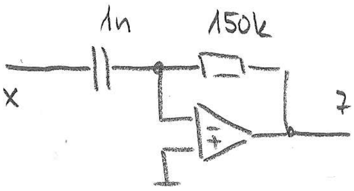
$$y = -\frac{1}{RC}\int x(t)\,dt = -\frac{1}{RC}\int \sin\omega t\,dt$$

$$= +\underbrace{\frac{1}{RC\omega}}_{\text{ojačenje}}\cos\omega t$$

tule je skrita faza: $+90^\circ$

$$A = \frac{1}{2\pi f RC} = \frac{1}{2\pi\cdot 10^{3}\cdot 10^{6}\cdot 10^{-8}} = 15.9\ \text{mV}$$

17b



$$T(p) = -\tau p = -10^{-9}\cdot 150\cdot 10^{3}p = -150\cdot 10^{-6}p$$

nekje računa odvod

$$T(i\omega) = -150\cdot 10^{-6}\cdot i\omega$$

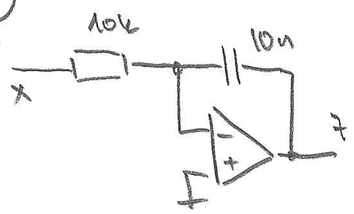
$$|T(i\omega)| = A = 150\cdot 10^{-6}\cdot 2\pi\cdot 10^{3} = 942\cdot 10^{-3} = 0.942$$

ampl. izh. signala je torej 0.942V

$$\operatorname{tg}\varphi = \frac{\operatorname{Im}(T(i\omega))}{\operatorname{Re}(T(i\omega))} = \frac{-150\cdot 10^{-6}\cdot\omega}{0} = -\infty \Rightarrow \underline{\underline{\varphi = -90^\circ}}$$

za vse frekvence

17c

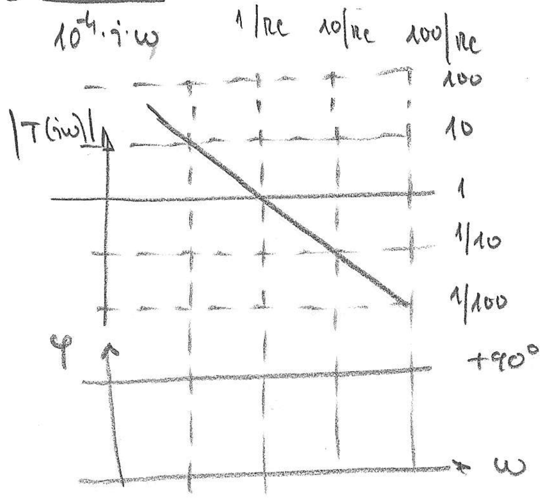


$$T(ip) = -\frac{1}{RCp} = -\frac{1}{10^4 \cdot 10^{-8} p} = -\frac{1}{10^{-4} p}$$

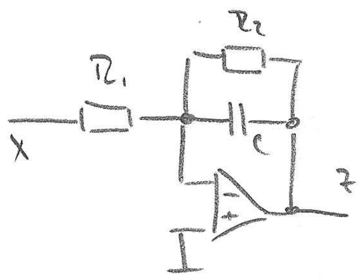
$$T(i\omega) = -\frac{1}{10^{-4} \cdot i \omega}$$

$$\left| T(i\omega) \right| = \frac{1}{10^{-4}\,\omega}$$

$$\operatorname{tg} \varphi = \frac{1}{10^{-4}\omega \cdot 0} \Rightarrow \varphi = 90^\circ$$



17d



natančno:

$$\frac{x}{R_1} + \frac{z}{R_2} + \frac{z}{\frac{1}{Cp}} = 0$$

$$-xR_2 = zR_1 + zR_1R_2Cp = zR_1(1+R_2Cp)$$

$$z = -x\frac{R_2}{R_1}\frac{1}{1+R_2Cp}$$

$$T(i\omega) = -\underbrace{\frac{R_2}{R_1}}_{10}\,\frac{1}{1+\underbrace{i\omega R_2 C}}$$

$$i \cdot 2\pi \cdot 10^4 \cdot 10^{-10} \cdot 10^3 = \underbrace{i \cdot 2\pi \cdot 10^{-3}}_{\text{zanemoni proti } 1}$$

$\Downarrow$ zato

$$T(i\omega) \doteq -\frac{R_2}{R_1} = \underline{\underline{-10}}$$

amplituda izh. signala znaša $0.1V \cdot 10 = \underline{\underline{1V}}$

približno:

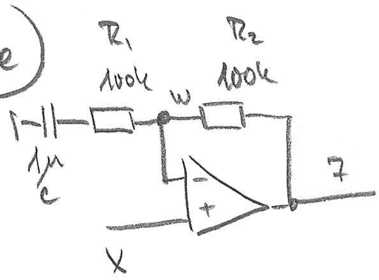
$$X_C = \frac{1}{2\pi f \cdot C} = \frac{1}{2\pi \cdot 10^3 \cdot 10^{-10}} \doteq 10^6\,\Omega$$

to je veliko več od upornosti $R_2 = 10k$

zato lahko pri tej frekvenci vhodnega signala vpliv kondenzatorja C zanemarim.

Ostane samo ojačevalnik z $A = -10$

(17e)



idealni OP $\Rightarrow$ $w = x$

$$\frac{w}{R_1 + \frac{1}{Cp}} + \frac{w - z}{R_2} = 0$$

$$\frac{x\,Cp}{1 + R_1 Cp} + \frac{x - z}{R_2} = 0$$

$$x R_2 Cp + x + x R_1 Cp = z(1 + R_1 Cp)$$

$$x\left(1 + (R_1 + R_2)Cp\right) = z(1 + R_1 Cp)$$

$$z = x\,\frac{1 + (R_1 + R_2)Cp}{1 + R_1 Cp}$$

$$T(p) = \frac{1 + (R_1 + R_2)Cp}{1 + R_1 Cp} \Rightarrow T(i\omega) = \frac{1 + (R_1 + R_2)\,i\omega C}{1 + i\omega R_1 C}$$

$$\Downarrow$$

$$|T(i\omega)| = \dots$$

pri 1kHz:

$$T(i\omega)\Big|_{f=1000} = \frac{1 + i\,200 \cdot \cancel{10^3} \cdot 2\cdot\pi \cdot \cancel{10^3} \cdot \cancel{10^{-6}}}{1 + i \cdot 2\pi \cdot \cancel{10^3} \cdot 100 \cdot \cancel{10^3} \cdot \cancel{10^{-6}}}$$

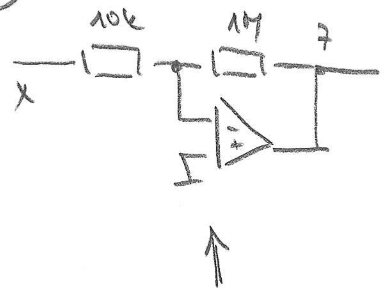
$$= \frac{1 + i \cdot 400\pi}{1 + i \cdot 200\pi}$$

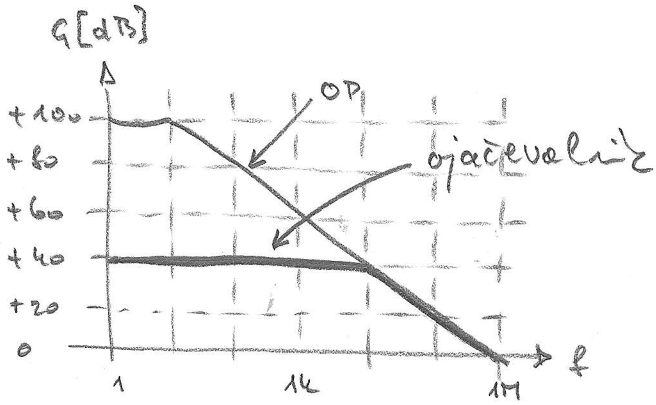
zgoraj in spodaj zanemari 1 (je premajhna proti 200π oz. 400π

$$T(i\omega)\Big|_{f=1kHz} \doteq 2$$

amplituda izhodnega signala znaša $\underline{\underline{2V}}$

17f





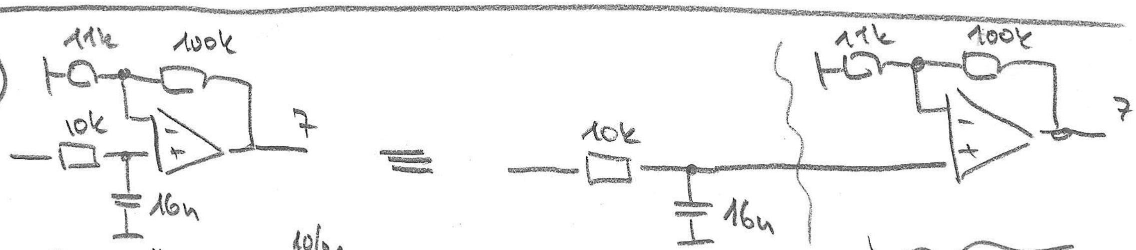
Za idealni OP

$$A = -\frac{1M}{10k} = -100 \equiv +40\,dB$$

iz predavanj: ojačenje definirata upornika, če le OP to zmore

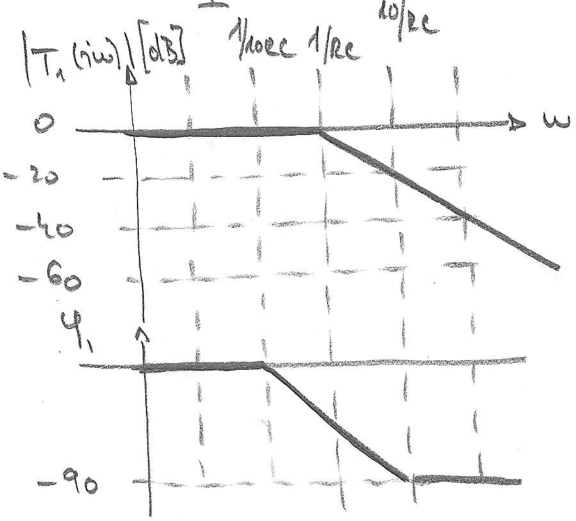
stopnja torej ojačuje s faktorjem -100 do frekvence približno $10\,kHz$, potem ojačenje pada z $20\,dB$/dekado !

17g

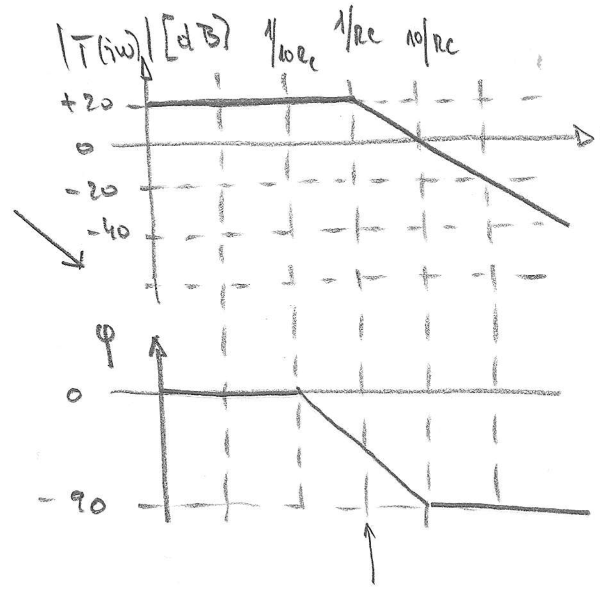


$T_1(i\omega)$

$$T_2(i\omega) = 1 + \frac{100k}{11k} = 10$$

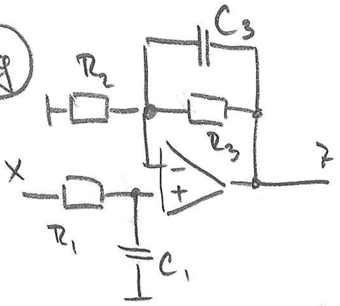


X10

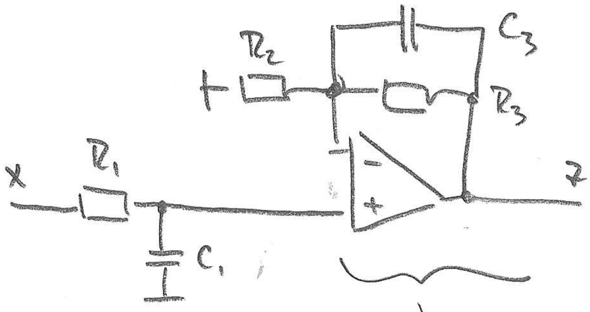


$\omega = 1/RC$

17g

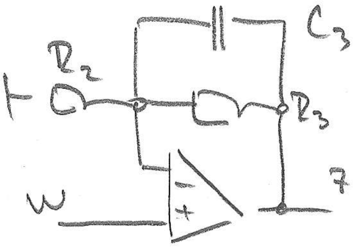


$\Rightarrow$



$$T(i\omega) = \frac{1}{1 + i\omega R_1 C_1}$$

samo tale del



idealni OP $\Rightarrow$ na neinvertiranem vhodu je tudi w

vozl. enačba za to vozlišče:

$$\frac{w}{R_2} + \frac{w - z}{R_3} + \frac{w - z}{\frac{1}{C_3 p}} = 0$$

$$wR_3 + wR_2 - zR_2 + wR_2R_3C_3p - zR_2R_3C_3p = 0$$

$$w\left[R_2 + R_3 + R_2R_3C_3p\right] = z\left[R_2 + R_2R_3C_3p\right]$$

$$T(p) = \frac{z}{w} = \frac{R_2\left[1 + \frac{R_3}{R_2} + R_3C_3p\right]}{R_2\left[1 + R_3C_3p\right]} = \frac{1 + \frac{R_3}{R_2} + R_3C_3p}{1 + R_3C_3p}$$

$$T(p) = 1 + \frac{R_3}{R_2(1 + R_3C_3p)} = 1 + \frac{R_3}{R_2}\underbrace{\frac{1}{1 + R_3C_3p}}$$

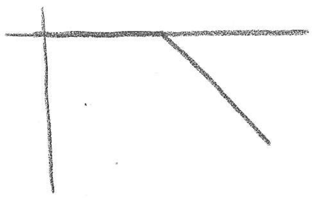
ponovi w + ojači skozi RC poslan signal

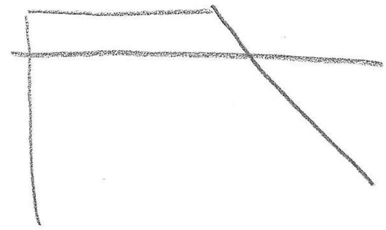
skupaj tako

$$T(i\omega) = T(i\omega)_{za\ RC} \cdot T(i\omega)_{za\ OP}$$

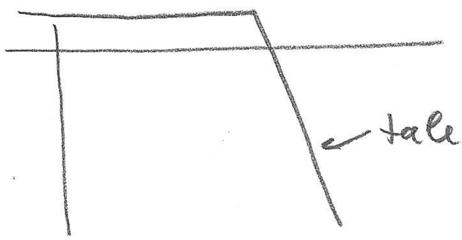
$$= \frac{1}{1+i\omega R_1 C_1} \cdot \left[1 + \frac{R_3}{R_2} \frac{1}{1+R_3C_3p} \right]$$

kaže, da je naloga postala precej bolj komplicirana, zato le približne rešitev





prelomni frekvenci sta enaki, tako sta izbrana RC člena



tale strmina je $-40\,dB/dek$