Frekvenčna analiza signala ➔ Fourier

$x(t) \leftrightarrow X(\omega)$: *isti signal, različno gledanje*

$x(t) \rightarrow F\big(x(t)\big) \Rightarrow c_k$ za periodično x(t), $-\infty < \mathrm{k} < \infty$

$\mathrm{c}(i\omega)$ za neperiodično x(t)

Zvezni periodični signal == razvoj v Fourierovo vrsto

Razvoj v Fourier-ovo vrsto:

$$F\big(x(t)\big) = \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, e^{-ik\omega t}\, dt = c_k \quad ; \ \omega = \frac{2\pi}{T}$$

Sinteza signala iz znanega razvoja v Fourier-ovo vrsto:

$$x(t) = \sum_{k=-\infty}^{\infty} c_k \ e^{ik\omega t}$$

Zvezni ne-periodični signal == Fourierova analiza

Razvoj v zvezni spekter:

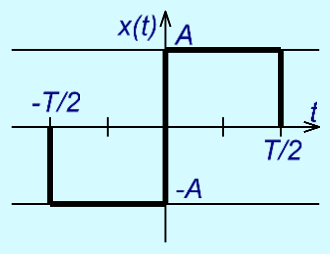
$$F\big(x(t)\big) = \int_{-\infty}^{\infty} x(t)\, e^{-i\omega t}\, dt = c(i\omega)$$

Sinteza iz znanega spektra:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} c(i\omega)\ e^{i\omega t} d\omega$$

Zgled za periodičen in pravokoten signal z amplitudo A: razvoj v vrsto

$$x(t) = \begin{cases} -A & za \quad -{}^{T}/_{2} < t < 0 \\ A & za \quad 0 < t < {}^{T}/_{2} \end{cases}$$



$$c_k = \frac{1}{T}\int_{-{}^{T}/_{2}}^{{}^{T}/_{2}} x(t)e^{-ik\omega t}\,dt = \frac{1}{T}\left[-A\int_{-{}^{T}/_{2}}^{0} e^{-ik\omega t}\,dt + A\int_{0}^{{}^{T}/_{2}} e^{-ik\omega t}\,dt\right] =$$

$$= \frac{A}{T}\left[-\left.\frac{e^{-ik\omega t}}{-ik\omega}\right|_{-{}^{T}/_{2}}^{0} + \left.\frac{e^{-ik\omega t}}{-ik\omega}\right|_{0}^{-{}^{T}/_{2}}\right] =$$

$$= \frac{iA}{Tk\omega}\left[-\left(\cos 0 + i\sin 0 - \cos k\frac{2\pi}{T}\frac{T}{2} - i\sin k\frac{2\pi}{T}\frac{T}{2}\right) + \left(\cos k\frac{2\pi}{T}\frac{T}{2} + i\sin k\frac{2\pi}{T}\frac{T}{2} - \cos 0 - i\sin 0\right)\right]$$

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$k = 0$	$c_0 = 0$	
$k = \pm 1$	$c_{-1} = \frac{i4A}{2\pi} = i\alpha$	$c_1 = -\frac{i4A}{2\pi} = -i\alpha$
$k = \pm 2$	$c_{-2} = 0$	$c_2 = 0$
$k = \pm 3$	$c_{-3} = \frac{-i4A}{3 * 2\pi} = -i\frac{\alpha}{3}$	$c_3 = -\frac{-i4A}{3 * 2\pi} = i\frac{\alpha}{3}$
$k = \pm 4$	$c_{-4} = 0$	$c_4 = 0$
$k = \pm 5$	$c_{-5} = \frac{-i4A}{5 * 2\pi} = -i\frac{\alpha}{5}$	$c_5 = -\frac{-i4A}{5 * 2\pi} = i\frac{\alpha}{5}$

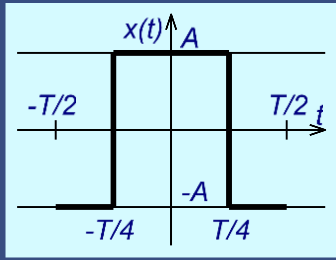
Zgled za periodičen in pravokoten signal z amplitudo A: sinteza iz razvoja v vrsto

$$x(t) = \sum_k c_k e^{ik\omega t} = \sum_k c_k(\cos k\omega t + i \sin k\omega t)$$

$k = 0$	$c_0 = 0$	
$k = 1$	$x(t, c_1) = -\frac{i4A}{2\pi}\left(\cos\frac{2\pi}{T}t + i\sin\frac{2\pi}{T}t\right)$	$x(t, c_{\pm 1}) = x(t, c_1) + x(t, c_{-1}) = \frac{4A}{\pi}\left(\sin\frac{2\pi}{T}t\right)$
$k = -1$	$x(t, c_{-1}) = \frac{i4A}{2\pi}\left(\cos\frac{2\pi}{T}t - i\sin\frac{2\pi}{T}t\right)$	
$k = 2, -2$	$x(t, c_2) = 0$, $x(t, c_{-2}) = 0$	$x(t, c_{\pm 2}) = 0$
$k = 3$	$x(t, c_3) = -\frac{i4A}{3 * 2\pi}\left(\cos 3\frac{2\pi}{T}t + i\sin 3\frac{2\pi}{T}t\right)$	$x(t, c_{\pm 3}) = x(t, c_3) + x(t, c_{-3}) = \frac{4A}{3 * \pi}\left(\sin 3\frac{2\pi}{T}t\right)$
$k = -3$	$x(t, c_{-3}) = \frac{i4A}{3 * 2\pi}\left(\cos 3\frac{2\pi}{T}t + i\sin 3\frac{2\pi}{T}t\right)$	
$k = \pm 4$	$x(t, c_4) = 0$, $x(t, c_{-4}) = 0$	$x(t, c_{\pm 4}) = 0$
$k = 5$	$x(t, c_5) = -\frac{i4A}{5 * 2\pi}\left(\cos 5\frac{2\pi}{T}t + i\sin 5\frac{2\pi}{T}t\right)$	$x(t, c_{\pm 5}) = x(t, c_5) + x(t, c_{-5}) = \frac{4A}{5 * \pi}\left(\sin 5\frac{2\pi}{T}t\right)$
$k = -5$	$x(t, c_{-5}) = \frac{i4A}{5 * 2\pi}\left(\cos 5\frac{2\pi}{T}t + i\sin 5\frac{2\pi}{T}t\right)$	

Zgled za periodičen in pravokoten signal z amplitudo A: razvoj v vrsto

$$x(t)=\begin{cases}-A & za \quad -{}^{T}/_{2}<t<-{}^{T}/_{4}\\ A & za \quad -{}^{T}/_{4}<t<{}^{T}/_{4}\\ -A & za \quad {}^{T}/_{4}<t<{}^{T}/_{2}\end{cases}$$



$$c_k=\frac{1}{T}\int_{-T/2}^{T/2}x(t)e^{-ik\omega t}\,dt=\frac{A}{T}\left[-\int_{-T/2}^{-T/4}e^{-ik\omega t}\,dt+\int_{-T/4}^{T/4}e^{-ik\omega t}\,dt-\int_{T/4}^{T/2}e^{-ik\omega t}\,dt\right]=$$

$$=\frac{A}{T}\left[-\left.\frac{e^{-ik\omega t}}{-ik\omega}\right|_{-T/2}^{-T/4}+\left.\frac{e^{-ik\omega t}}{-ik\omega}\right|_{-T/4}^{T/4}-\left.\frac{e^{-ik\omega t}}{-ik\omega}\right|_{T/4}^{T/2}\right]=$$

$$=\frac{iA}{k2\pi}\left[-\left(\overbrace{\cos k\frac{\pi}{2}}^{0}+i\sin k\frac{\pi}{2}\right)+(\cos k\pi+\overbrace{i\sin k\pi}^{0})+\left(\overbrace{\cos k\frac{\pi}{2}}^{0}-i\sin k\frac{\pi}{2}\right)-\left(\overbrace{\cos k\frac{\pi}{2}}^{0}+i\sin k\frac{\pi}{2}\right)-(\cos k\pi-\overbrace{i\sin k\pi}^{0})+\left(\overbrace{\cos k\frac{\pi}{2}}^{0}-i\sin k\frac{\pi}{2}\right)\right]=$$

$$=\frac{4A}{k2\pi}\sin k\frac{\pi}{2}$$

$k=0$	$c_0=0$	
$k=\pm1$	$c_{-1}=\frac{4A}{2\pi}=\alpha$	$c_1=\frac{4A}{2\pi}=\alpha$
$k=\pm2$	$c_{-2}=0$	$c_2=0$
$k=\pm3$	$c_{-3}=-\frac{4A}{3*2\pi}=-\frac{\alpha}{3}$	$c_3=-\frac{4A}{3*2\pi}=-\frac{\alpha}{3}$
$k=\pm4$	$c_{-4}=0$	$c_4=0$
$k=\pm5$	$c_{-5}=\frac{4A}{5*2\pi}=\frac{\alpha}{5}$	$c_5=\frac{4A}{5*2\pi}=\frac{\alpha}{5}$

Zgled za periodičen in pravokoten signal z amplitudo A: sinteza iz razvoja v vrsto

$$x(t) = \sum_k c_k \, e^{ik\omega t} = \sum_k c_k \, (\cos k\omega t + i \sin k\omega t)$$

$k = 0$	$c_0 = 0$	
$k = 1$	$x(t, c_1) = \frac{4A}{2\pi}\left(\cos\frac{2\pi}{T}t + i\sin\frac{2\pi}{T}t\right)$	$x(t, c_{\pm 1}) = x(t, c_1) + x(t, c_{-1}) = \frac{4A}{\pi}\left(\cos\frac{2\pi}{T}t\right)$
$k = -1$	$x(t, c_{-1}) = \frac{4A}{2\pi}\left(\cos\frac{2\pi}{T}t - i\sin\frac{2\pi}{T}t\right)$	
$k = 2, -2$	$x(t, c_2) = 0$, $x(t, c_{-2}) = 0$	$x(t, c_{\pm 2}) = 0$
$k = 3$	$x(t, c_3) = -\frac{4A}{3 * 2\pi}\left(\cos 3\frac{2\pi}{T}t + i\sin 3\frac{2\pi}{T}t\right)$	$x(t, c_{\pm 3}) = x(t, c_3) + x(t, c_{-3}) = -\frac{4A}{3 * \pi}\left(\cos 3\frac{2\pi}{T}t\right)$
$k = -3$	$x(t, c_{-3}) = -\frac{4A}{3 * 2\pi}\left(\cos 3\frac{2\pi}{T}t + i\sin 3\frac{2\pi}{T}t\right)$	
$k = \pm 4$	$x(t, c_4) = 0$, $x(t, c_{-4}) = 0$	$x(t, c_{\pm 4}) = 0$
$k = 5$	$x(t, c_5) = \frac{4A}{5 * 2\pi}\left(\cos 5\frac{2\pi}{T}t + i\sin 5\frac{2\pi}{T}t\right)$	$x(t, c_{\pm 5}) = x(t, c_5) + x(t, c_{-5}) = \frac{4A}{5 * \pi}\left(\cos 5\frac{2\pi}{T}t\right)$
$k = -5$	$x(t, c_{-5}) = \frac{4A}{5 * 2\pi}\left(\cos 5\frac{2\pi}{T}t + i\sin 5\frac{2\pi}{T}t\right)$	

Zgled za razvoj: poljuben kosinusni signal, m je naravno število

$$x(t) = A\cos(m\omega t + \varphi) = A\,\frac{e^{i(m\omega t+\varphi)} + e^{-i(m\omega t+\varphi)}}{2} \qquad ; \qquad \omega = \frac{2\pi}{T}$$

$$c_k = \frac{A}{2T}\int_{-T/2}^{T/2}\left(e^{i(m\omega t+\varphi)} + e^{-i(m\omega t+\varphi)}\right)e^{-ik\omega t}\,dt = \frac{A}{2T}\left[\int_{-T/2}^{T/2}\left[e^{i\varphi}e^{i(m-k)\omega t} + e^{-i\varphi}e^{-i(m+k)\omega t}\right]dt\right] = \frac{A}{2T}\left[\frac{e^{i\varphi}e^{i(m-k)\omega t}}{i(m-k)\omega} - \frac{e^{-i\varphi}e^{-i(m+k)\omega t}}{i(m+k)\omega}\right]\Bigg|_{-T/2}^{T/2}$$

0! 0! 0! 0!

$$= \frac{A}{2T}\left[e^{i\varphi}\,\frac{\cancel{\cos}(m-k)\pi + i\sin(m-k)\pi - \cancel{\cos}(m-k)\pi + i\sin(m-k)\pi}{i(m-k)\frac{2\pi}{T}} - e^{-i\varphi}\,\frac{\cancel{\cos}(m+k)\pi - i\sin(m+k)\pi - \cancel{\cos}(m+k)\pi - i\sin(m+k)\pi}{i(m+k)\frac{2\pi}{T}}\right]$$

$$c_k = 0 \quad !!!!!$$

Izjema: k= $+m$ $\quad\Longrightarrow\quad c_{k=+m} = \frac{A\,e^{i\varphi}}{2}\;\frac{\sin 0}{0} = \frac{A}{2}\,e^{i\varphi}$

Izjema: k= $-m$ $\quad\Longrightarrow\quad c_{k=-m} = \frac{Ae^{-i\varphi}}{2}\;\frac{\sin 0}{0} = \frac{A}{2}\,e^{-i\varphi}$

Zgled za sintezo: poljuben kosinusni signal, m je naravno število

$$x(t) = A\cos(m\omega t + \varphi) \quad \Longrightarrow \quad c_{k=\pm m} = \frac{A}{2}\, e^{\pm i\varphi} \quad ; \quad \omega = \frac{2\pi}{T}$$

$$x(t) = \sum_{k=\pm m} c_k\, e^{ik\omega t} = c_m\, e^{im\omega t} + c_{-m}\, e^{-im\omega t} = \frac{A}{2}\left(e^{i\varphi} e^{im\omega t} + e^{-i\varphi} e^{-im\omega t}\right) = \frac{A}{2}\left(e^{i(m\omega t+\varphi)} + e^{-i(m\omega t+\varphi)}\right) =$$

$$= \frac{A}{2}[\cos(m\omega t + \varphi) + i\sin(m\omega t + \varphi) + \cos(m\omega t + \varphi) - i\sin(m\omega t + \varphi)] =$$

$$= A\cos(m\omega t + \varphi)$$

Čista ($\varphi = 0$) kosinusni signal in sinusni signal sta torej:

$$x(t) = A\cos m\omega t \qquad \Longrightarrow \qquad c_{k=+m} = \frac{A}{2} \quad , \quad c_{k=-m} = \frac{A}{2}$$

$$x(t) = \sum_{k=\pm m} c_k\, e^{ik\omega t} = c_m\, e^{im\omega t} + c_{-m}\, e^{-im\omega t} = \frac{A}{2}\left(e^{im\omega t} + e^{-im\omega t}\right) =$$

$$= \frac{A}{2}\left[\cos m\omega t + i\sin m\omega t + \cos m\omega t - i\sin m\omega t\right] = A\cos m\omega t$$

$$x(t) = A\sin m\omega t = A\cos\left(m\omega t - \frac{\pi}{2}\right) \qquad \Longrightarrow \qquad c_{k=+m} = -i\,\frac{A}{2} \quad , \quad c_{k=-m} = i\,\frac{A}{2}$$

$$x(t) = \sum_{k=\pm m} c_k\, e^{ik\omega t} = c_m\, e^{im\omega t} + c_{-m}\, e^{-im\omega t} = i\,\frac{A}{2}\left(-e^{im\omega t} + e^{-im\omega t}\right) =$$

$$= i\,\frac{A}{2}\left[-\cos m\omega t - i\sin m\omega t + \cos m\omega t - i\sin m\omega t\right] = A\sin m\omega t$$