obnova: uvod v vzorčenje

Fourier

$x(t) \longleftrightarrow x(\omega)$

$x(t) \xrightarrow{F} F(x(t)) \Rightarrow c_k$ za periodične $x(t)$

$c(i\omega)$ za neperiodične $x(t)$

periodično

tja: $$F(x(t)) = c_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\cdot e^{-ik\omega t}\,dt$$

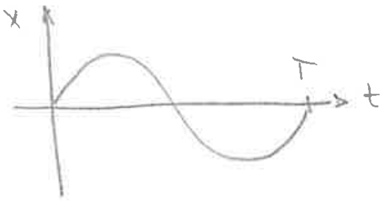
meji = faza

c_k ≡ kompleksen: cos i sin

nazaj: $$x(t) = \sum_k c_k e^{ik\omega t}$$

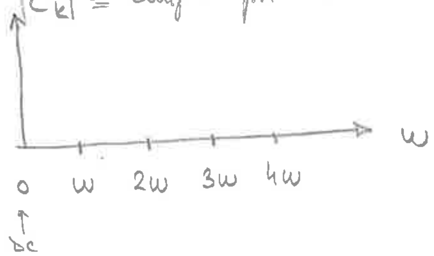
$2\pi f = \frac{2\pi}{T}$

samo realni del!



→

$|c_k|$ = ampl. pri $k\omega$



ω: diskretna

neperiodično

tja: $$F(x(t)) = c(i\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-i\omega t}\,dt$$

nazaj: $$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} c(i\omega)\, e^{i\omega t}\,d\omega$$

ω: zvezna

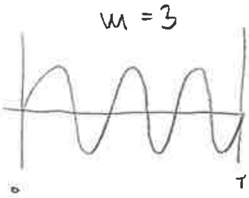
samo realni del!

Zgled za Fourier | zvezno

$$x(t) = A \cdot \cos(m\omega t + \varphi) = A\,\frac{e^{i(m\omega t+\varphi)} + e^{-i(m\omega t+\varphi)}}{2}$$

$\varphi = \pi/2$

$m = 3$



$$F(x(t)) = c(k\omega) = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-ik\omega t}\, dt = \frac{A}{2T}\int_{-T/2}^{T/2}\left[e^{i((m-k)\omega t+\varphi)} + e^{-i((m+k)\omega t+\varphi)}\right] dt$$

$$= \frac{A}{2T}\left[\frac{e^{i\varphi}\cdot e^{i(m-k)\omega t}}{i(m-k)\omega} + \frac{e^{-i\varphi}\, e^{-i(m+k)\omega t}}{-i(m+k)\omega}\right]_{-T/2}^{T/2}$$

$$= \frac{A}{2T}\left[\frac{T\cdot e^{i\varphi}\cdot e^{i(m-k)\frac{2\pi}{T}t}}{i(m-k)\cdot 2\pi} + \frac{T\cdot e^{-i\varphi}\cdot e^{-i(m+k)\frac{2\pi}{T}t}}{-i(m+k)\,2\pi}\right]_{-T/2}^{T/2}$$

$$= \frac{A}{2}\left[\frac{e^{i\varphi}\left(e^{i(m-k)\pi} - e^{-i(m-k)}\right)}{i(m-k)\,2\pi} + \frac{e^{-i\varphi}\left(e^{-i(m+k)\pi} - e^{i(m+k)\pi}\right)}{-i(m+k)\,2\pi}\right]$$

$\underbrace{}_{2i\sin(m-k)\pi}$ $\qquad$ $\underbrace{}_{-2i\sin(m+k)\pi}$

$$= \frac{A}{2}\left[\frac{e^{i\varphi}\cdot \cancel{2}\cancel{i}\sin(m-k)\pi}{\cancel{i}(m-k)\cancel{2}\pi} + \frac{-e^{-i\varphi}\cdot \cancel{2}\cancel{i}\sin(m+k)\pi}{-\cancel{i}(m+k)\cancel{2}\pi}\right]$$

$$= \frac{A}{2}\left[\underbrace{\frac{e^{i\varphi}\cdot\sin(m-k)\pi}{(m-k)\pi}}_{\neq 0 \text{ za } m=k} + \underbrace{\frac{e^{-i\varphi}\cdot\sin(m+k)\pi}{(m+k)\pi}}_{\neq 0 \text{ za } m=-k}\right] \longrightarrow$$

$$\longrightarrow \frac{A}{2}\left[\underset{\uparrow\, m=k}{e^{i\varphi}} + \underset{\uparrow\, m=-k}{e^{-i\varphi}}\right] = \underset{\uparrow\, m=k}{\frac{A}{2}\left[\cos\varphi + i\sin\varphi\right]} + \underset{\uparrow\, m=-k}{\frac{A}{2}\left[\cos\varphi - i\sin\varphi\right]}$$

2011/3a

vzorčenje

zajemanje

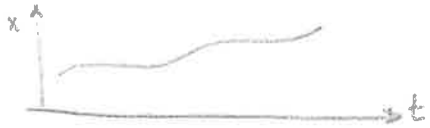
① načini: SW, HW direktno, HW/SW prekinitve
- timer, IRQ

② teorija

izgubljamo informacijo, ker vidimo signal le ob kT_v, $k \in \mathbb{Z}$

demo LV

|||

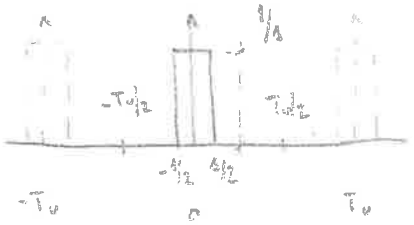


niz δ   X : produkt

niz $\delta(kT_v)$ $\Rightarrow$ niz $\delta = \sum_{k=-\infty}^{\infty} \delta[kT_v]$

$$x_a(t) \rightarrow x_d[kT_v] = \underbrace{x_a(t) \cdot \sum_{k=-\infty}^{\infty} \delta[kT_v]}_{\text{slabo zapisano}} = x_a(t) \sum_{k=-\infty}^{\infty} \delta(t-kT_v)$$

spekter niza δ



$$F[\text{niz } \delta] = \frac{1}{T_v} \int_{-T_v/2}^{T_v/2} \delta(t) e^{-i\omega k t} dt =$$

$$= \frac{1}{T_v} \int_{-\Delta/2}^{\Delta/2} \frac{1}{\Delta} e^{-ik\omega t} dt =$$

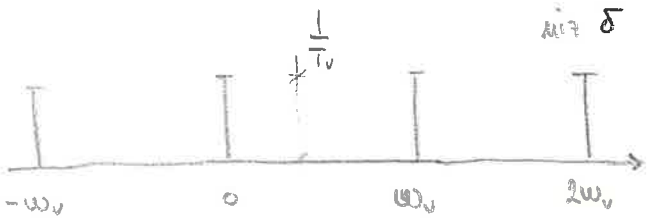
$$= \frac{1}{\Delta T_v} \frac{1}{-ik\omega} e^{-ik\omega t} \Big|_{-\Delta/2}^{\Delta/2} = \frac{i}{\Delta T_v k\omega} \left[\cos k\omega \frac{\Delta}{2} - i \sin k\omega \frac{\Delta}{2} - \cos k\omega \frac{\Delta}{2} - i \sin k\omega \frac{\Delta}{2} \right] =$$

$$= \frac{2 \sin k\omega \frac{\Delta}{2}}{T_v 2 k\omega \frac{\Delta}{2}} \Rightarrow F[\text{niz } \delta] = \lim_{\Delta \to 0} \frac{1}{T_v} \frac{\sin k\omega \Delta/2}{k\omega \Delta/2} = \frac{1}{T_v}$$

$$\Downarrow$$

$$\text{niz } \delta = \frac{1}{T_v} \sum_k e^{ik\omega t} = \frac{1}{T_v} \sum_k e^{ik2\pi \frac{t}{T_v}}$$

↑ velikost!



demo DA disp

Zato: spekter niza δ sunkov (periodičnih) je sestavljen iz niza harmonskih komponent, vse imajo velikost $\frac{1}{T_v}$

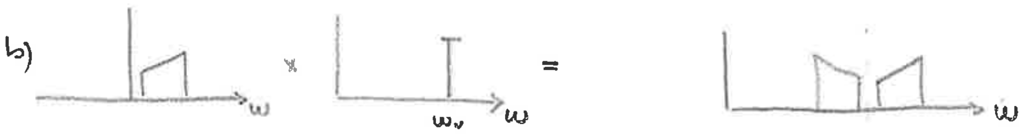
---

nazorno

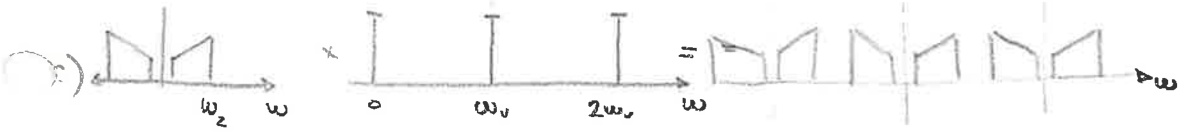
a)

$$\cos \omega_A t \times \cos \omega_v t = \frac{1}{2}\left[\cos(\omega_A+\omega_v)t + \cos(\omega_v-\omega_A)t\right]$$

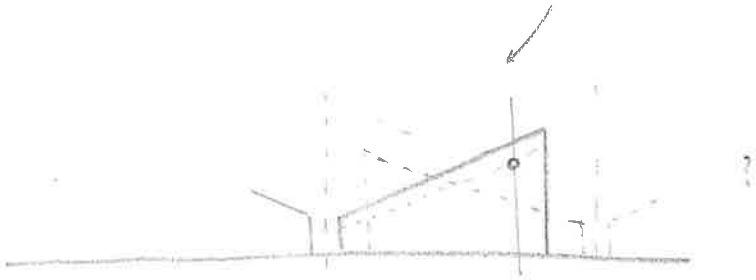
b)



c)

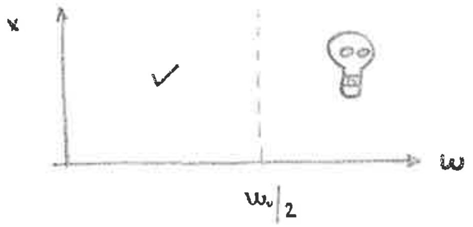


pozor! $\omega_z \le \frac{\omega_v}{2}$ če $\omega_z > \frac{\omega_v}{2} \Rightarrow$ prekrivanje spektrov!
iz spektra ne vemo kam spada označena vrednost!

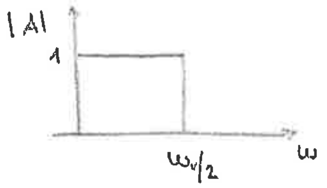


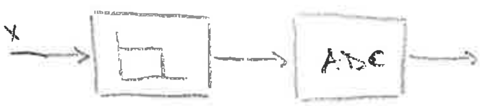
demo VB6 za spektre
demo LV za zajem

omeji obseg frekvence signala, ki ga zajemaš!



⇒ potrebujemo filter: analogni





OK!

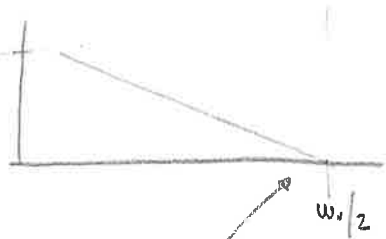
← GRRR! žal takega filtra ni!

kakšen analogni filter?

1. red?

-20dB/dek
1/10 na dekado!

} tole



$w_0/2$

zanemarljiva napaka ≡
≡ dušenje na manj od 1LSB
10BITNI ADC → 0.001
-60dB

pozabi ⇐ ! začeti je treba 3 dekade prej! ←

2. red? začeti je treba 1.5 dekade prej! ⇒ pozabi

6. red? začeti je treba pri $0.2\ w_0/2$ za Bessel
$0.5\ w_0/2$ za Čebišev 1 dB !

(pozor: Čebišev nima lin. faze!)

nujno je, da strmo filtriramo pred ($w_0/2$) ADC! FilterPro demo

---

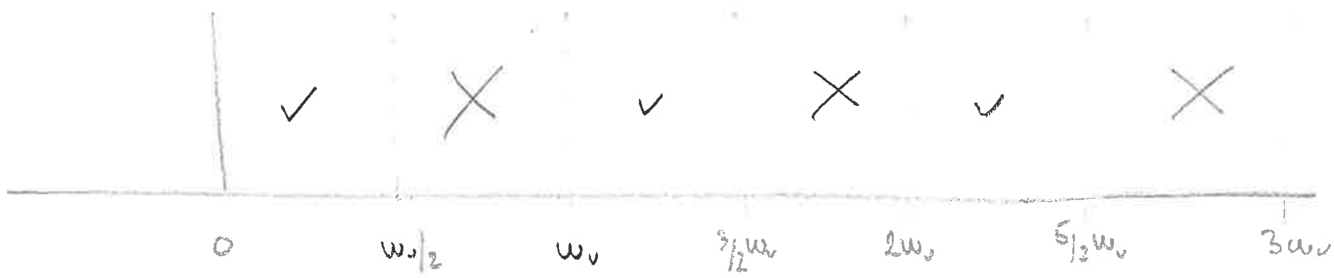
neki pojav: kaj pa, če je frekvenčni spekter omejen
na w_0 do $\frac{3}{2} w_0$?
na $10 w_0$ do $10 w_0 + \frac{1}{2} w_0$?
} gre ok!

⇓

bandpass sampling

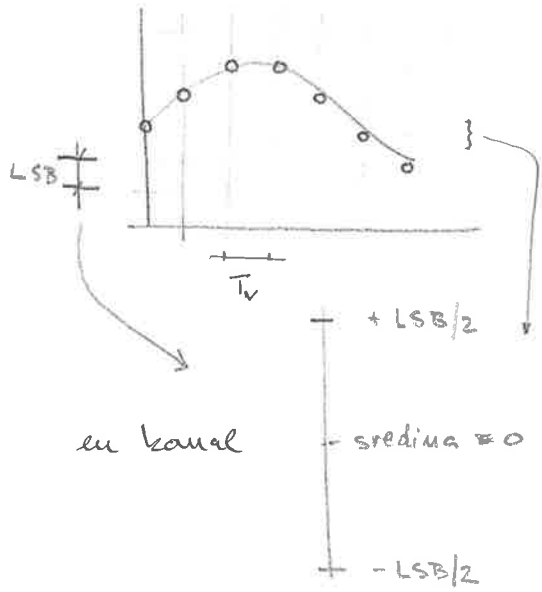
⇓

zelo koristno za sprejemnike



! samo v enem območju naenkrat
✓ → pravi vrstni red frekvenc
X → obrnjeni

Težave pri kvantizaciji ≡ šum



včasih preveč, včasih premalo!
↓
naključno

Zanima me $SNR = 10 \cdot \log \frac{U_{SRMS}^2}{U_{NRMS}^2}$ signal/noise

⇓

določi RMS signala }
določi RMS šuma } nazadnje 10 log /

a) RMS signala

$$U_{RMS}^2 = \frac{1}{2\pi}\int_0^{2\pi} A^2 \sin^2\varphi \, d\varphi = \frac{A^2}{2\pi}\int_0^{2\pi}\frac{1-\cos 2\varphi}{2}\, d\varphi =$$

signal: $A \sin\varphi$

$$= \frac{A^2}{2 \cdot 2\pi}\cdot 2\pi = \frac{A^2}{2} \Rightarrow \boxed{U_{RMS} = \frac{A}{\sqrt{2}}}$$

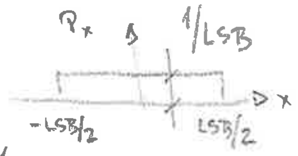
b) RMS šuma

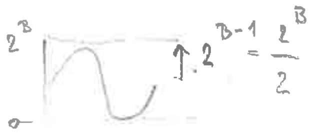
$$z^2 = \int_{-LSB/2}^{+LSB/2} (x-\bar{x})^2 \cdot P_x \, dx = \int_{-LSB/2}^{LSB/2} x^2 \cdot \frac{1}{LSB}\, dx = \frac{x^3}{3\,LSB}\Bigg|_{-LSB/2}^{LSB/2} =$$

$\bar{x} \to 0$

vse napake so enako verjetne }

$$\int_{-\infty}^{\infty} P_x \, dx = 1 \Rightarrow P_x = \frac{1}{LSB}$$





$$= \frac{1}{3\,LSB}\left[\frac{LSB^3}{8} + \frac{LSB^3}{8}\right] = \boxed{\frac{LSB^2}{12} = z^2}$$

tako: $SNR = 10 \cdot \log \frac{U_{SRMS}^2}{U_{NRMS}^2} = 10 \log \frac{A^2 \cdot 12}{2 \cdot LSB^2} = 10 \log \frac{2^{2B} \cdot 12 \cdot LSB^2}{2 \cdot LSB^2 \cdot 4} =$

B-bitni pretvornik: $2^B \times LSB \equiv$ obseg

$A = \frac{2^B \times LSB}{2}$

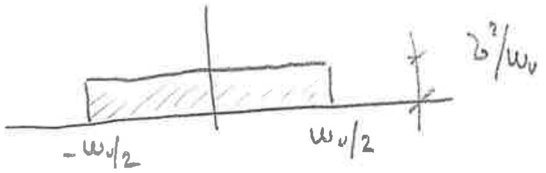
$$= 10 \log \frac{3}{2} + 10 \cdot 2B \cdot \log 2 = \boxed{1.76\,dB + 6.02\,B\,dB}$$

2011/4b

B	SNR
8	50dB
16	98dB
24	146dB

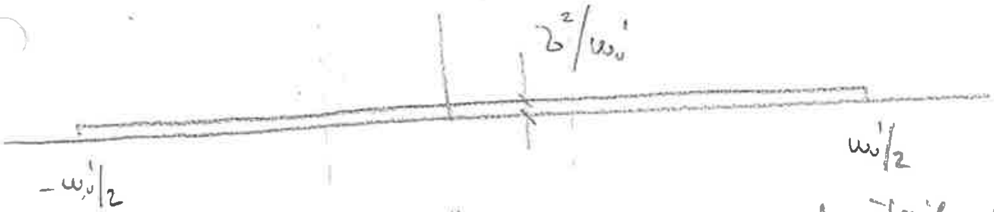
} malo bitov = majhen SNR = veliko šuma!

na srečo je šum enakomerno porazdeljen po kompletnem spektru



⇓

vzorči z veliko frekvenco → isti šum porazdeli po širšem območju!



malo število bitov, hitrejše vzorčenje

⇓

manj šuma v območju $-\omega_v/2$ do $\omega_v/2$

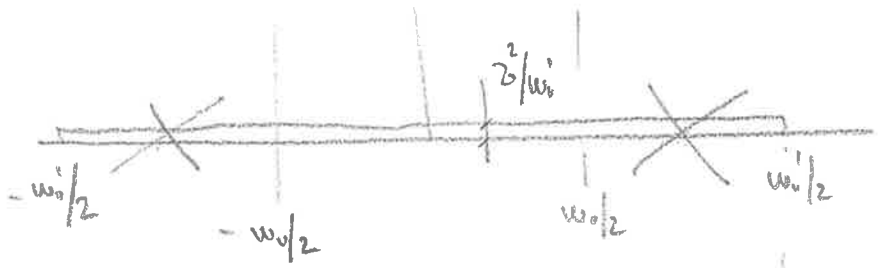
↓

torej

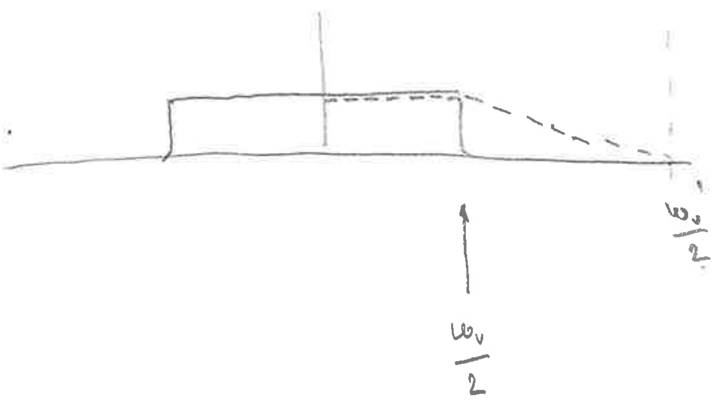
↓

vzorči hitro & digitalno filtriraj!

⇓



oversampling!



2012

oversampling: dobro!