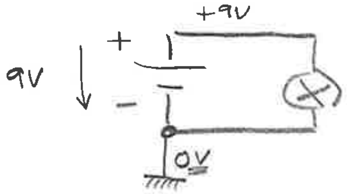

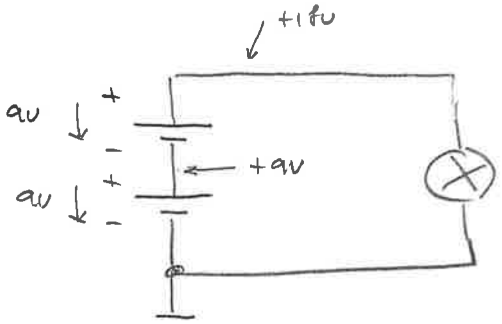

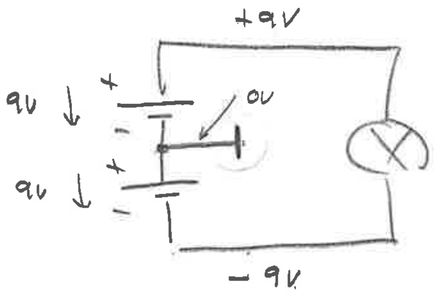

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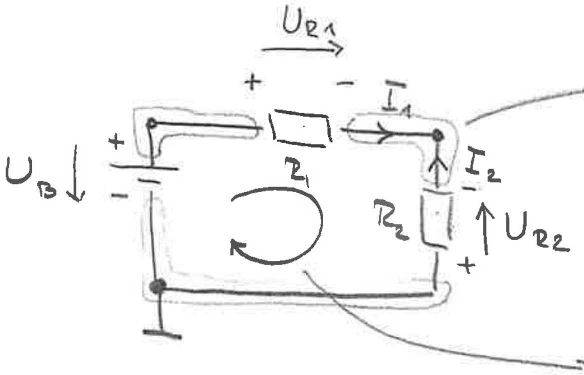
1 . 1,2 . 1,5 . 1,8 . 2,2 . 2,7 . ~~3,3~~ 3,9

4,7 6,8 8,2 E6 E12 , E24 , E48

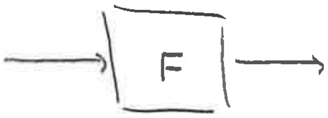
$0.1\Omega \leq R \leq 10M\Omega$



$$\sum_k I_k = 0 \Rightarrow \underline{\underline{I_1 + I_2 = 0}}$$

$$\sum_k U_k = 0 \Rightarrow \underline{+U_{R1} - U_{R2} - U_B = 0}$$

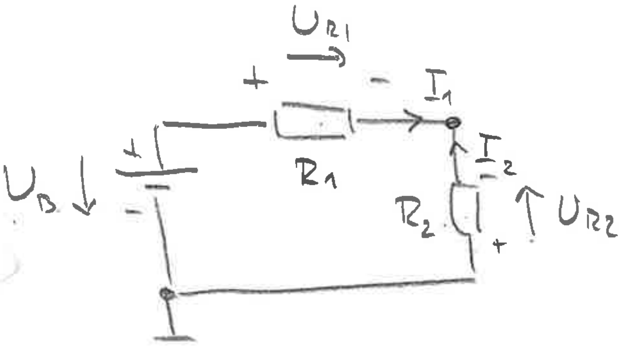
$$U_{R1} = U_B + U_{R2}$$



$$\underbrace{\begin{matrix} a \longrightarrow F(a) \\ b \longrightarrow F(b) \end{matrix}}$$

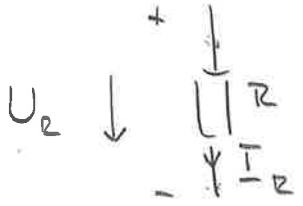
$$a+b \longrightarrow F(a) + F(b)$$

$$F(a+b)$$



$$I_1 + I_2 = 0$$

$$\frac{U_{R1}}{R_1} + \frac{U_{R2}}{R_2} = 0$$

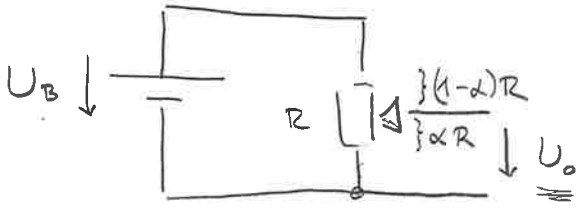


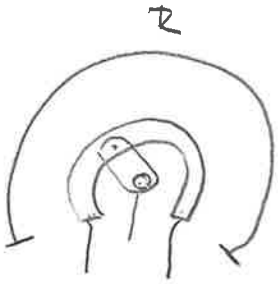
ohmov zakon: $I_R = \frac{U_R}{R}$

$$\frac{U_B + U_{R2}}{R_1} + \frac{U_{R2}}{R_2} = 0$$

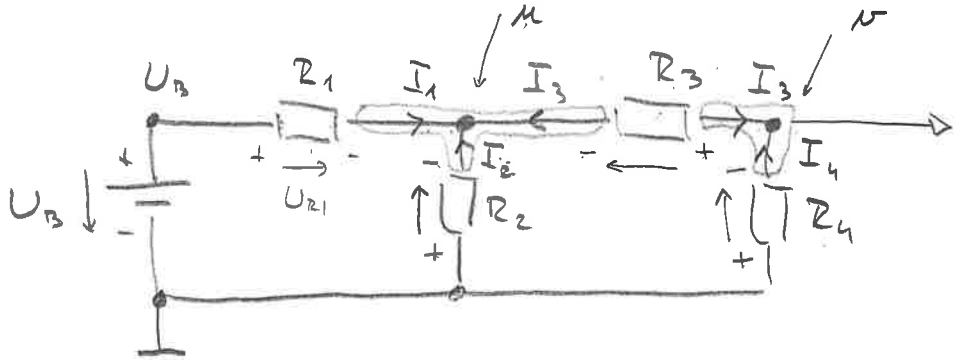
$$U_B R_2 + \underline{U_{R2}} \cdot R_2 + \underline{\underline{U_{R2}}} R_1 = 0$$

$$\underline{\underline{U_{R2} = -U_B \frac{R_2}{R_1 + R_2}}} \Bigg\} \text{ delilnik napetosti}$$





$$U_0 = U_B \frac{\alpha R}{(1-\alpha)R + \alpha R} = \underline{\underline{U_B \cdot \alpha}} \quad ; \quad 0 \le \alpha \le 1$$



$$I_1 + I_2 + I_3 = 0$$

$$\boxed{\frac{U_B - \mu}{R_1} + \frac{0 - \mu}{R_2} + \frac{\nu - \mu}{R_3} = 0}$$

$$I_3 + I_4 = 0$$

$$\boxed{\frac{\mu - \nu}{R_3} + \frac{0 - \nu}{R_4} = 0}$$

$$y_k = \sum_{m=0}^{M-1} x_{k-m} \cdot f_m$$

Pri tem velja, da je izdelani FIR filter kavzalnega značaja, saj njegov izhodni signal y_k temelji le na trenutnem in preteklih vzorcih vhodnega signala x. Izpeljava sistema za fitriranje bo enostavnejša, če konvolucijsko enačbo zapišemo v matrični obliki:

$$\boldsymbol{X_k} = \begin{bmatrix} x_k \\ x_{k-1} \\ \vdots \\ x_{k-M+1} \end{bmatrix} \qquad \boldsymbol{F} = \begin{bmatrix} f_0 \\ f_1 \\ \vdots \\ f_{M-1} \end{bmatrix} \qquad \Longrightarrow \qquad y_k = \boldsymbol{X_k^T} \cdot \boldsymbol{F} = \boldsymbol{F}^T \cdot \boldsymbol{X_k}$$

V takem zapisu napako e_k izračunamo:

$$e_k = d_k - y_k = d_k - \boldsymbol{X_k^T F} = d_k - \boldsymbol{F^T X_k}$$

Za kriterij o velikosti napake v posameznem koraku pa smo raje zapisali kvadrat zgornjega izraza. Kadar je napaka velika, želimo bolj posegati v sistem in s tem hitreje izboljševati lastnosti sistema. Zato računamo:

$${e_k}^2 = (d_k - y_k)^2 = (d_k - \boldsymbol{F^T X_k})^2 = {d_k}^2 - 2d_k\boldsymbol{F^T X_k} + (\boldsymbol{F^T X_k})^2$$

Prilagajanje je dobro, če je povprečna vrednost napake čim manjša, zato za kriterij kakovosti prilagajanja definiramo povprečno vrednost kvadrata napake (kriterijsko finkcijo J, ki je pravzaprav kvadrat efektivne vrednosti napake RMS):

$$J_k = \langle {e_k}^2 \rangle = \langle {d_k}^2 \rangle - \langle 2d_k \overset{X_k^T \cdot F}{\widetilde{\boldsymbol{F^T X_k}}} \rangle + \langle \boldsymbol{F^T X_k X_k^T F} \rangle$$

$$J_k = \langle {d_k}^2 \rangle - \langle 2d_k \boldsymbol{X_k^T} \rangle \boldsymbol{F} + \boldsymbol{F^T} \langle \boldsymbol{X_k X_k^T} \rangle \boldsymbol{F}$$

Kriterijska funkcija je torej kvadratna funkcija koeficientov filtrskega jedra, taki pa lahko analitično poiščemo minimum in na ta način ob poznavanju vseh lastnosti vhodnih signalov izberemo optimalne vrednosti filtrskih koeficientov $\boldsymbol{F}$. Gradient kriterijske funkcije je:

$$\nabla J_k = \frac{\partial J_k}{\partial \boldsymbol{F}} = \langle -2d_k \boldsymbol{X_k} \rangle + \boldsymbol{F^T} \langle \boldsymbol{X_k X_k^T} \rangle$$

Od tod izračunamo optimalne vrednosti filtrskih koeficientov $\boldsymbol{F_{OPT}}$:

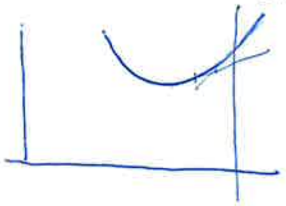
$$\langle -2d_k \boldsymbol{X_k} \rangle + \boldsymbol{F^T} \langle \boldsymbol{X_k X_k^T} \rangle = 0 \qquad \Longrightarrow \qquad \boldsymbol{F_{OPT}^T} = \langle 2d_k \boldsymbol{X_k} \rangle \ \langle \boldsymbol{X_k X_k^T} \rangle^{-1}$$

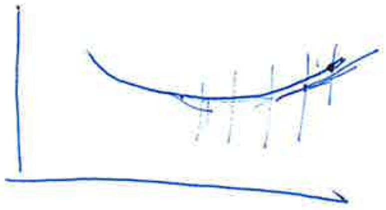
Če bi torej natančno poznali vse signale, bi optimalne vrednosti filtrskih koeficientov lahko izračunali z množenjem, transponiranjem, povprečenjem in inverzijo matrik. Take operacije bi morali izvajati dokaj pogosto, kar je računsko zelo intenzivno in zato praktično neizvedljivo. Treba bo najti drugo, enostavnejšo pot za izbiro optimalnih vrednosti koeficientov filtra.

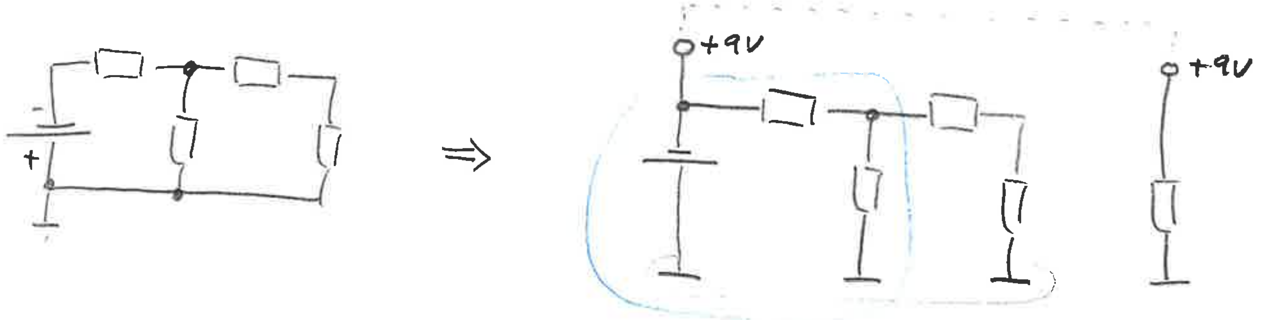
Minimum poljubne kvadratne funkcije $f(x)$ lahko iščemo tudi iterativno (po korakih). Pri tem v vsakem koraku velja:

$$x_{MIN\,k} = x_{MIN\,k-1} - \mu \cdot \frac{d\,f(x)}{dx}$$

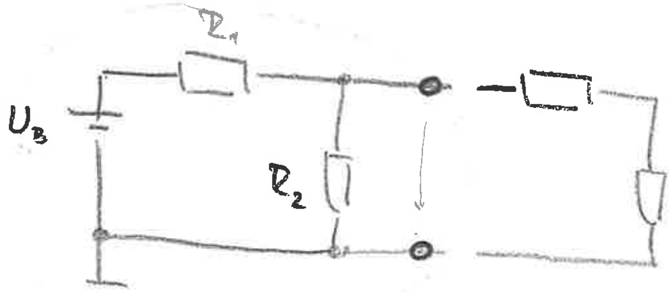
Hitrost približevanja minimumu določa velikost konstante μ (in strmina funkcije v točki preverjanja). Ta mora biti dovolj majhna zato, da minimuma funkcije ne preskočimo s predolgim korakom in hkrati dovolj velika, da je približevanje minimumu zmerno hitro. V našem primeru, ko korakoma iščemo



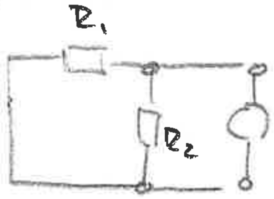




Thevenin teorem

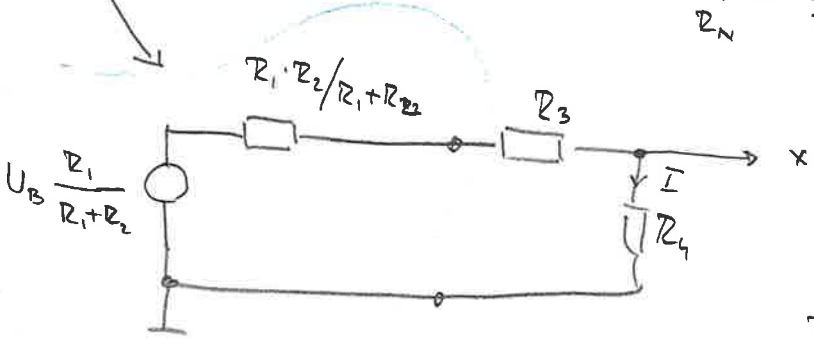


$$\Rightarrow \underline{U_N} = U_B \frac{R_2}{R_1+R_2}$$



$$\Omega \longrightarrow R_N = R_1 \parallel R_2 = \frac{R_1 \cdot R_2}{R_1+R_2}$$

$$\frac{1}{R_N} = \frac{1}{R_1} + \frac{1}{R_2}$$

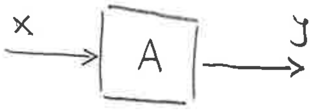


$$\underline{I} = \frac{U_B \frac{R_1}{R_1+R_2}}{\frac{R_1 \cdot R_2}{R_1+R_2} + R_3 + R_4}$$

$$x = \underline{I} \cdot R_4$$

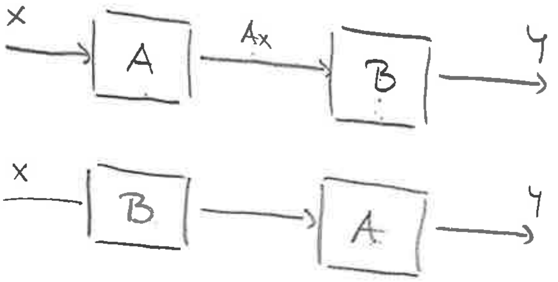
lepe lastnosti lin. vezij

–



$y = Ax$

–



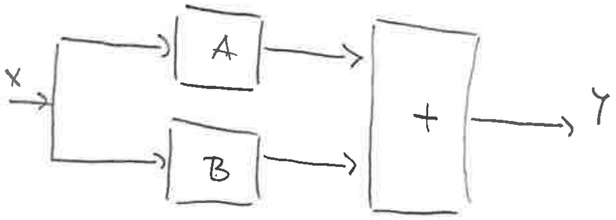
$\rightarrow y = B(Ax)$

↗ za lin vezja

↘ za nelin vezja

$y = A(Bx)$

–

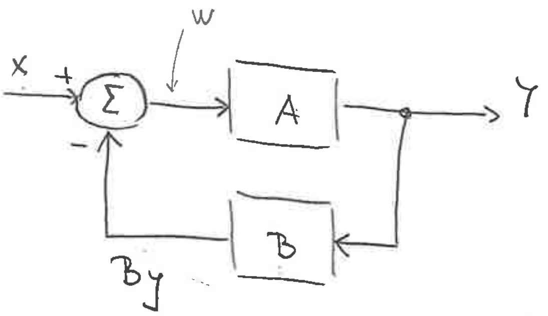


$y = Ax + Bx$

$= (A+B)x$; za lin. op.

–

–



$w = x - By$

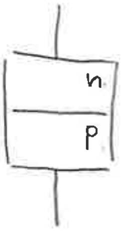
$y = Aw = Ax - ABy$

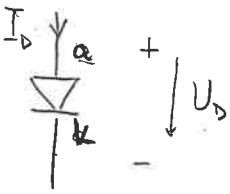
$y = x \frac{A}{1+AB}$

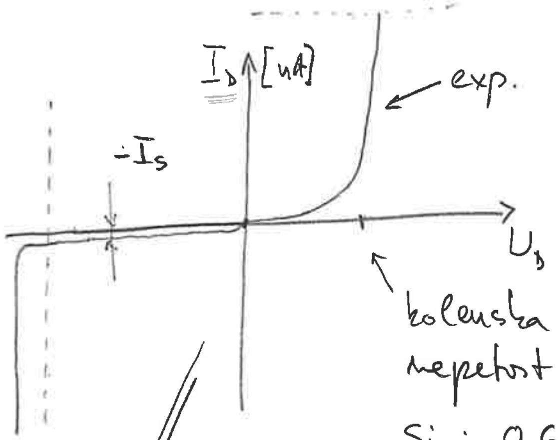
$y = x \frac{1}{\frac{1}{A}+B}$; $A \equiv$ ojačevalnik gain $\rightarrow \infty$

$y = \frac{x}{B} = xB^{-1}$

dioda







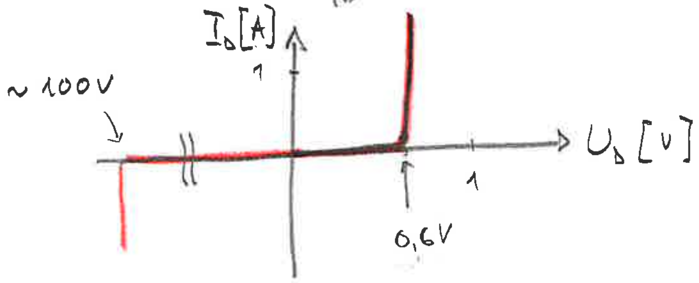
kolenska napetost

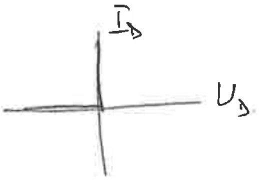
Si :	0,6V
Ge :	0,2V
shottky :	0,3V

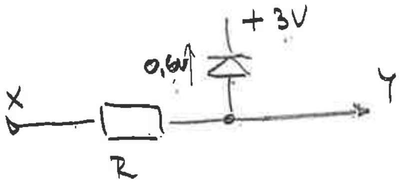
$$I_D = I_s\left(e^{U_D/U_T} - 1\right)$$

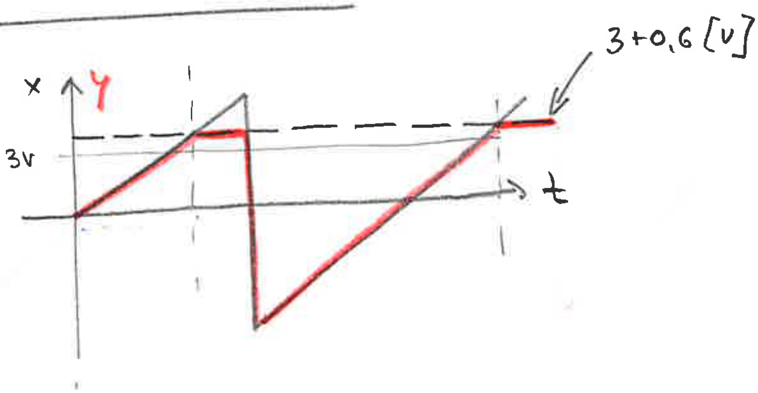
$$U_T = \text{termična napetost} = k \cdot T/e_0 \doteq 26mV \Big|_{T=20°}$$

$$I_s = \text{tok nasičenja} \doteq 10^{-10} A$$



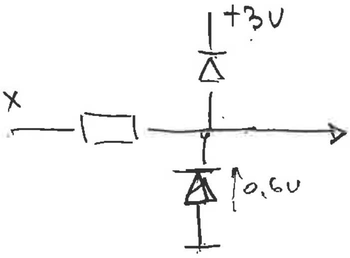






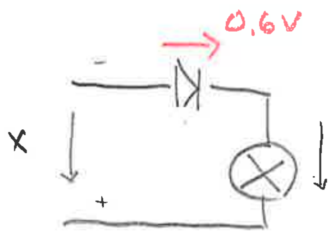
① } brez diode
$y = x$

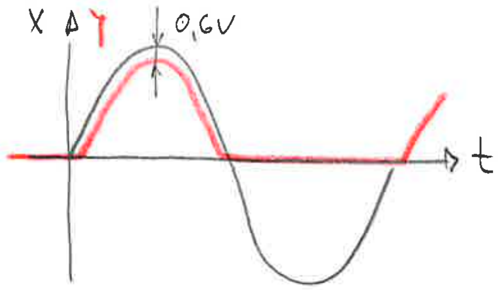
② če je $y \geq +3V + 0,6V$ ⇒ dioda prevaja
-"- $y <$ -"- ⇒ dioda ne prevaja

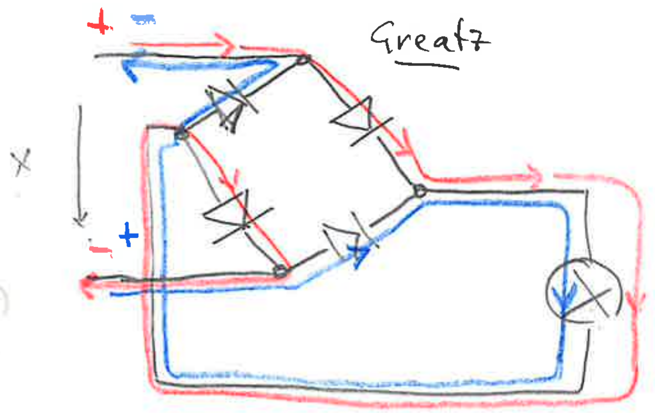


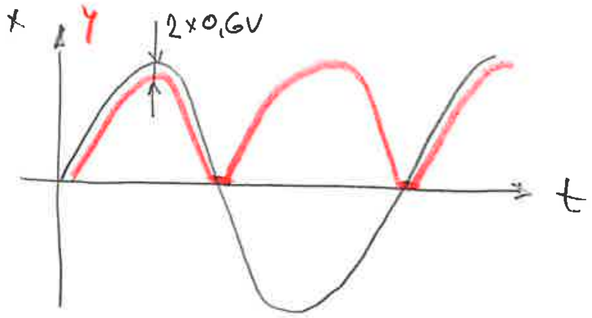
občutljivo vezje

usmerjanje

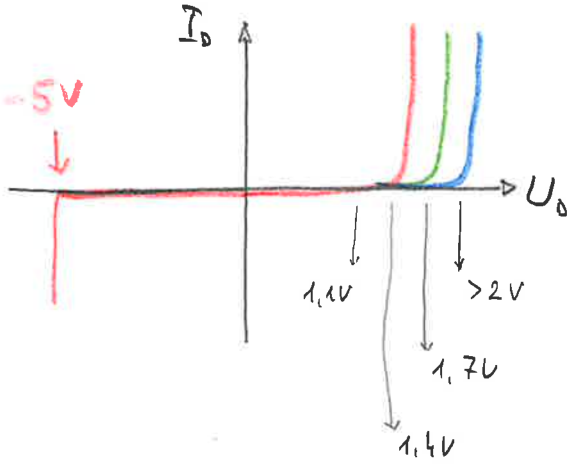


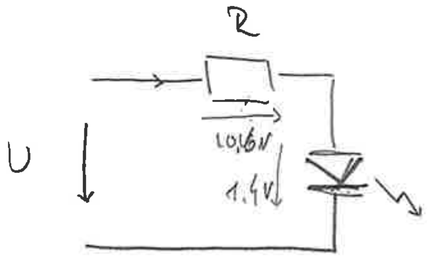






LED



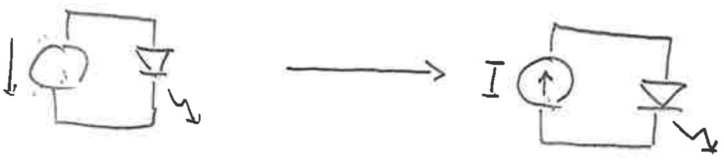


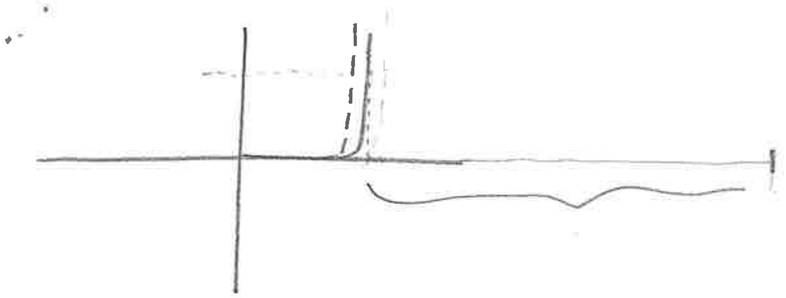
$I_{LEDTY} = 20mA = 0{,}02A$

$U = 12V$

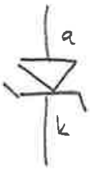
$U_R = U - U_{LED} = 12 - 1{,}4\,[V] = \underline{10{,}6\,V}$

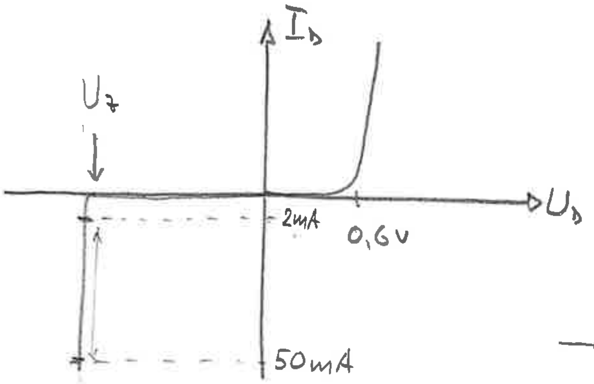
$$I_{LEDTY} = \frac{U_R}{R} \Rightarrow R = \frac{U_R}{I_{LEDTY}} = \frac{10.6}{0{,}02} \doteq 500\,\Omega$$

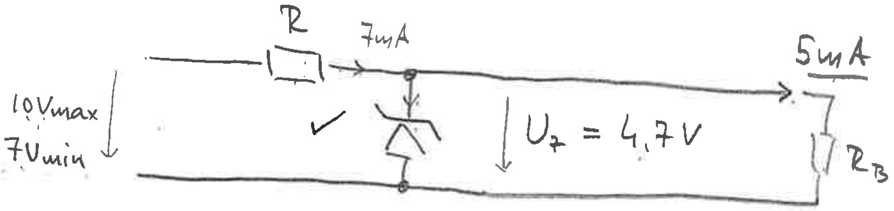




Zenerjeva dioda







a) min vh. napetost ⇒ tok skozi ZD > min. toka skozi ZD

$$U_R = 7V - 4{,}7V = 2{,}3V \Rightarrow R = \frac{2{,}3V}{0{,}007A} \doteq \underline{\underline{330\,\Omega}}$$

b) preveri tok skozi ZD ob max $U_{VHODNA} = 10V$

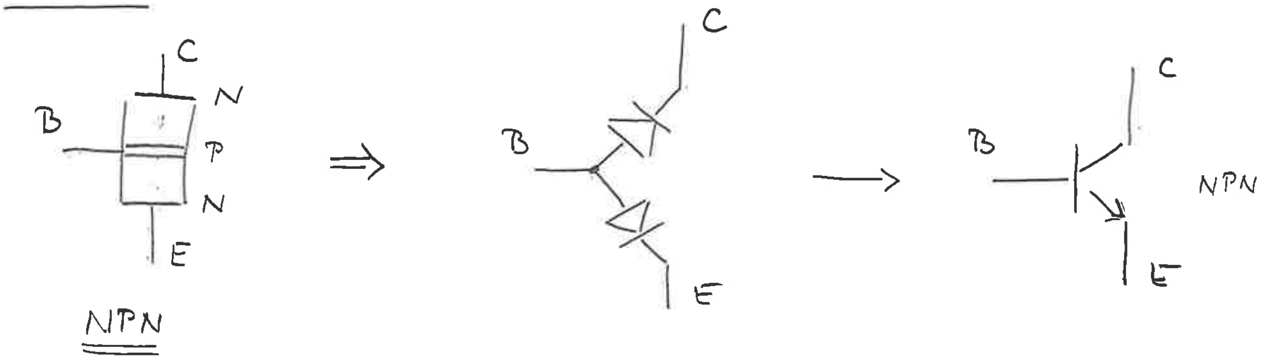
$$I_R = \frac{10V - 4{,}7V}{R} = \frac{5{,}3}{330} = 16mA$$

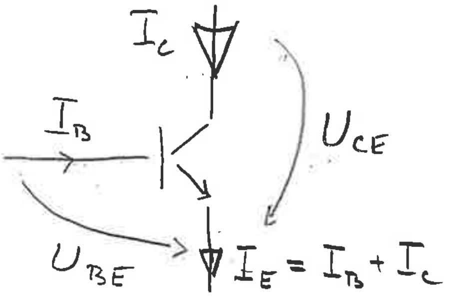
$$I_{ZD} = 16 - 5\,[mA] = \underline{\underline{11mA}} \quad ✓$$

c) moč na ZD : $P_{ZD} = U_{ZD} \cdot I_{ZDmax}$

$$= 4{,}7V \cdot 11mA \doteq \underline{\underline{52mW}}$$

tranzistor





$$I_E = I_B + I_C$$

$$I_C = \beta \cdot I_B$$

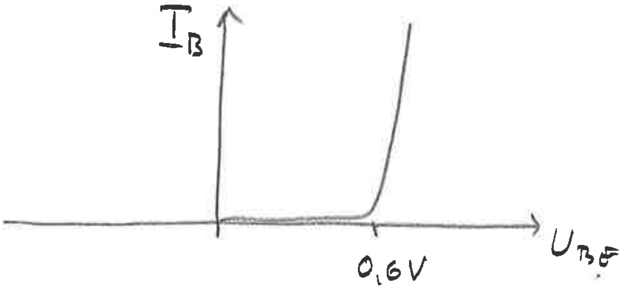
↓

~ 100

$$I_B = I_{B0} \cdot \left(e^{U_{BE}/U_T} - 1 \right)$$

$$U_T \approx 26 mV \Big|_{T = 20°C}$$

$$I_E = I_B (1 + \beta) \doteq \beta I_B$$



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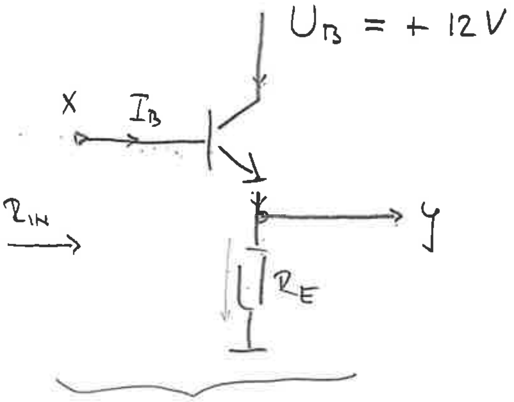
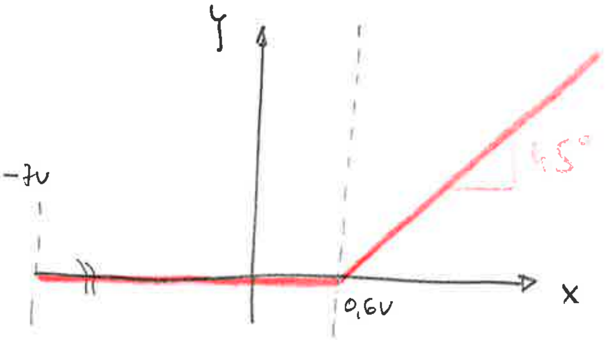
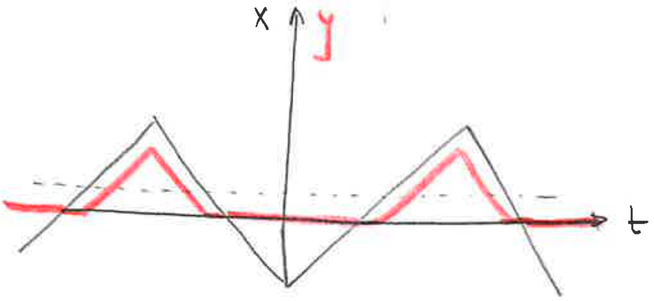
emitorski sledilnik



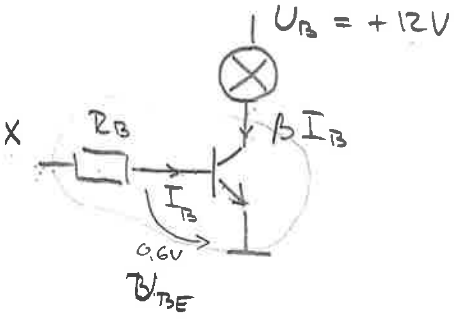


$\beta \cdot R_E \equiv$ veliko vh. upornost

$R_E \equiv$ majhno izh. upornost



tranzistor kot stikalo



$$X = \begin{cases} 0V \longrightarrow I_B = 0 \Rightarrow I_c = \beta I_B = 0 \\ +5V \longrightarrow \end{cases}$$

$$I_B = \frac{+5V - 0{,}6V}{R_B \to 440\Omega} = 10mA \Rightarrow I_c = 1A$$

izberi R_B tako, da znaša $I_B = 10mA$

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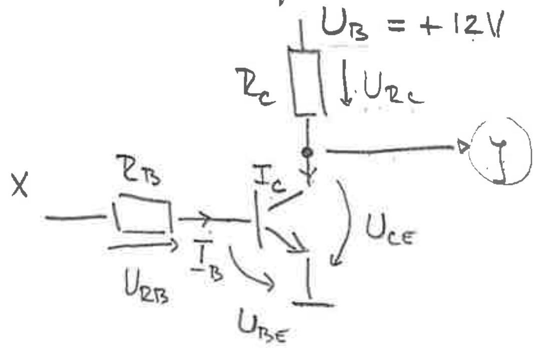
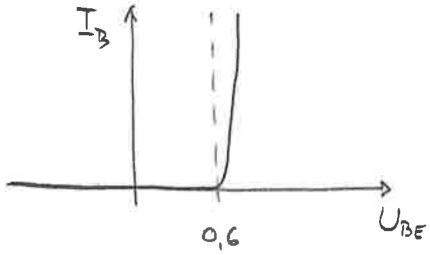
tranzistor kot ojačevalnik



$$y = U_B - U_{Rc}$$
$$= U_B - I_c \cdot R_c$$
$$= U_B - \beta I_B \cdot R_c$$



a) I_B za $U_{BE} \leq 0{,}6V$

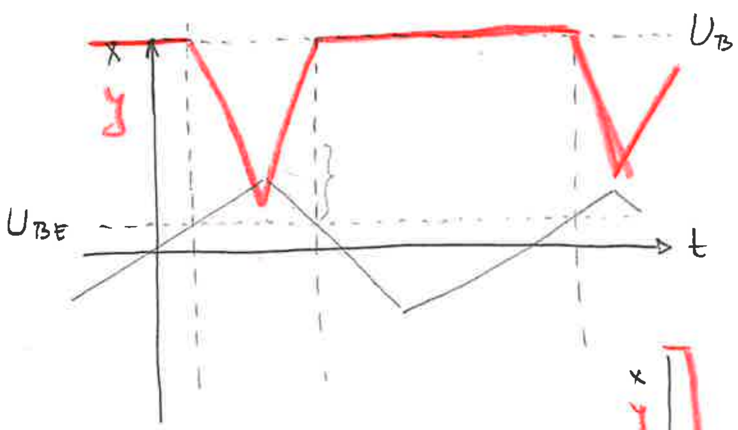
b) I_B za $U_{BE} > 0{,}6V$

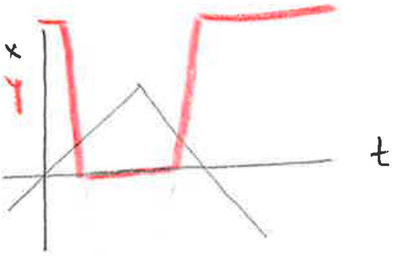
a) $I_B = 0 \to I_c = 0 \Rightarrow \underline{\underline{y = U_B}}$

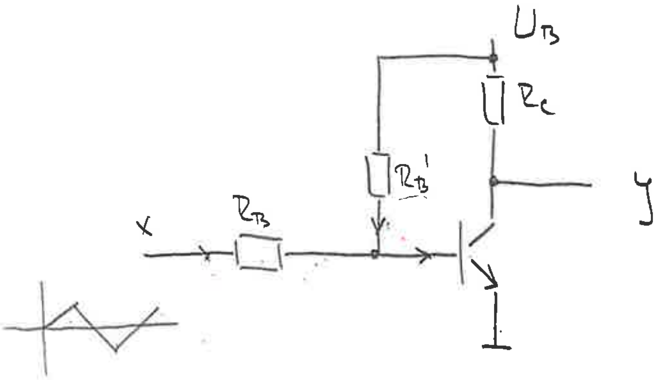
b) $I_B = I_{RB} = \frac{X - U_{BE}}{R_B} = \frac{X - 0{,}6}{R_B} \Rightarrow y = U_B - \beta \cdot \frac{X - U_{BE}}{R_B} \cdot R_c$

$$y = U_B - X \cdot \beta \frac{R_c}{R_B} + U_{BE} \cdot \beta \cdot \frac{R_c}{R_B}$$

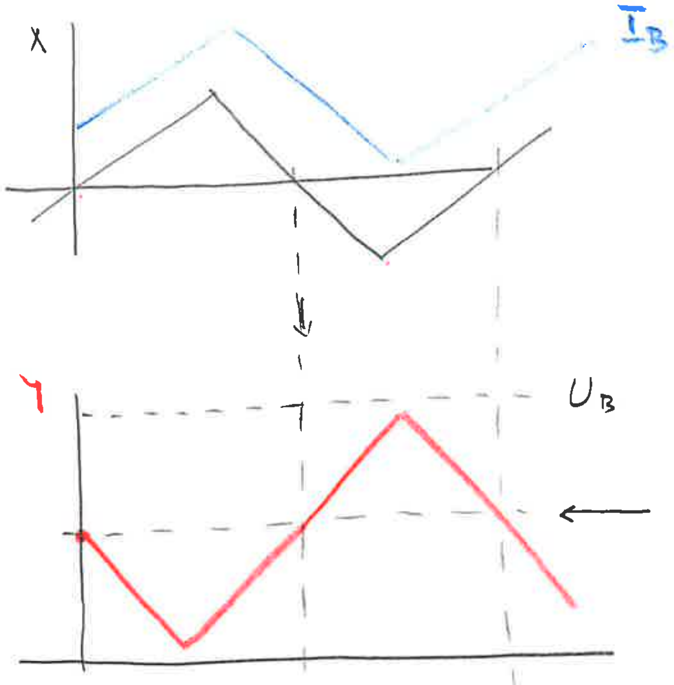
$R_c = 10k$, $R_B = 10k$







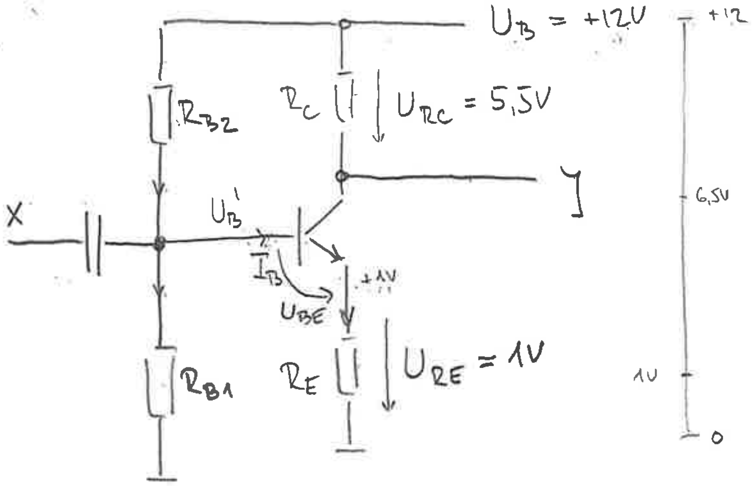
dimenzioniraj R_B' tako, da bo $I_{RB} + I_{RB'}$ ves čas > 0



← ko je $x = 0$ naj bo $y = U_B/2$

R_B' izberi tako ↗

"bias" ≡ prednapetost
predtok



① delovna točka ≡ y ko ni vh. signala

$$U_B' = U_B \cdot \frac{R_{B1}}{R_{B1}+R_{B2}} \quad \text{za } I_{RB2} \gg I_B$$

$U_B' = 1{,}6V$ tipično

$U_{BE} = 0{,}6V$

$U_{RE} = 1{,}6V - 0{,}6V = 1V$

izberi tok skozi tranzistor $I_C \doteq I_E = 1mA$

$$\Downarrow$$

$$R_E = \frac{U_{RE}}{I_E} = 1000\Omega$$

y = sredina med U_B in U_{RE}

$= 6{,}5V$

$$R_C = \frac{U_{RC}}{I_C} = \frac{5{,}5V}{1mA} = 5{,}5k\Omega$$

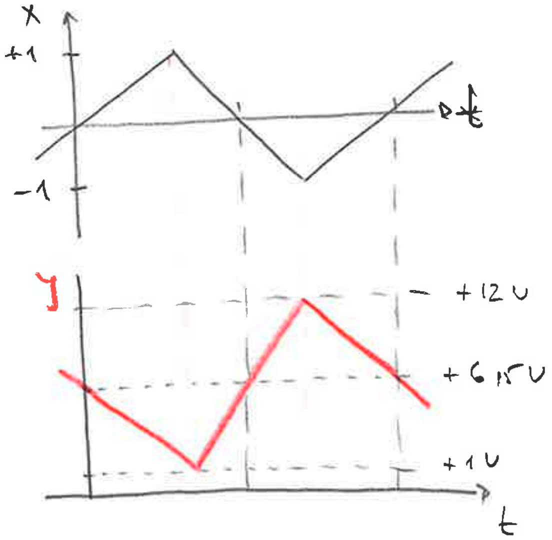
$$I_C \doteq I_E = \beta I_B \Rightarrow I_B \doteq \frac{I_C}{\beta} = \frac{1mA}{100} \doteq 10\mu A$$

izberi tok skozi $R_{B1} = 10\times 10\mu A = 10 \times I_B = 100\mu A$

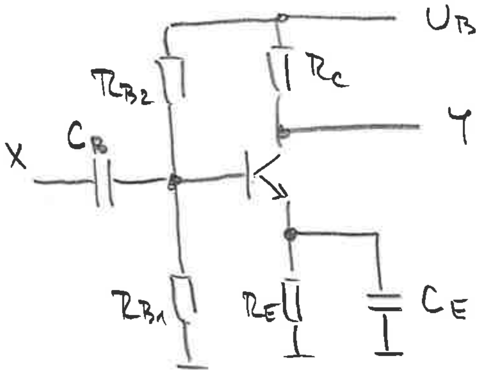
zato: $R_{B1} = \frac{U_B'}{I_{RB1}} = \frac{1{,}6V}{0.1mA} = 16k\Omega$

podobno $R_{B2} = \frac{U_{RB2}}{I_{RB2}} = \frac{U_B - U_B'}{0{,}1mA} = \frac{12-1{,}6\,[V]}{0{,}1mA} = 104k\Omega$

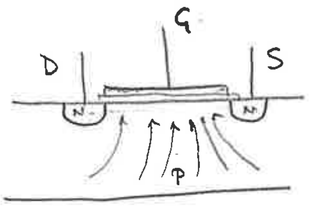
ojačanje $G \doteq \frac{R_C}{R_E} \longrightarrow G = 5{,}5$ ⇐ izpeljava na vajah

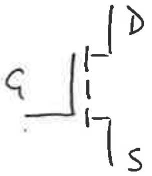


končna verzija ojačevalnika

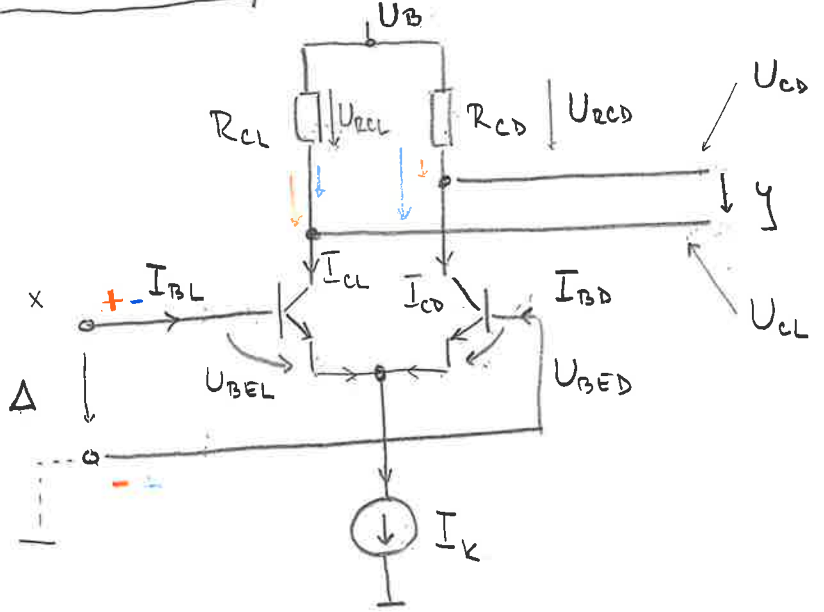


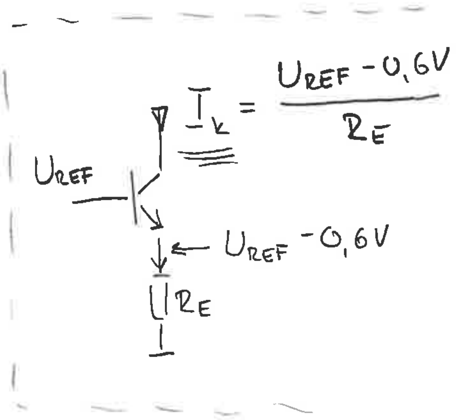
MOS





diferencialni par tr.





$$I_k = \frac{U_{REF} - 0{,}6V}{R_E}$$

① $\underline{\underline{\Delta = 0}} \Rightarrow I_{EL} = I_{ED} = I_k/2 = I_{CL} = I_{CD}$

$U_{RCL} = I_k/2 \cdot R_{CL}$

$U_{RCD} = I_k/2 \cdot R_{CD}$

$$y = U_{CD} - U_{CL}$$

$$= U_B - U_{RCL} - (U_B - U_{RCD}) = U_{RCD} - U_{RCL} = \underline{\underline{0}}$$

② $\underline{\Delta > 0} \Longrightarrow y < 0$

③ $\underline{\Delta < 0} \Longrightarrow y > 0$

} predznak!

$$I_{BL} = I_{BL0} \cdot \left(e^{U_{BEL}/U_T} - 1 \right) \qquad U_{BEL} = U_{BE0} + \frac{\Delta}{2}$$

$$I_{CL} = \beta_L \cdot I_{BL}$$

$$I_{BD} = I_{BD0} \cdot \left(e^{U_{BED}/U_T} - 1 \right) \qquad U_{BED} = U_{BE0} - \frac{\Delta}{2}$$

$$I_{CD} = \beta_D \cdot I_{BD}$$

$$\frac{I_{CL}}{I_{CD}} = \frac{\cancel{\beta_L} \cdot \cancel{I_{BL0}} \cdot \left(e^{\frac{U_{BE0} + \frac{\Delta}{2}}{U_T}} \cancel{-1} \right)}{\cancel{\beta_D} \cdot \cancel{I_{BD0}} \cdot \left(e^{\frac{U_{BE0} - \Delta/2}{U_T}} \cancel{-1} \right)} = \frac{\cancel{e^{\frac{U_{BE0}}{U_T}}} \cdot e^{\frac{\Delta}{2U_T}}}{\cancel{e^{\frac{U_{BE0}}{U_T}}} \cdot e^{-\frac{\Delta}{2U_T}}}$$

$$\underline{I_{CL} = I_{CD} \cdot e^{\Delta/U_T}}$$

$$\underline{I_{CL} + I_{CD}} \doteq I_{EL} + I_{ED} = \underline{I_k}$$

$$I_{CL} = \left(I_k - I_{CL} \right) \cdot e^{\Delta/U_T}$$

$$I_{CL} \left(1 + e^{\Delta/U_T} \right) = I_k e^{\Delta/U_T} \Rightarrow \boxed{I_{CL} = I_k \frac{e^{\Delta/U_T}}{1 + e^{\Delta/U_T}}}$$

$$I_{CD} = I_{CL} \cdot \frac{1}{e^{\Delta/U_T}}$$

$$\boxed{I_{CD} = I_k \frac{1}{1 + e^{\Delta/U_T}}}$$

$$\underline{\underline{y}} = \left(U_B - I_{CD} \cdot R_{CD} \right) - \left(U_B - I_{CL} \cdot R_{CL} \right) =$$

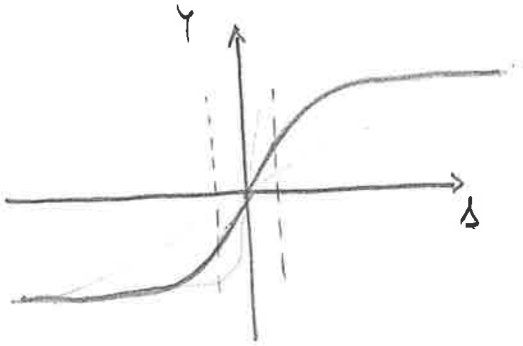
$$= \underline{\underline{R_C}} \underline{\underline{\left(I_{CL} - I_{CD} \right)}}$$

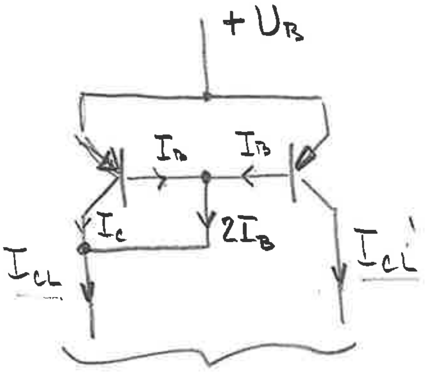
$$\downarrow$$

$$I_k \frac{e^{\Delta/U_T}}{1 + e^{\Delta/U_T}} - I_k \frac{1}{1 + e^{\Delta/U_T}} = \underline{\underline{I_k \frac{e^{\Delta/U_T} - 1}{1 + e^{\Delta/U_T}}}}$$

$$y = R_c \cdot I_k \frac{e^{s/U_T} - 1}{e^{s/U_T} + 1} = R_c \cdot I_k \frac{e^{s/2U_T} - e^{-s/2U_T}}{e^{s/2U_T} + e^{-s/2U_T}}$$

$$\equiv \underbrace{\qquad\qquad}_{\text{arc tg hyp}}$$





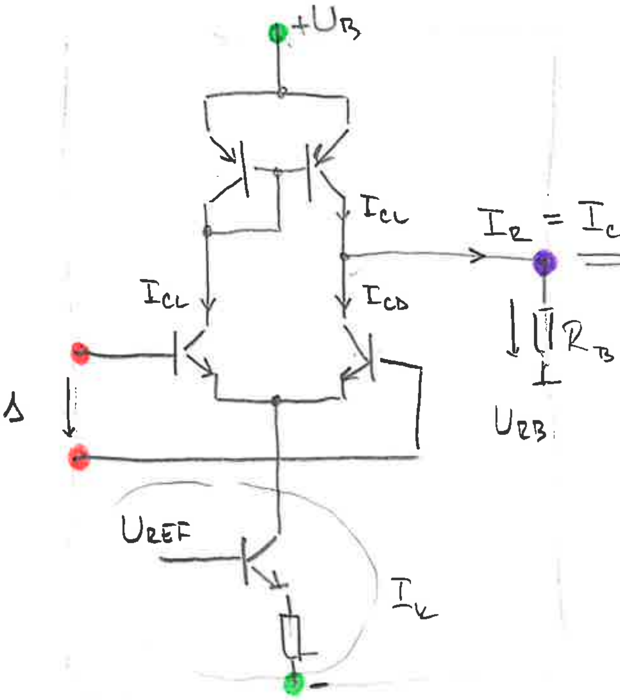
tokovno zrcalo

$$I_{CL} = 2I_B + I_c = 2I_B + \beta I_B$$

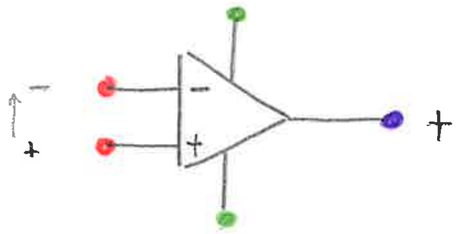
$$= I_B(\beta + \cancel{2}) \; ; \; \beta \approx 100$$

$$I_{CL} = I_B \cdot \beta$$

$$I_{CL}' = I_{CL}$$



$$\underline{\underline{I_R = I_{CL} - I_{CD}}}$$

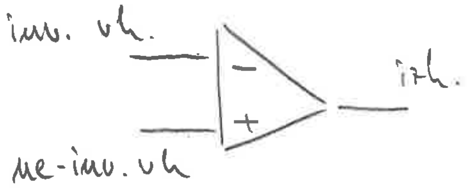


OP AMP

$$\underline{\underline{U_{RB}}} = I_R \cdot R_B = \underline{\underline{(I_{CL} - I_{CD}) \cdot R_B}}$$

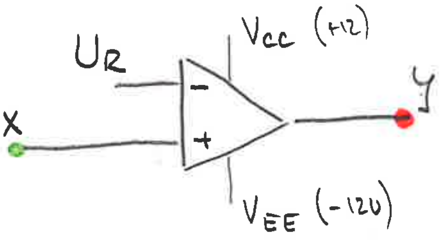
OP AMP

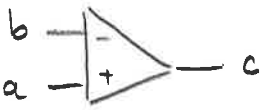
parameter	Realne vr.	Idealne vr.	
ojačanje	10^5	∞	
vhodni tok	10^{-15}A .... 10^{-4}A	0	$I_B \to I_{BIAS}$
offset nap.	10^{-6}V ... 10^{-2}V	0	U_{OFF}
max izh. nap	omejena z napajanjem	/	
max. izh. tok	omejen ~ +/- 20mA	/	I_{omax}



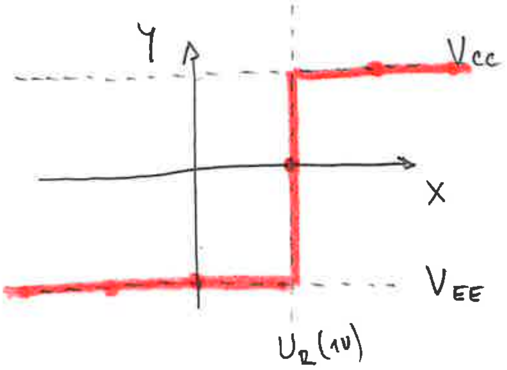
vaja 7 OPAMP

① komparator

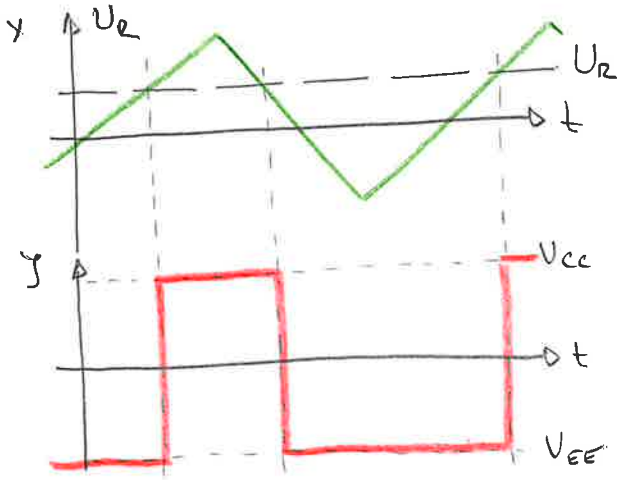


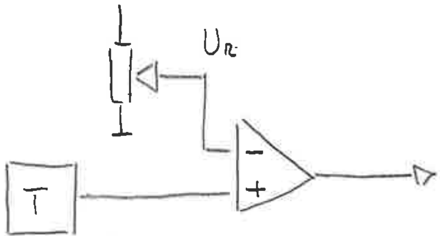


$c = G \cdot (a - b) \rightarrow \infty \cdot (a-b)$!



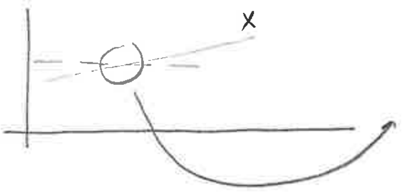
X	Y
U_R	0
$>U_R$	Vcc
$<U_R$	V_{EE}

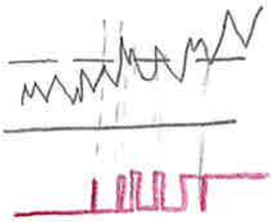


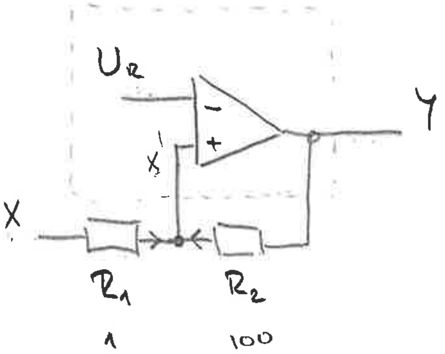


nap.
sorazmerna T

② komparator s histerezo







pozitivna
povratna vezava

$$I_{R1} + I_{R2} = 0$$

$$\frac{x-x'}{R_1} + \frac{y-x'}{R_2} = 0$$

$$x\cdot R_2 - x'R_2 + yR_1 - x'R_1 = 0$$

$$x'(R_1+R_2) = xR_2 + yR_1$$

$$x' = x\frac{R_2}{R_1+R_2} + y\frac{R_1}{R_1+R_2}$$

a) $y = V_{cc}$

$$U_R = x\frac{R_2}{R_1+R_2} + V_{cc}\frac{R_1}{R_1+R_2}$$

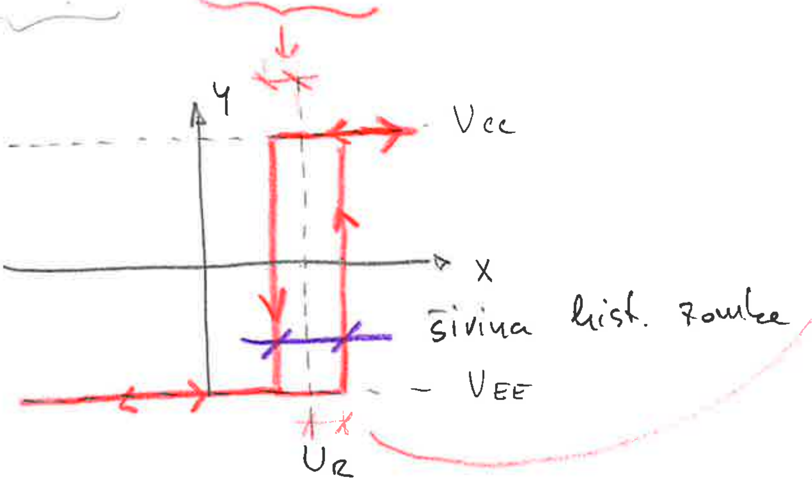
$$x = \underbrace{U_R\frac{R_1+R_2}{R_2}} - \underbrace{V_{cc}\frac{R_1}{R_2}}$$

b) $y = V_{EE}$

$$U_R = x\frac{R_2}{R_1+R_2} + V_{EE}\frac{R_1}{R_1+R_2}$$

$$x = U_R\frac{R_1+R_2}{R_2} - \underbrace{V_{EE}\frac{R_1}{R_2}}$$

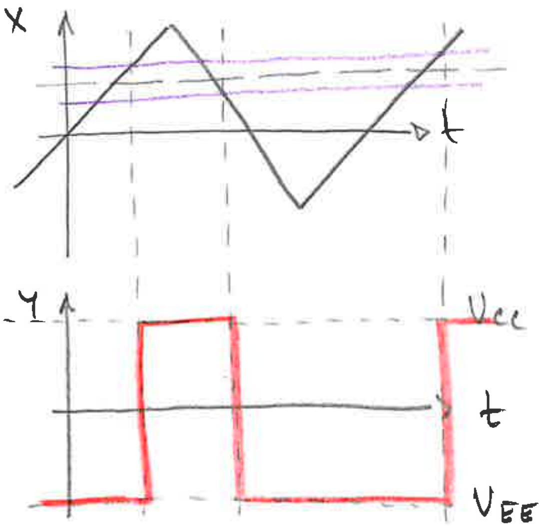
$V_{EE} = -12V$

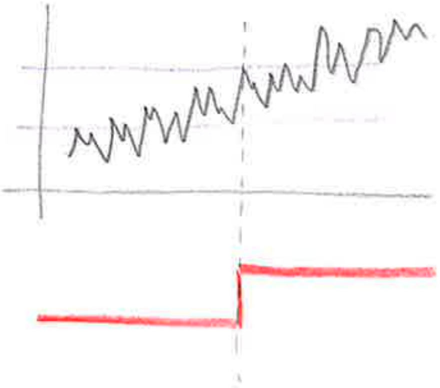


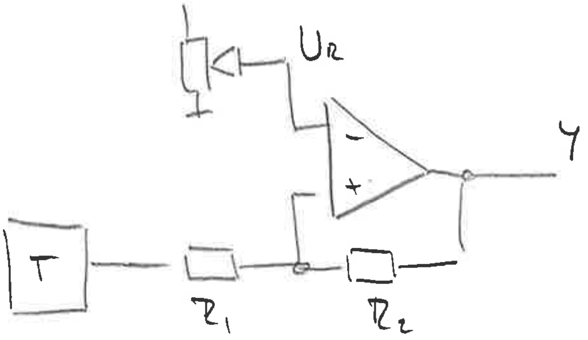
širina histerezne zanke $= 2\cdot V_{cc}\frac{R_1}{R_2} = \check{S}HZ$

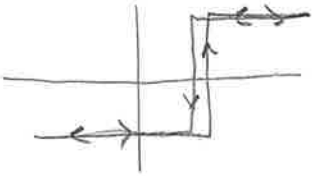
tipično: za $V_{cc} = -V_{EE} = 12V$

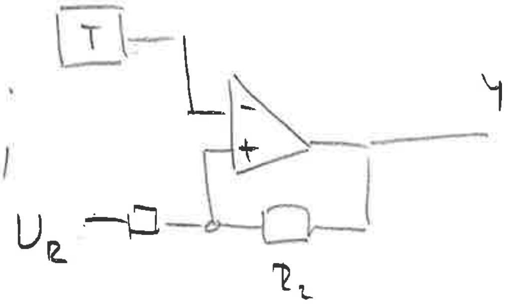
$R_1/R_2 = 1/100 \Rightarrow \check{S}HZ = 0{,}24V$

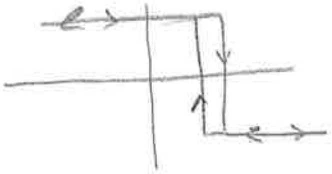

X
t
Y
Vcc
t
VEE




UR
-
+
Y
T
R1
R2




T
-
+
Y
UR
R2



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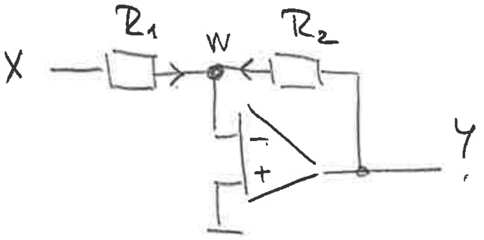
③ negativna p.v. ⟶ ojačevalnik



$$y = G(0 - w) = -G \cdot w$$

$$w = -\frac{y}{G}$$

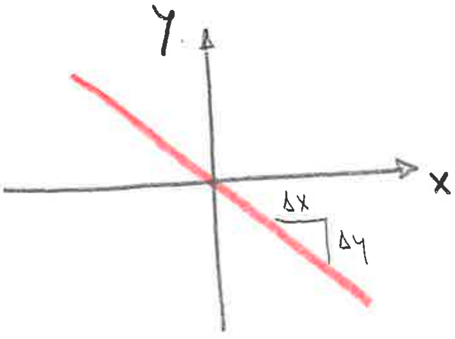
$$I_{R1} + I_{R2} = 0$$

$$\frac{x-w}{R_1} + \frac{y-w}{R_2} = 0 \Rightarrow xR_2 - \underline{\underline{w}}(R_1+R_2) + \underline{\underline{y}}R_1 = 0$$

$$xR_2 + \frac{y}{G}(R_1+R_2) + yR_1 = 0$$

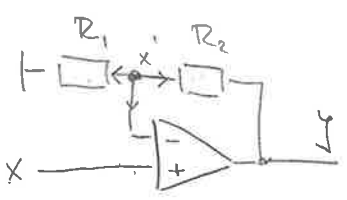
$$y\left(R_1 + \frac{R_1+R_2}{G}\right) = -xR_2$$

$$\underbrace{y = -x\frac{R_2}{R_1 + \frac{R_1+R_2}{G}}}\ ;\ G \to \infty$$



$$\boxed{y = -x\frac{R_2}{R_1}}$$

$$\frac{\Delta y}{\Delta x} \equiv \text{ojačanje} = -\frac{R_2}{R_1}$$



$$y = (x - x')\cdot G \Rightarrow \frac{y}{G} = x - x' \Big|_{G \to \infty}$$

$$\Downarrow$$

$$x = x'$$

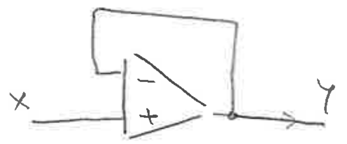
$$I_{R1} + I_{R2} = 0$$

$$\frac{x}{R_1} + \frac{x - y}{R_2} = 0 \Rightarrow x(R_1 + R_2) = yR_1$$

$$\underline{\underline{y = x\,\frac{R_1 + R_2}{R_1} = x\left(1 + \frac{R_2}{R_1}\right)}}$$

neobračajoči ojač.

- velika vh. upornost ≡ majhen vhodni tok



$\Rightarrow \underline{\underline{y = x}}$: buffer

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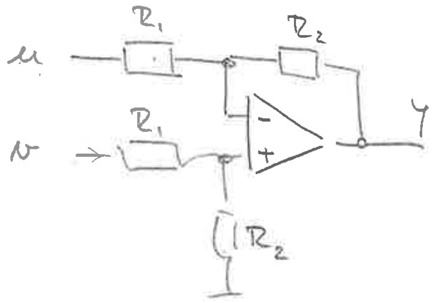
diferenčni ojačevalnik



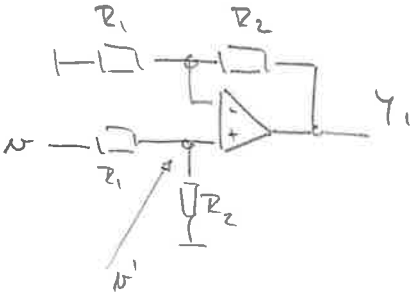
$\Rightarrow$ linearno vezje

① $u = 0,\ v \neq 0 \Rightarrow y_1$

② $u \neq 0,\ v = 0 \Rightarrow y_2$

$u \neq 0$ in $v \neq 0 \Rightarrow y = y_1 + y_2$

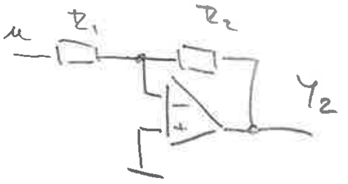
①



$$v' = v\,\frac{R_2}{R_1 + R_2} \Rightarrow y_1 = v\,\frac{R_2}{\cancel{R_1 + R_2}}\cdot\frac{\cancel{R_1 + R_2}}{R_1}$$

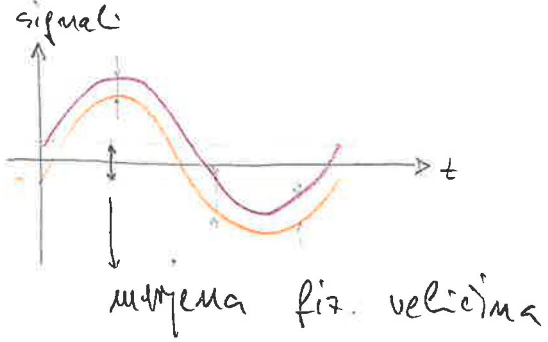
$$\underline{y_1 = v\,\frac{R_2}{R_1}}$$

②



$$\underline{y_2 = -u\,\frac{R_2}{R_1}}$$

$$\Sigma : \boxed{y = \frac{R_2}{R_1}(v - u)}$$



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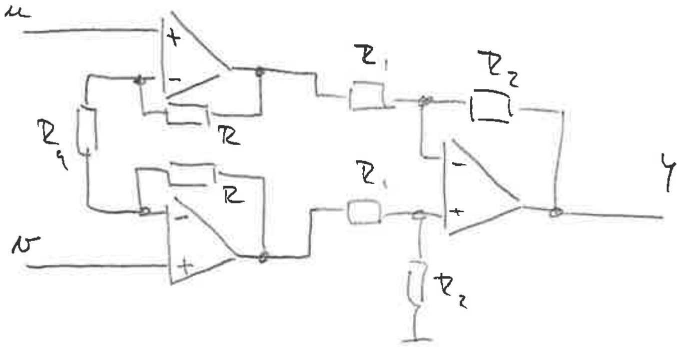
instrumentacijski ojačevalnik



$$Y = (v - u)\cdot\frac{R_2}{R_1}\cdot\frac{R}{2R_4}$$

?

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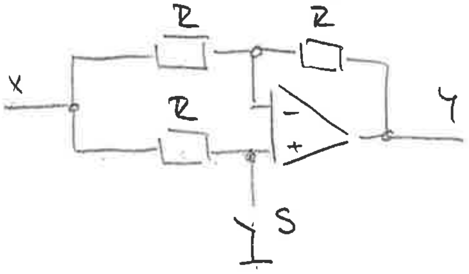
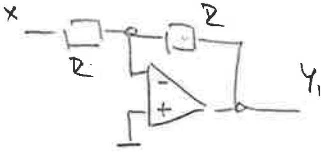
nelinearna vezja

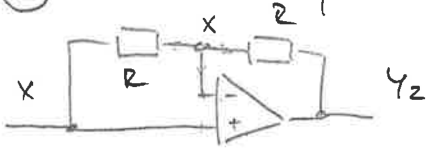


① S sklenjeno



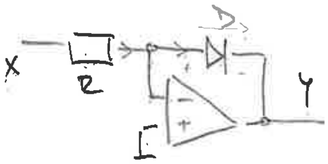
$Y_1 = -X$

② S razklenjeno



$Y_2 = X$

$$Y = \begin{cases} \text{s sklenjeno: } Y = -X \\ \text{s razklenjeno: } Y = X \end{cases}$$



① D prevaja ⟶ za $X > 0 : Y < 0$

② D ne prevaja → za $X < 0 : Y = +U_{cc}$

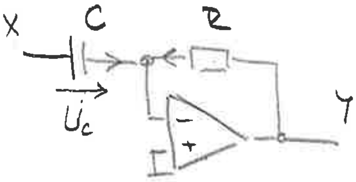
① $I_R = I_D = \frac{X}{R} = I_{D0}\cdot\left(e^{U_D/U_T}\cancel{-1}\right) = I_{D0}\cdot e^{-Y/U_T}$

$$\boxed{Y = -U_T \ln\frac{X}{R\cdot I_{D0}}}$$

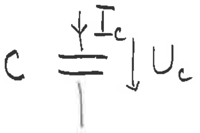
$Y = F(x)$ za karakterizacijo vezja me zanima le F

→ prenosna funkcija F

$$\boxed{F = \frac{Y}{X}}$$



diferenciator



$$I_c = C\frac{dU_c}{dt}$$

$$I_c dt = C dU_c \Big/ \int$$

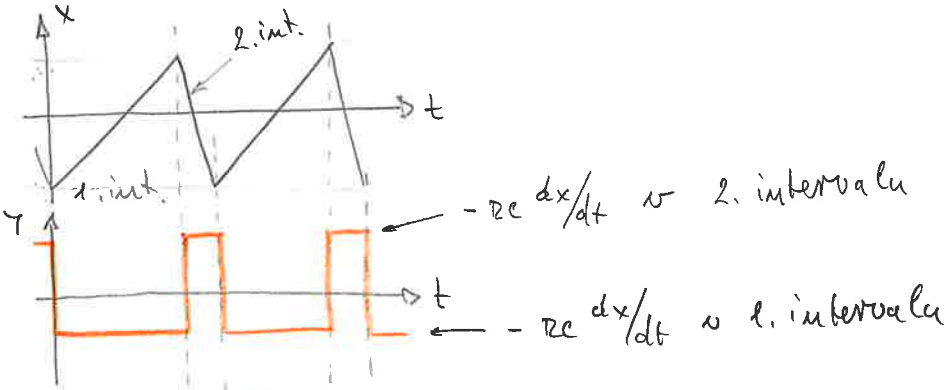
$$\int I_c dt = C U_c$$

$$U_c = \frac{1}{C}\int_0^T I_c dt$$

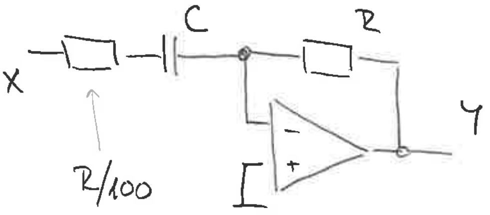
$$I_c + I_R = 0$$

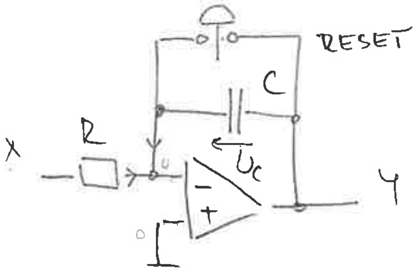
$$C\frac{dU_c}{dt} + \frac{Y}{R} = 0$$

$$\boxed{Y = -RC\frac{dx}{dt}} \quad ; \quad RC = \tau$$



$$\boxed{Y = -RC\dot{x}}$$





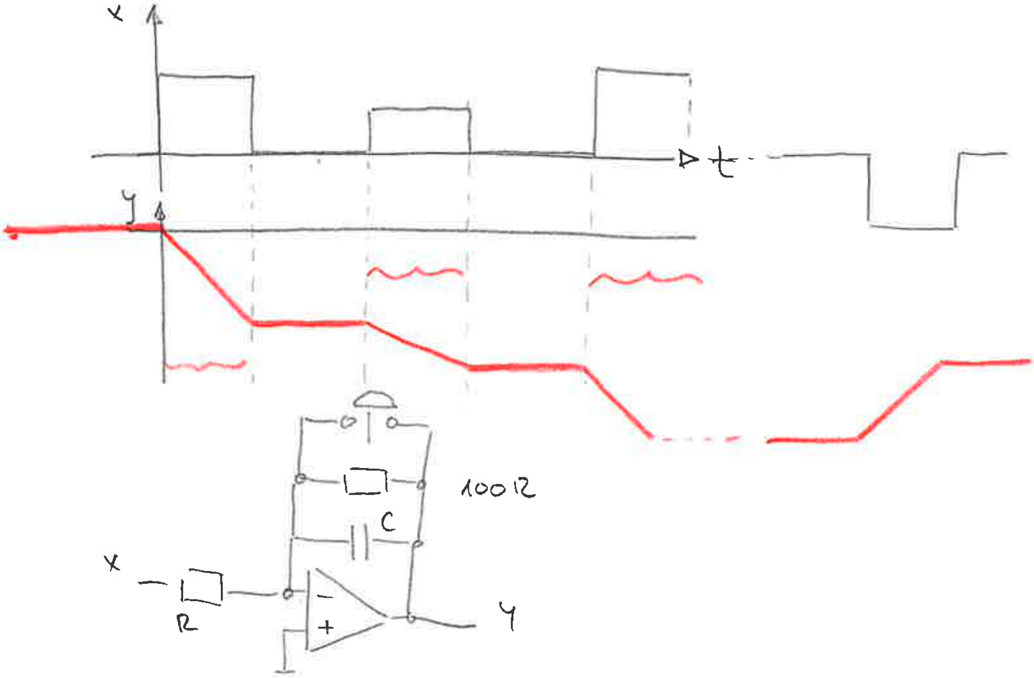
integrator

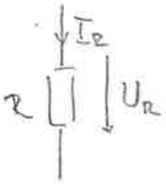
$$I_R + I_c = 0$$

$$\frac{x}{R} + C\frac{dU_c}{dt} = 0$$

$$\frac{x}{R} + C\frac{dy}{dt} = 0 \Rightarrow dy = -\frac{1}{RC}\,x\,dt \;/\!/\int$$

$$y = -\frac{1}{RC}\int_0^t x\,dt + \text{konst.}$$

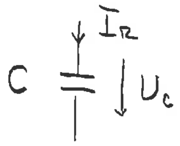




$$\boxed{I_R = \frac{U_R}{R}}$$

↑

upornost R



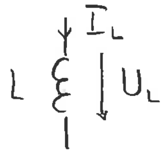
$$I_c = C\frac{dU_c}{dt} \quad \boxed{\frac{d}{dt} = p}$$

$$I_c = CpU_c$$

$$\boxed{I_c = \frac{U_c}{\frac{1}{Cp}}}$$

↑

upornost $\frac{1}{Cp}$

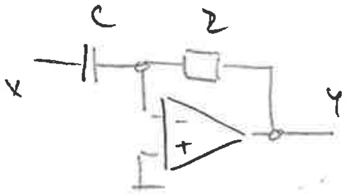


$$U_L = L\cdot\frac{dI_L}{dt}$$

$$U_L = L\cdot pI_L$$

$$\boxed{I_L = \frac{U_L}{Lp}}$$

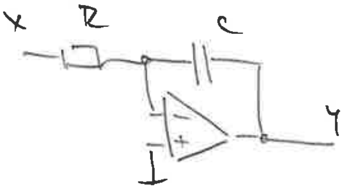
upornost Lp



$$I_c + I_R = 0$$

$$\frac{X}{\frac{1}{Cp}} + \frac{Y}{R} = 0 \Rightarrow \underline{\underline{Y = -XRCp}}$$

$$\frac{Y}{X} = -RCp = T(p)$$

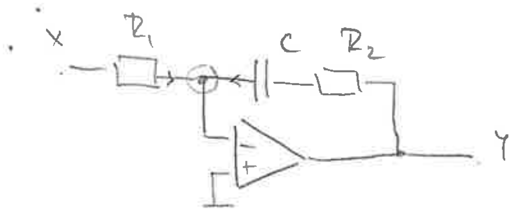


$$I_c + I_R = 0$$

$$\frac{Y}{\frac{1}{Cp}} + \frac{X}{R} = 0 \Rightarrow \underline{\underline{Y = -\frac{1}{RCp}X}} = -\frac{1}{RC}p^{-1}X$$

$$\frac{Y}{X} = -\frac{1}{RCp} = T(p)$$

ob 13^{00}

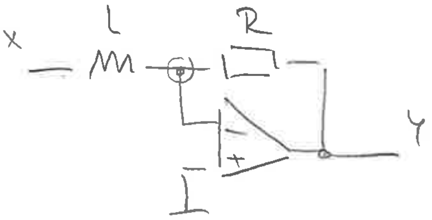


$$I_{R1} + I_{R2C} = 0$$

$$\frac{X}{R_1} + \frac{Y}{R_2 + \frac{1}{Cp}} = 0$$

$$\frac{X}{R_1} + \frac{YCp}{1+R_2Cp} = 0$$

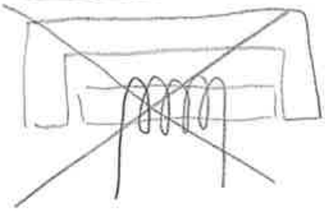
$$Y = -X\,\frac{1+R_2Cp}{R_1Cp} \Rightarrow T(p) = \frac{Y}{X} = -\left[\underset{\substack{\uparrow\\ \text{integral}}}{\frac{1}{R_1Cp}} + \underset{\substack{\uparrow\\ \text{ojačanje}}}{\frac{R_2}{R_1}}\right]$$

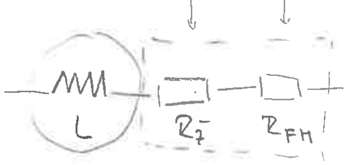


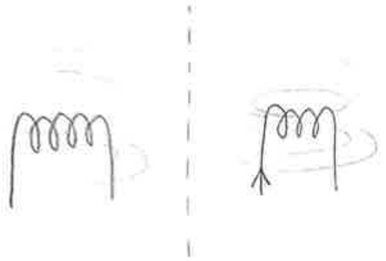
$$I_L + I_R = 0$$

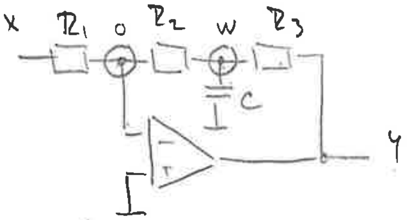
$$\frac{X}{Lp} + \frac{Y}{R} = 0 \Rightarrow Y = -X\,\frac{R}{Lp}$$

$$\underline{T(p) = -\frac{R}{Lp}} = \frac{Y}{X}$$







- 

$$I_{R1} + I_{R2} = 0 \qquad I_{R2} + I_{R3} + I_c = 0$$

$$\underbrace{\frac{x}{R_1} + \frac{w}{R_2} = 0 \qquad \frac{w}{R_2} + \frac{w-y}{R_3} + \frac{w}{\frac{1}{Cp}} = 0}$$

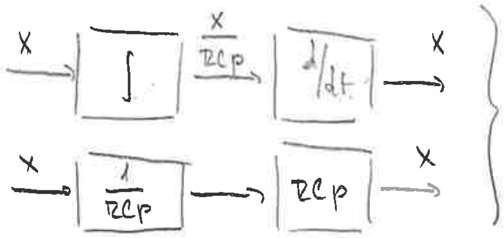
$$T(p) = -\left[\frac{R_3}{R_1} + \frac{R_2}{R_1} + \frac{R_2 R_3}{R_1} Cp\right]$$

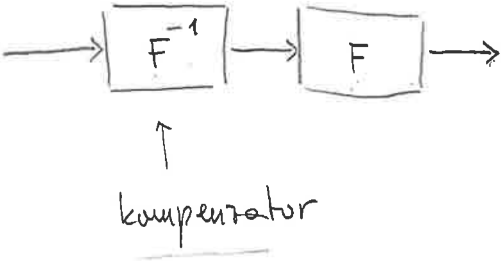
---

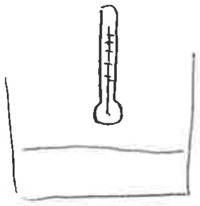
p : zraven <u>vedno</u> RC

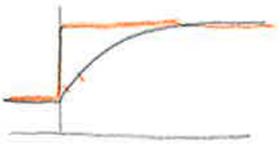
p^2 : — " — R^2C^2

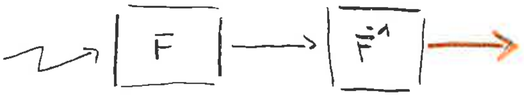
stopnja p = število kondenzatorjev

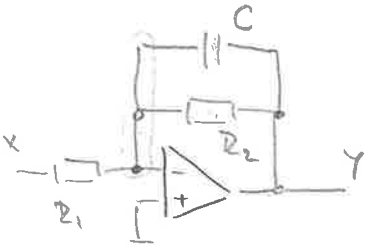










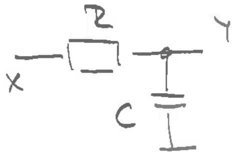


$$I_{R1} + I_{R2} + I_c = 0$$

$$\frac{X}{R_1} + \frac{Y}{R_2} + \frac{Y}{\frac{1}{Cp}} = 0$$

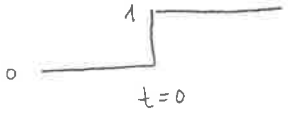
$$xR_2 + yR_1 + yCpR_1R_2 = 0$$

$$yR_1(1 + R_2Cp) = -xR_2 \Rightarrow \underline{\underline{\frac{Y}{X} = T(p) = -\frac{R_2}{R_1}\frac{1}{1+R_2Cp}}}$$

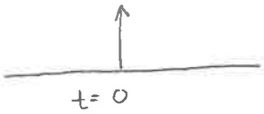


$$y = x\frac{\frac{1}{Cp}}{R + \frac{1}{Cp}} = x\frac{1}{1+RCp}$$

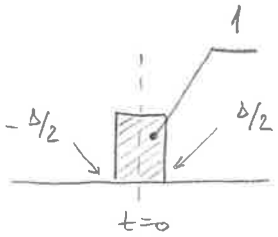
$$\underline{\underline{T(p) = \frac{Y}{X} = \frac{1}{1+RCp}}}$$

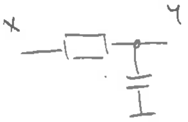


nap. <u>stopnica</u> $u(t)$



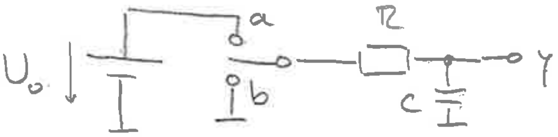
nap. <u>sunek</u> $\delta(t)$ $\Rightarrow$





$$T(p) = \frac{1}{1+\tau p} = \frac{Y}{X} \Rightarrow x = y + \tau p y$$

$$x = y + \tau y' = y + \tau\frac{dy}{dt}$$



$t<0$: stikalo v položaju a
$t=0$: stikalo preklopimo
$t>0$: stikalo v položaju b

$t<0$: $y = U_0$ $\equiv$ stacionarno stanje

$t=0$: preklopimo : $\underline{y = U_0}$

$t > 0$ $\qquad \underbrace{y + \tau \frac{dy}{dt}} = x = \underbrace{0}$

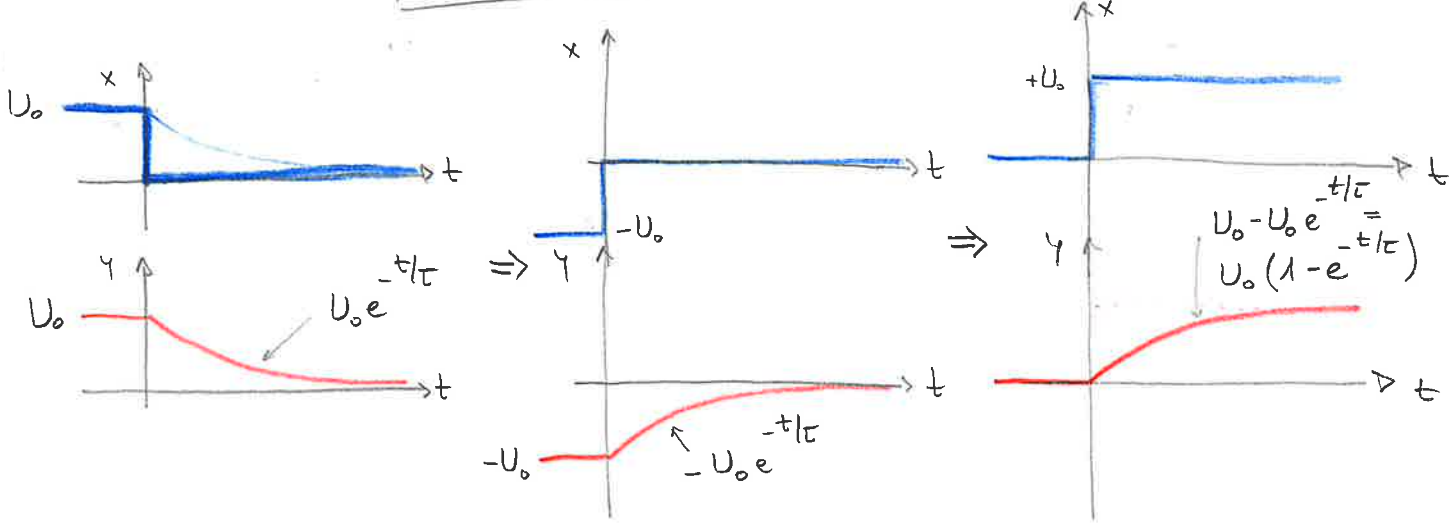
$$\frac{dy}{y} = -\frac{dt}{\tau} \quad \Big| \int$$

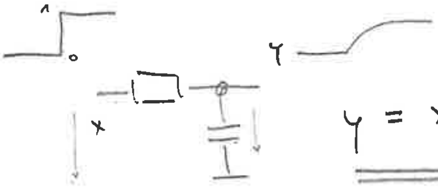
$$\ln y = -\frac{t}{\tau} + \ln k \quad \Big| \exp$$

$$y = k \cdot e^{-t/\tau} \; * \quad ; \quad y(t=0) = \underline{U_0} = \underline{k}\, \underbrace{e^{-0/\tau}}_{1}$$

$$\Downarrow$$

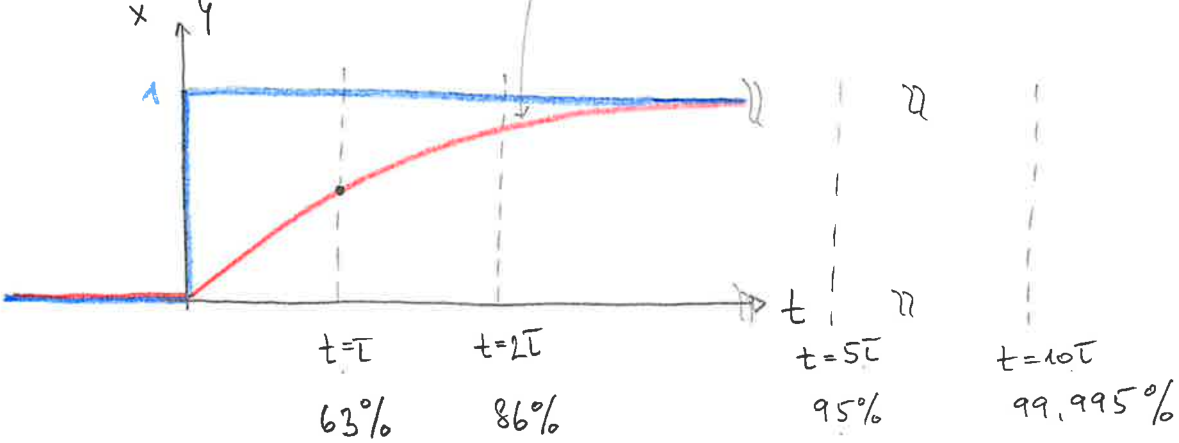
$$\boxed{y = U_0 \cdot e^{-t/\tau}}$$





$$\underline{\underline{y = x \cdot \left(1 - e^{-t/RC}\right)}}$$

$t = \tau \Rightarrow y = x \cdot (1 - e^{-1})$



raba RC člena $\left(\frac{1}{1+\tau p} \right)$

- povprečevalnik : $\tau = RC \gg$ periode opazovanega vhodnega signala

$$\underline{\underline{\tau \geq 20\, T_p}}$$

- $\tau \approx T_p$: exponentno približevanje končni vrednosti

- $y \ll x$: približni integrator

$$T(p) = \frac{1}{1+\tau p} = \frac{Y}{X} \quad \Rightarrow \quad y + \tau \frac{dy}{dt} = x$$

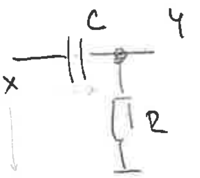
$$\frac{dy}{dt} = (x-y)\frac{1}{\tau}$$

$$dy = \frac{1}{\tau}(x-y)\,dt \;/ \int$$

$$y = \frac{1}{\tau}\int (x-y)\,dt$$

(pod pogojem, da je $\underline{y \ll x}$

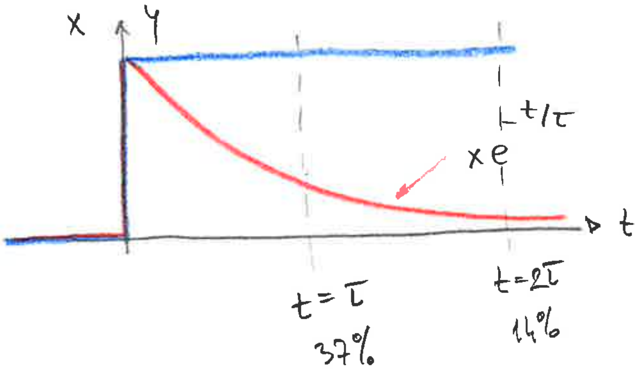
$$\underline{\underline{y = \frac{1}{\tau}\int x\,dt}}$$



$$y = x \frac{R}{R + \frac{1}{Cp}} = \frac{\tau p}{1+\tau p} x \Rightarrow T(p) = \frac{\tau p}{1+\tau p}$$

$$y = x - \left(x\left(1 - e^{-t/\tau}\right)\right)$$

$$= \cancel{x} - \cancel{x} + x e^{-t/\tau} = x e^{-t/\tau}$$

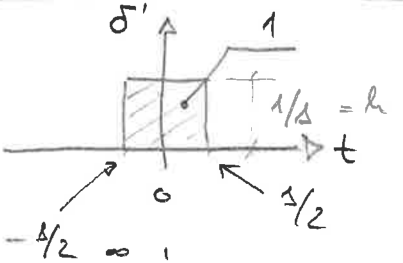


$$T(p) = \frac{y}{x} = \frac{1}{1+\tau p}$$

$$\equiv$$

$$y + \tau \frac{dy}{dt} = x$$

vzb : δ



$$\int_{-\infty}^{\infty} \delta' dt = 1 \Rightarrow h = \frac{1}{\Delta}$$

a) vpliv vzbujanja od $-\Delta/2$ do $\Delta/2$

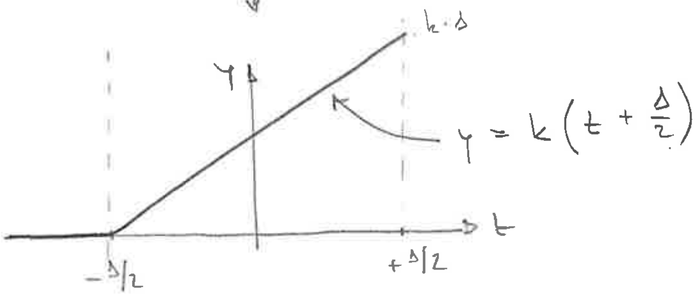
b) po vzbujanju : izzvenevanje

a)

a) $$y + \tau \frac{dy}{dt} = \delta' \Big/ \int_{-\Delta/2}^{\Delta/2} dt$$

$$\underbrace{\int_{-\Delta/2}^{\Delta/2} y\,dt} + \underbrace{\tau \int_{-\Delta/2}^{\Delta/2} \frac{dy}{\cancel{dt}} \cdot \cancel{dt}} = \underbrace{\int_{-\Delta/2}^{\Delta/2} \delta' dt}_{\downarrow \; 1}$$

$$\tau \int_{-\Delta/2}^{\Delta/2} dy = \tau \left(y\left(+\tfrac{\Delta}{2}\right) - \underbrace{y\left(-\tfrac{\Delta}{2}\right)}_{=0} \right) = \underline{\underline{\tau \, y\left(\tfrac{\Delta}{2}\right)}}$$



$$k \cdot \int_{-\Delta/2}^{\Delta/2} \left(t + \frac{\Delta}{2}\right) \cdot dt = k \cdot \left[\frac{t^2}{2} + \frac{\Delta t}{2} \right]_{-\Delta/2}^{\Delta/2} =$$

$$= k \left[\cancel{\frac{\Delta^2}{8}} + \frac{\Delta^2}{4} - \cancel{\frac{\Delta^2}{8}} + \frac{\Delta^2}{4} \right] = k \frac{\Delta^2}{2}$$

za pravo δ uzbujanje moram $\Delta \to 0 \equiv$ limita

za pravo δ = $\int_{-\Delta/2}^{\Delta/2} y\,dt = \lim_{\Delta \to 0} k \frac{\Delta^2}{2} = \underline{\underline{\underline{0}}}$

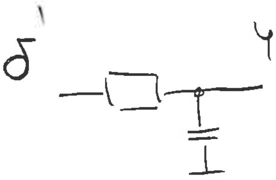
$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

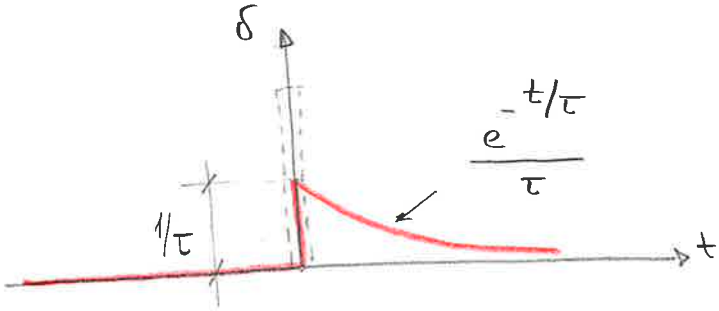
$$\tau \cdot y\left(\tfrac{\Delta}{2}\right) = 1 \Rightarrow \boxed{y^+ = \frac{1}{\tau}}$$

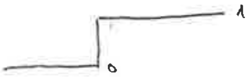
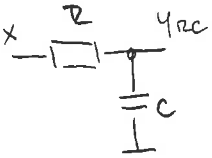
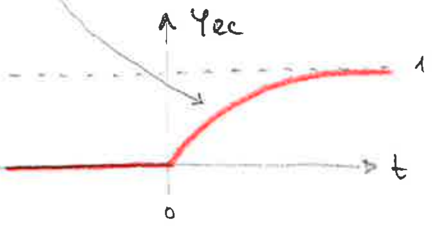
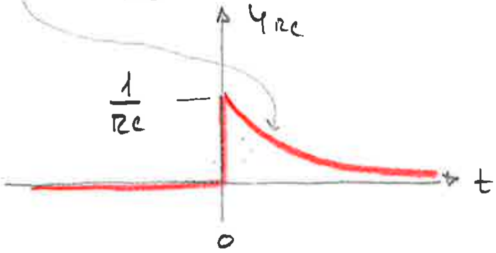
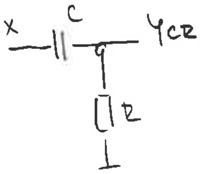
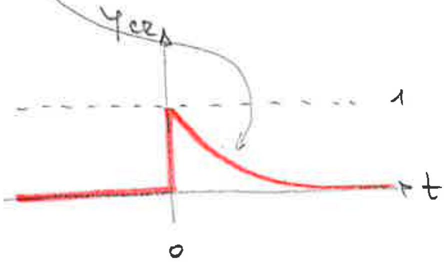
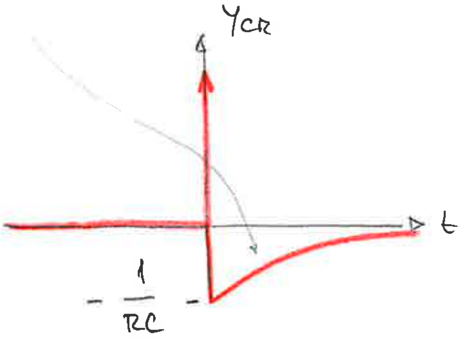
b) $y + \tau \frac{dy}{dt} = 0$

prepišemo od prej*: $$\underbrace{y = k \cdot e^{-t/\tau} \qquad y^{+} = \frac{1}{\tau}}_{k = \frac{1}{\tau}}$$

$$\boxed{y = \frac{e^{-t/\tau}}{\tau}}$$





	$u(t)$ 	$\delta(t)$  pl = 1
	$y_{RC} = u(t)\left(1-e^{-t/RC}\right)$ 	$y_{RC} = \frac{\delta(t)}{RC}\, e^{-t/RC}$; $\delta(t) = 1$ 
	$y_{CR} = u(t)\; e^{-t/RC}$ 	$y_{CR} = \delta - \frac{1}{RC}\, e^{-t/RC}$ 

$w(t)$
utežnostna funkcija

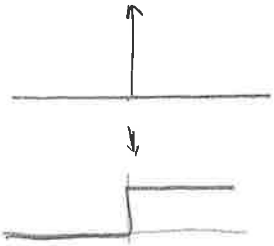
$h(t)$
impulzni odziv

$u(t) \rightarrow \boxed{?} \xrightarrow{w(t)}$

$\delta \rightarrow \boxed{?} \xrightarrow{h(t)}$

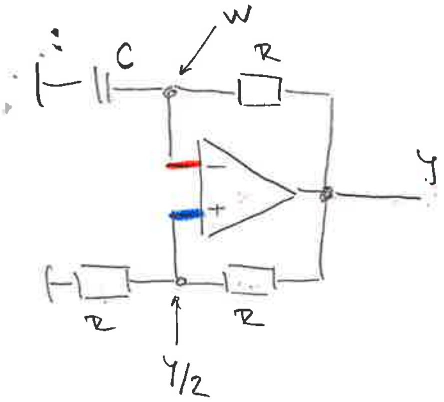
$\delta(t) \rightarrow \boxed{\int} \xrightarrow{u(t)} \boxed{?} \xrightarrow{w(t)}$

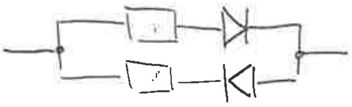
$\delta(t) \rightarrow \boxed{?} \underbrace{\xrightarrow{h(t)} \boxed{\int}} \xrightarrow{w(t)}$



$w(t) \rightarrow \boxed{d/dt} \xrightarrow{h(t)} \quad \Rightarrow \quad w(t) = \frac{1}{p} \cdot h(t)$

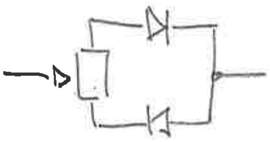
$h(t) = p \cdot w(t)$

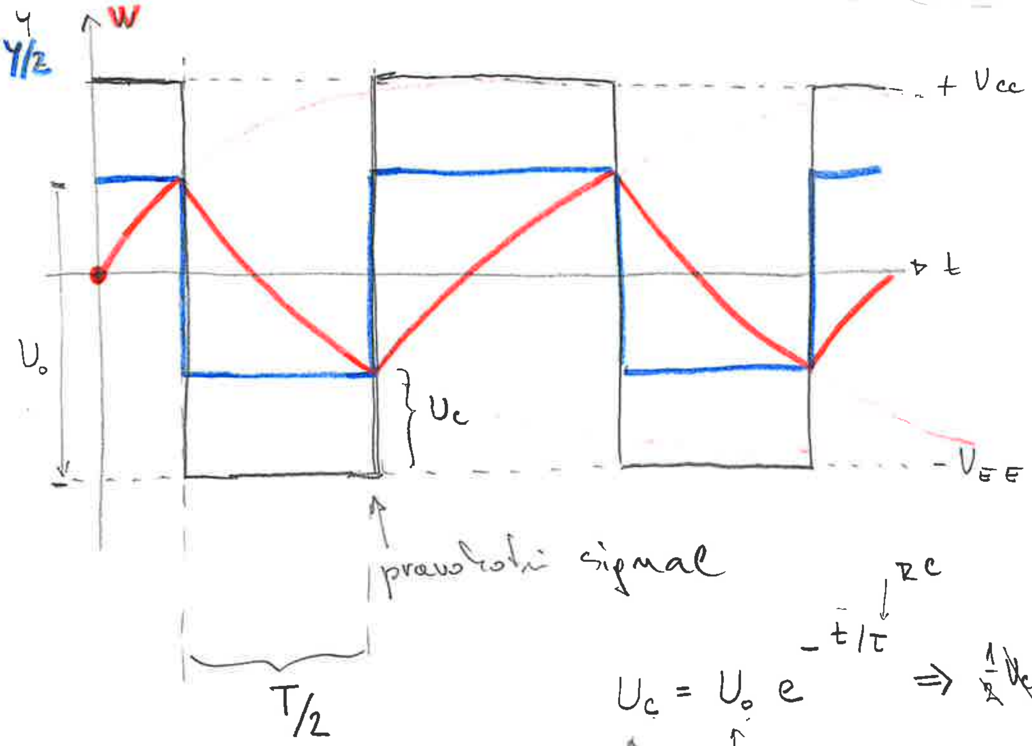




nes. DUTY CYCLE

PWM





$$U_c = U_0\, e^{-t/\overset{RC}{\tau}} \Rightarrow \frac{1}{\cancel{2}}\cancel{V_{cc}} = \frac{3}{\cancel{2}}\cancel{V_{cc}}\, e^{-\frac{T}{2\tau}}$$

U_c ← $\frac{1}{2}V_{cc}$

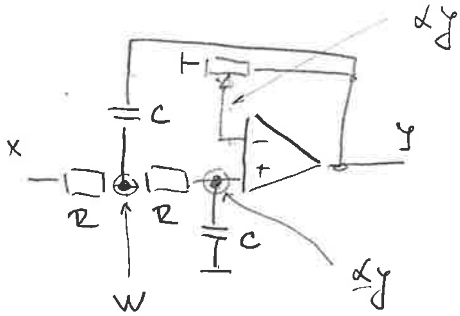
U_0 ← $\frac{3}{2}V_{cc}$

$$\ln\frac{1}{3} = -\frac{T}{2RC}$$

$$\frac{T}{2} = RC\ln 3 \Rightarrow T = 2RC\ln 3$$

$$\boxed{f = \frac{1}{T} = \frac{1}{2RC\ln 3}}$$

relaksacijski oscilator



αy ; $0 \le \alpha \le 1$

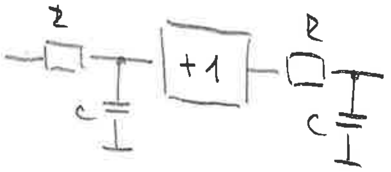
$$\boxed{\begin{aligned} &\frac{w-x}{R} + \frac{w-y}{\frac{1}{Cp}} + \frac{w-\alpha y}{R} = 0 \\ &\frac{\alpha y - w}{R} + \frac{\alpha y}{\frac{1}{Cp}} = 0 \end{aligned}}$$

$$\Swarrow$$

$$y = x\,\frac{1}{\tau^2 p^2 + \tau p\left(3 - \frac{1}{\alpha}\right) + 1} \cdot \frac{1}{\alpha} \quad ; \quad \tau = RC$$

$$T(p) = \frac{Y}{X} = \boxed{\frac{1}{\alpha} \quad \frac{1}{\tau^2 p^2 + \tau p\left(3 - 1/\alpha\right) + 1} = T(p)}$$

$$\alpha = 1 \rightarrow T(p) = \frac{1}{\tau^2 p^2 + \tau p \cdot 2 + 1} = \frac{1}{(1+\tau p)^2}$$



$$= \frac{1}{1+\tau p} \cdot \frac{1}{1+\tau p}$$

$$T(p) = \underbrace{\frac{Dp^2 + Ep + F}{Ap^2 + Bp + C} = \frac{Y}{X}}$$

$$\downarrow$$

$$D\ddot{x} + E\dot{x} + Fx = A\ddot{y} + B\dot{y} + Cy$$

potem, ko je x že povsem 0
po vzbujanju

$$A\ddot{y} + B\dot{y} + Cy = 0 \qquad \left\{\begin{aligned} y &= ke^{\alpha t} \\ \dot{y} &= \alpha k e^{\alpha t} \\ \ddot{y} &= \alpha^2 k e^{\alpha t} \end{aligned}\right.$$

$$\downarrow$$

$$A\alpha^2 ke^{\alpha t} + B\alpha ke^{\alpha t} + Cke^{\alpha t} = 0$$

$$\underbrace{ke^{\alpha t}}_{\neq 0}\underbrace{(A\alpha^2 + B\alpha + C)}_{0} = 0$$

$$\Downarrow$$

$$A\alpha^2 + B\alpha + C = 0$$

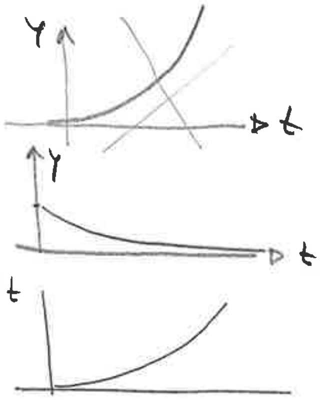
$$\alpha_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$\Downarrow$$

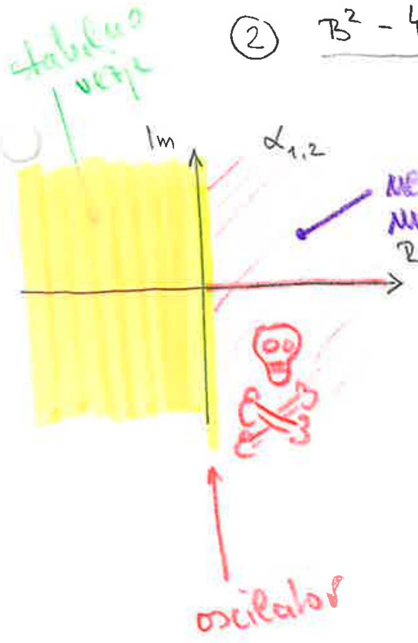
$$y = k \cdot e^{\alpha_{1,2} \cdot t}$$

① $B^2 - 4AC > 0$

- $\alpha_{1,2}$: ~~pozitivna~~ → $y = ke^{+?\cdot t}$
- $\alpha_{1,2}$: negativna → $y = ke^{-?\cdot t}$
- ~~$\alpha_1 > 0$ in $\alpha_2 < 0$ ali obratno~~ → $y = k_1 e^{+?t} + k_2 e^{-?t}$



② $B^2 - 4AC < 0$



$$-\frac{B}{2A} \pm i\sqrt{4AC - B^2}/2A$$

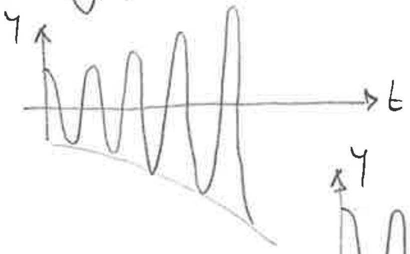
$$y = ke^{-B/2A \pm i\frac{\sqrt{}}{2A}}$$

$$= ke^{-B/2A} \cdot \underbrace{e}$$

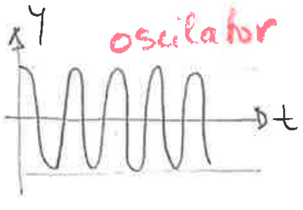
$$ke^{-B/2A} \quad 2\cos\frac{\sqrt{}}{2A} + \cancel{i\sin\frac{\sqrt{}}{2A}}$$

če $B/2A > 0$ ⇒

če ~~$B/2A < 0$~~ ⇒

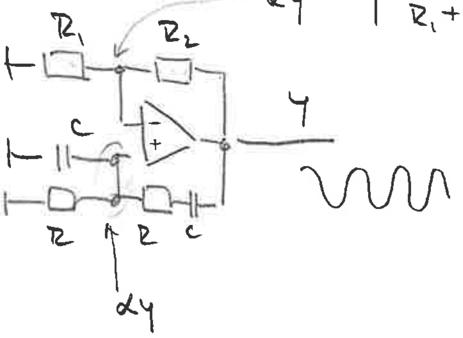


če $B/2A = 0$ ⇒



Wienov oscilator

$$\alpha y = y\frac{R_1}{R_1+R_2}$$



$$\frac{\alpha y}{\frac{1}{Cp}} + \frac{\alpha y}{R} + \frac{\alpha y - y}{R + \frac{1}{Cp}} = 0$$

$$\alpha y RCp + \alpha y + \frac{(\alpha y - y)\cdot Cp \cdot R}{1 + RCp} = 0$$

$$\alpha y \tau p\cdot(1 + RCp) + \alpha y(1+RCp) + \alpha y RCp - yRCp = 0$$

$$\alpha y \tau p + \alpha y \tau^2 p^2 + \alpha y + \alpha y \tau p + \alpha y \tau p - y\tau p = 0$$

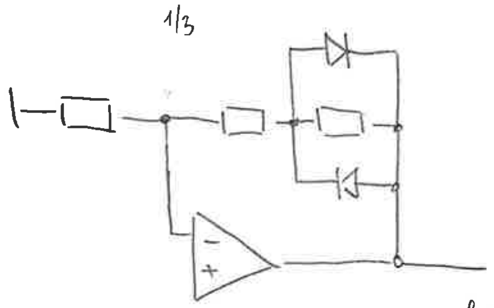
$$\alpha y \tau^2 p^2 + \alpha y \tau p\left(3 - \frac{1}{\alpha}\right) + \alpha y = 0$$

$$\alpha y\left[\tau^2 p^2 + \underbrace{\tau p\left(3 - \frac{1}{\alpha}\right)}_{B=0} + 1\right] = 0$$

$3 - \frac{1}{\alpha} = 0 \Rightarrow$ oscilator

$$3 = \frac{1}{\alpha} \Rightarrow \alpha = \frac{1}{3}$$

$\boxed{\text{gre za } R_2 = 2R_1}$



$$f = \frac{1}{RC\cdot 2\pi}$$

$$y\tau^2p^2 + 1y = 0 \longrightarrow \tau^2\ddot{y} + y = 0$$

$x = \cos \omega t$

$y = A \cdot \cos(\omega t + \varphi)$

} lastnosti vezja podaja frekvenčna p.f. $T(\omega)$

1\. poskus : diferenciator $T(p) = \tau p = \frac{y}{x}$

$y = \tau p x = \tau x'$; $x = \cos \omega t$

$y = -\tau \cdot \omega \sin \omega t$

$T(\omega) = \frac{y}{x} = \frac{-\tau\omega \sin\omega t}{\cos \omega t}$

$x' \longrightarrow y'$, $x'' \longrightarrow y''$ $\Rightarrow$ $x' + x'' \longrightarrow y' + y''$

$\mathrm{Re} + i\,\mathrm{Im} \longrightarrow \mathrm{Re} + i\,\mathrm{Im}$

$x = \cos \omega t + i \sin \omega t = e^{i\omega t}$

$y = \tau x' = \tau i\omega e^{i\omega t}$

} $T(i\omega) = \frac{y}{x} = \frac{\tau i\omega \cancel{e^{i\omega t}}}{\cancel{e^{i\omega t}}} = i\omega\tau$

$T(i\omega) = |T(i\omega)| \cdot e^{i\varphi}$

$|T(i\omega)| = \sqrt{T(i\omega) \cdot T(-i\omega)}$

$\varphi = \mathrm{arctg} \frac{\mathrm{Im}[T(i\omega)]}{\mathrm{Re}[T(i\omega)]}$

vh. signal : $x = e^{i\omega t} = \cos \omega t + i \sin \omega t$

za vezje : $T(i\omega) = \frac{y}{x}$

$\Downarrow$

$y = T(i\omega) \cdot x = |T(i\omega)| \cdot e^{i\varphi} \cdot x$

$= |T(i\omega)| \cdot e^{i\omega t} \cdot e^{i\varphi}$

$= |T(i\omega)| \cdot e^{i(\omega t + \varphi)}$

$= |T(i\omega)| \cdot [\cos(\omega t + \varphi) + i \sin(\omega t + \varphi)]$

povezava med $T(p)$ in $T(i\omega)$

$$\boxed{T(p)=\frac{Dp^2+Ep+F}{Ap^2+Bp+C}}=\frac{y}{x}$$

$$\Downarrow$$

$$A\ddot{y}+B\dot{y}+Cy=D\ddot{x}+E\dot{x}+Fx$$

$$x=e^{i\omega t}\qquad y=T(i\omega)\cdot x=T(i\omega)\cdot e^{i\omega t}$$

$$\dot{x}=i\omega e^{i\omega t}\qquad \dot{y}=i\omega\cdot T(i\omega)\cdot e^{i\omega t}$$

$$\ddot{x}=(i\omega)^2e^{i\omega t}\qquad \ddot{y}=(i\omega)^2\cdot T(i\omega)\cdot e^{i\omega t}$$

$$A(i\omega)^2\cdot T(i\omega)\cdot\cancel{e^{i\omega t}}+B(i\omega)\cdot T(i\omega)\cdot\cancel{e^{i\omega t}}+C\cdot T(i\omega)\cdot\cancel{e^{i\omega t}}=$$

$$=D\cdot(i\omega)^2\cancel{e^{i\omega t}}+E(i\omega)\cdot\cancel{e^{i\omega t}}+F\cancel{e^{i\omega t}}$$

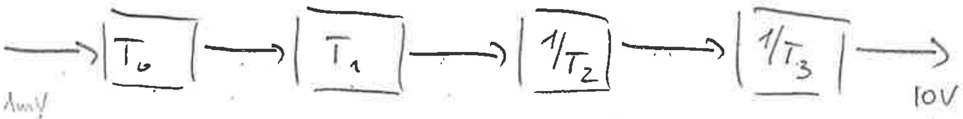
$$T(i\omega)\left[A(i\omega)^2+B(i\omega)+C\right]=D(i\omega)^2+E(i\omega)+F$$

$$\boxed{T(i\omega)=\frac{D\cdot(i\omega)^2+E\cdot(i\omega)+F}{A\cdot(i\omega)^2+B\cdot(i\omega)+C}}$$

$$\boxed{T(i\omega)=T(p)\Big|_{p\to i\omega}}$$

$$T(i\omega)=\frac{D\cdot(i\omega)^2+E\cdot(i\omega)+F}{A\cdot(i\omega)^2+B\cdot(i\omega)+C}=\frac{\left(1-i\frac{\omega}{\omega_0}\right)\left(1-i\frac{\omega}{\omega_1}\right)}{\left(1-i\frac{\omega}{\omega_2}\right)\left(1-i\frac{\omega}{\omega_3}\right)}\cdot\text{konst.}$$

$$=\frac{T_0(i\omega)\cdot T_1(i\omega)}{T_2(i\omega)\cdot T_3(i\omega)}=T_0(i\omega)\cdot T_1(i\omega)\cdot\frac{1}{T_2(i\omega)}\cdot\frac{1}{T_3(i\omega)}$$



ojačanja se množijo

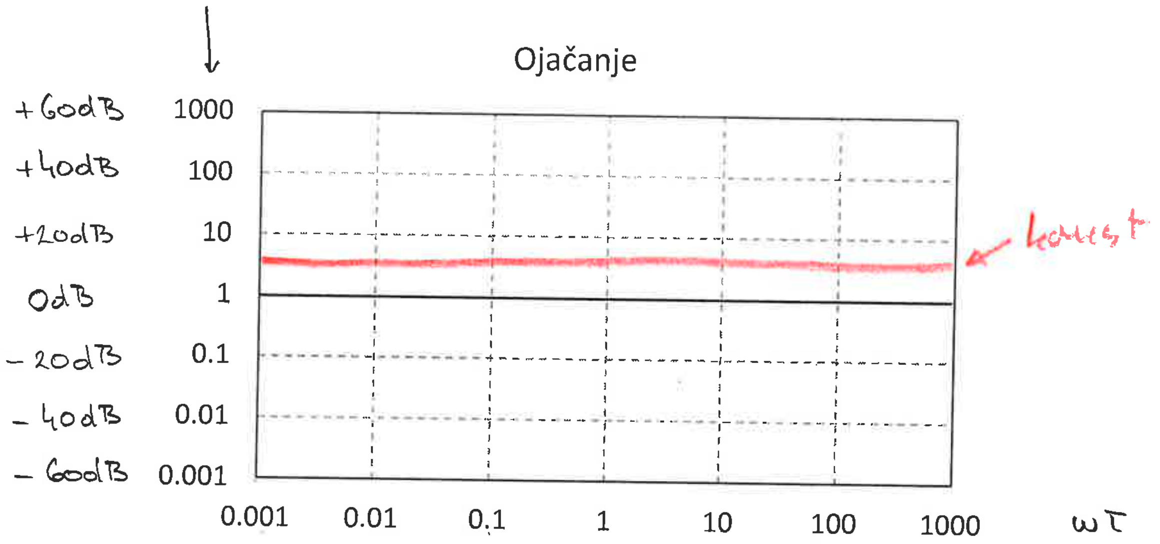
fazni zasuki se seštevajo

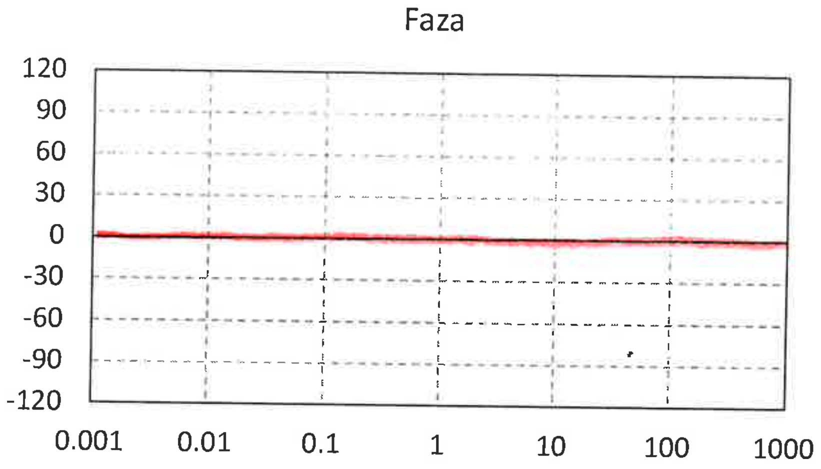
① $\Rightarrow$ $T(i\omega) = \text{konst.}$

$$|T(i\omega)| = \text{konst}$$

$$\varphi = \text{arc tg} \frac{\text{Im}(T(i\omega))}{\text{Re}(T(i\omega))} = 0$$

$$\text{decibel} = 20 \cdot \log \frac{|Y|}{|X|} \, [\text{dB}]$$



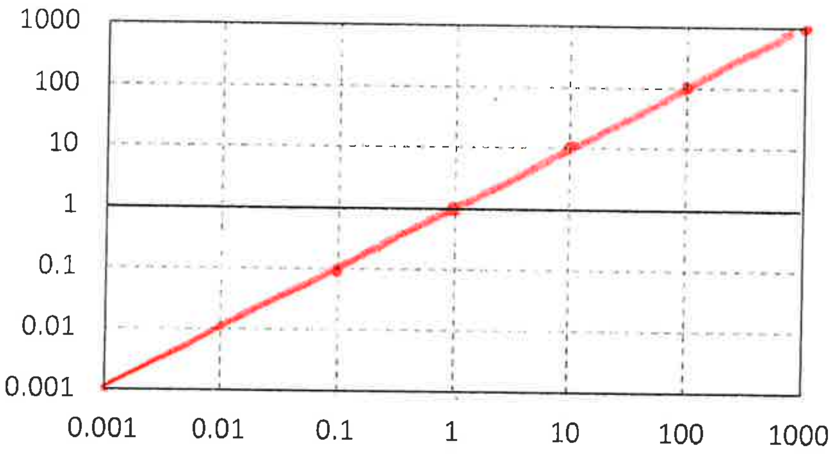


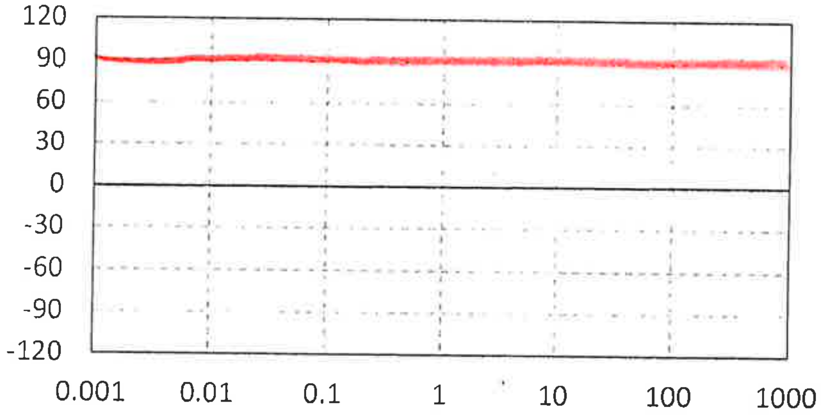
② diferenciator : $T(i\omega) = i\omega\tau$

$$|T(i\omega)| = \omega\tau$$

$$\varphi = \text{arc tg} \frac{\text{Im}[T(i\omega)]}{\text{Re}[T(i\omega)]} = \text{arc tg} \frac{\omega\tau}{0} = \pi/2$$

ω	$\lvert T(i\omega)\rvert$
$1/10\tau$	$1/10$
$1/\tau$	1
$10/\tau$	10

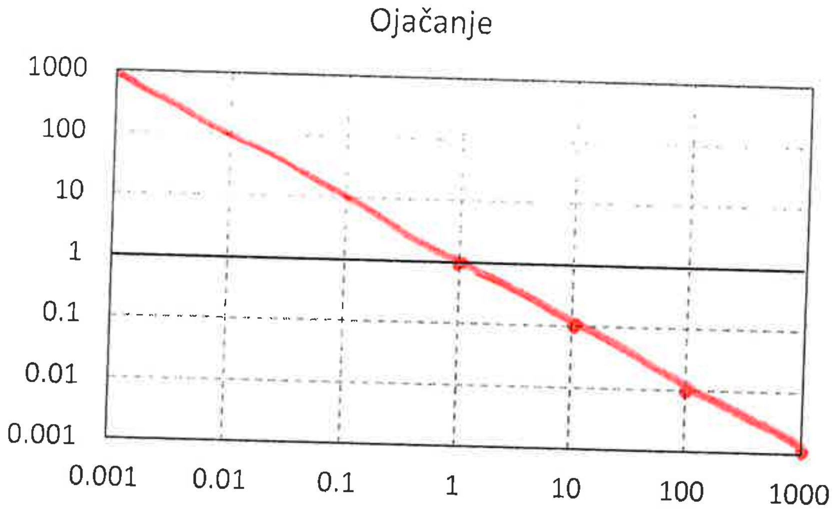

Ojačanje

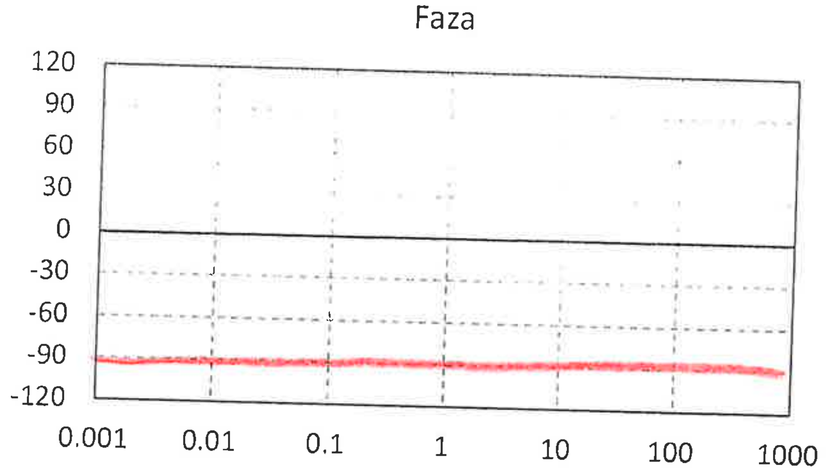

Faza

③ integrator : $T(i\omega) = \frac{1}{i\omega\tau} = \frac{-i}{\omega\tau}$

$$|T(i\omega)| = \frac{1}{\omega\tau}$$

$$\varphi = \operatorname{arctg}\frac{\operatorname{Im}(T(i\omega))}{\operatorname{Re}(T(i\omega))} = \operatorname{arctg}\frac{-\frac{1}{\omega\tau}}{0} = -\pi/2$$





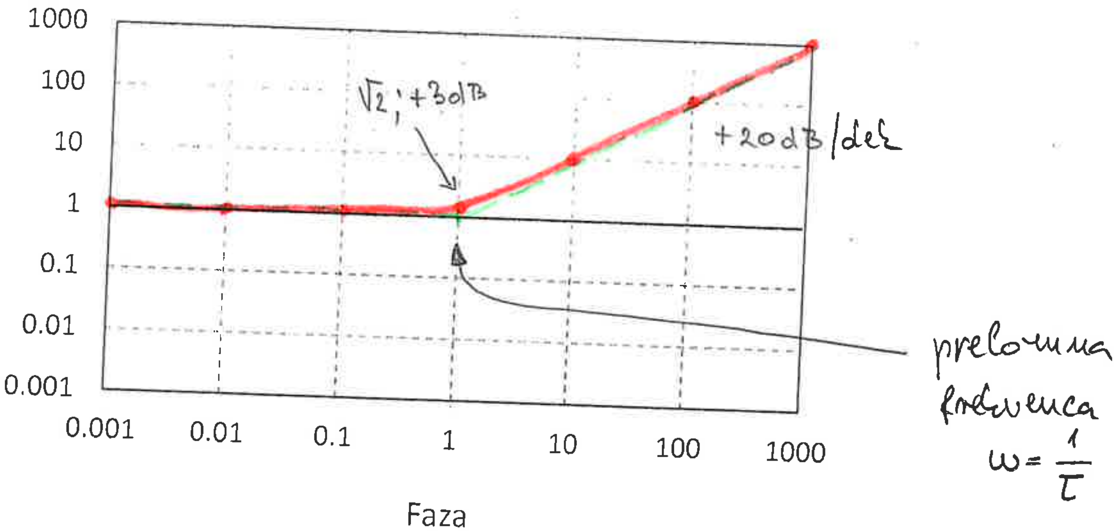
4. $T(i\omega) = 1 + i\omega\tau$

$$|T(i\omega)| = \sqrt{(1+i\omega\tau)\cdot(1-i\omega\tau)} = \sqrt{1+\omega^2\tau^2}$$

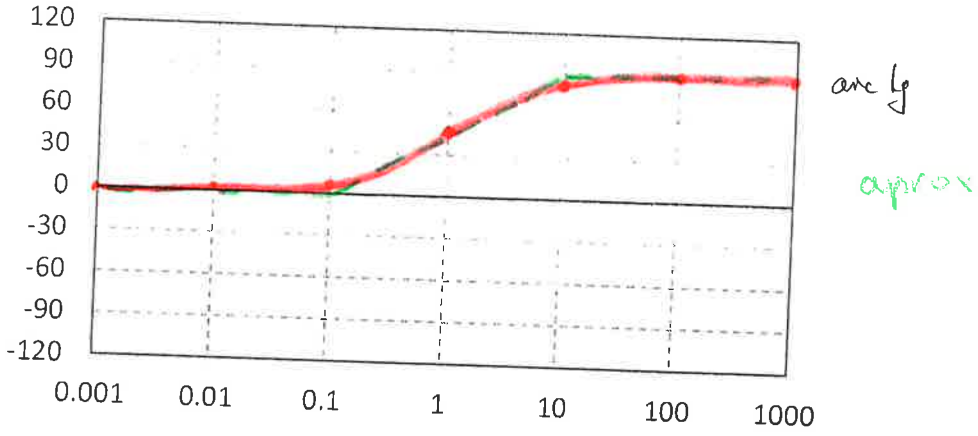
$$\varphi = \operatorname{arc\,tg}\frac{Im}{Re} = \operatorname{arc\,tg}\frac{\omega\tau}{1}$$

ω	$\lvert T(i\omega)\rvert$	φ
$1/100\tau$	$\doteq 1$	0,573°
$1/10\tau$	$\sqrt{1{,}01} \doteq 1$	5,71°
$1/\tau$	$\sqrt{2} = 1{,}41$ [+3dB]	$\pi/4$
$10/\tau$	$\sqrt{101} \doteq 10$	84,29°
$100/\tau$	$\doteq 100$	89,43°

Ojačanje



Faza



$$\alpha y\left[\tau^2 p^2 + \tau p\left(3-\frac{1}{\alpha}\right)+1\right]=0$$

$$\Downarrow$$

$$\alpha\left[\tau^2\ddot{y}+\tau\dot{y}\left(3-\frac{1}{\alpha}\right)+1\right]=0$$

$$y = k\,e^{\beta t}$$
$$\dot{y} = k\beta e^{\beta t}$$
$$\ddot{y} = k\beta^2 e^{\beta t}$$

$$\Downarrow$$

$$\alpha k\left[\tau^2\beta^2+\tau\beta\left(3-\frac{1}{\alpha}\right)+1\right]e^{\beta t}=0$$

$$\beta_{1,2} = \frac{-\cancel{\tau}\left(3-\frac{1}{\alpha}\right)\pm\sqrt{\cancel{\tau^2}\left(3-\frac{1}{\alpha}\right)^2-4\cancel{\tau^2}}}{2\tau^{\cancel{2}}}$$

$$= \frac{-\left(3-\frac{1}{\alpha}\right)\pm\sqrt{9-\frac{6}{\alpha}+\frac{1}{\alpha^2}-4}}{2\tau}$$

resitev za oscilator: pod $\sqrt{\ }$ je neg. vrednost

dušenje $= 0$

$$y = k\Big[\underbrace{e^{Re(\beta)t}}_{\text{ovojnica}}\cdot\underbrace{e^{Im(\beta)t}}_{\text{nihanje}}\Big]$$

ovojnica
⇓
konst za $Re[\beta]=0$

dušenje $=0 \Rightarrow 3-\frac{1}{\alpha}=0 \Rightarrow \underline{\underline{\alpha=\frac{1}{3}}}$

takrat je $Im(\beta)$

$$Im(\beta)=\sqrt{\frac{\cancel{9}-\cancel{6\cdot 3}+\cancel{9}-4}{4\tau^2}} = \frac{\cancel{2}i}{\cancel{2}\tau} = \frac{i}{\tau}$$

torej je nihanje zapisan

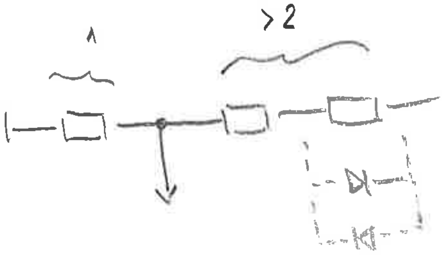
$$\left.\begin{aligned} e^{i\,t/\tau} &= \cos\frac{t}{\tau}+i\sin\frac{t}{\tau}\\ &+\cos t/\tau - i\sin t/\tau \end{aligned}\right\} 2\cos\frac{t}{\tau}$$

če $3-\frac{1}{\alpha} > 0$ ⇒ v exponentu je neka negativnega amplituda upada

$\underline{\alpha > 1/3}$

prevelika amplituda
↓
žari
↓
velika R
↓
$\alpha > 1/3$

alternativa



amplituda narašča
+
diodi
=
ustali se na dovolj veliki amplitudi

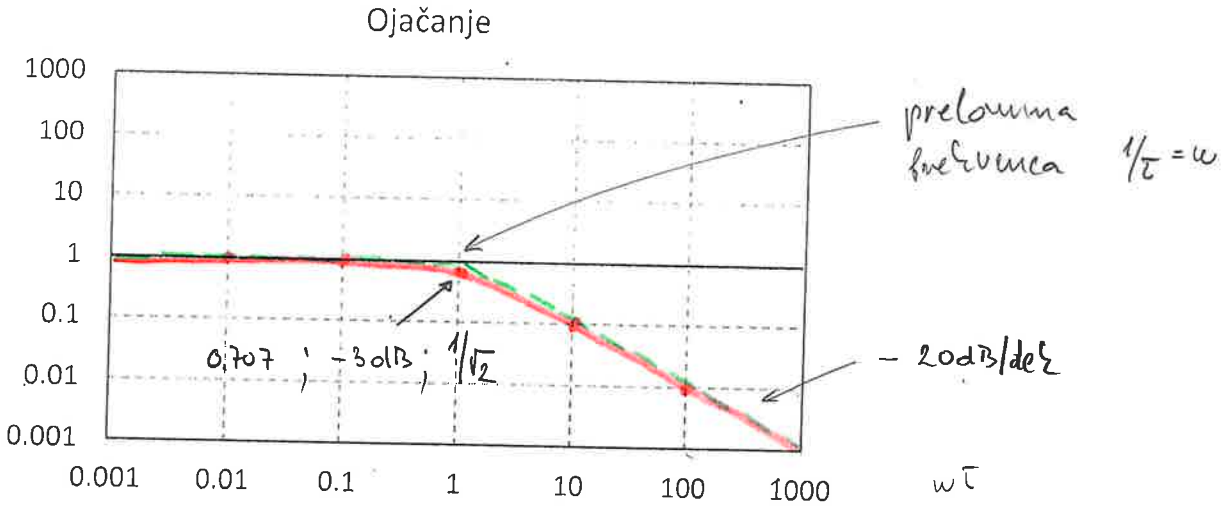
/ 5.

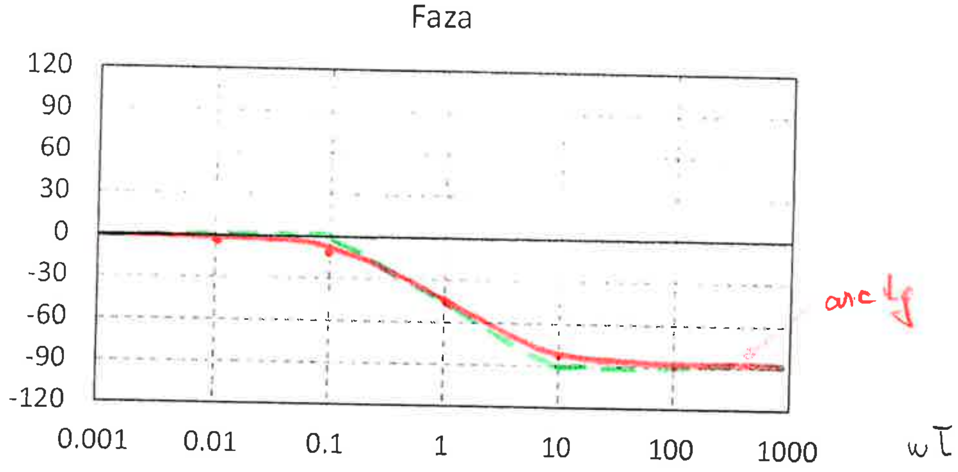
$$T(i\omega) = \frac{1}{1+i\omega\tau} = \frac{1-i\omega\tau}{1+\omega^2\tau^2}$$

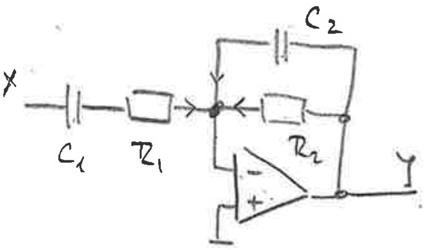
$$|T(i\omega)| = \frac{1}{|1+i\omega\tau|} = \frac{1}{\sqrt{1+\omega^2\tau^2}}$$

$$\varphi = \mathrm{arctg}\frac{Im}{Re} = \mathrm{arctg}\frac{-\omega\tau^2}{1}$$

ω	$\lvert T(i\omega)\rvert$	φ
$1/100\tau$	1	$-0{,}573°$
$1/10\tau$	$\frac{1}{\sqrt{1+0{,}01}} \doteq 1$	$-5{,}71°$
$1/\tau$	$1/\sqrt{2} \equiv -3dB$	$-45°$
$10/\tau$	$\frac{1}{\sqrt{1+100}} \doteq 1/10$	$-84{,}3°$
$100/\tau$	$1/100$	$-89{,}4°$







$$\frac{X}{R_1+\frac{1}{C_1p}}+\frac{Y}{R_2}+\frac{Y}{\frac{1}{C_2p}}=0$$

$\tau_{21}=R_2C_1$
$\tau_{11}=R_1C_1$
$\tau_{22}=R_2C_2$

$$\frac{XC_1p}{1+R_1C_1p}+\frac{Y}{R_2}+YC_2p=0$$

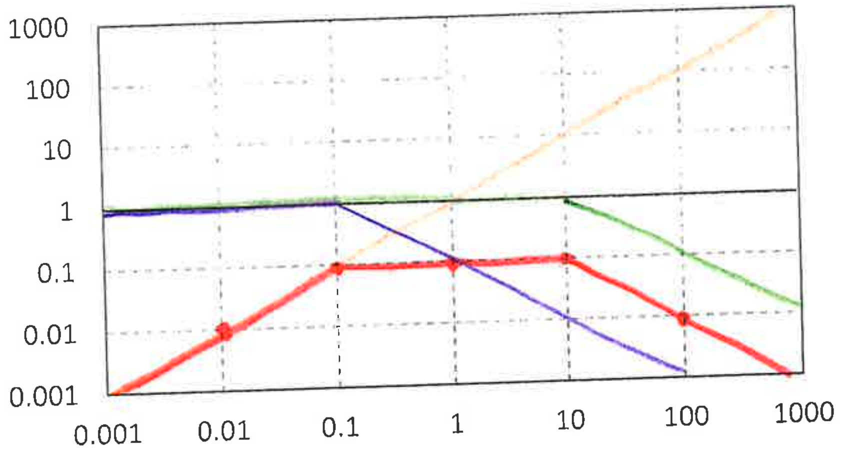
$$\frac{XC_1R_2p}{1+R_1C_1p}=-Y\left(1+R_2C_2p\right)\Rightarrow T(p)=-\frac{R_2C_1p}{(1+R_1C_1p)(1+R_2C_2p)}$$

$$\Downarrow$$

$$T(i\omega)=-\frac{i\omega\tau_{21}}{(1+i\omega\tau_{11})(1+i\omega\tau_{22})}$$

$$T(i\omega)=-\underbrace{\underline{\left(i\omega\tau_{21}\right)}\cdot\underline{\left(\frac{1}{1+i\omega\tau_{11}}\right)}\cdot\underline{\left(\frac{1}{1+i\omega\tau_{22}}\right)}}$$

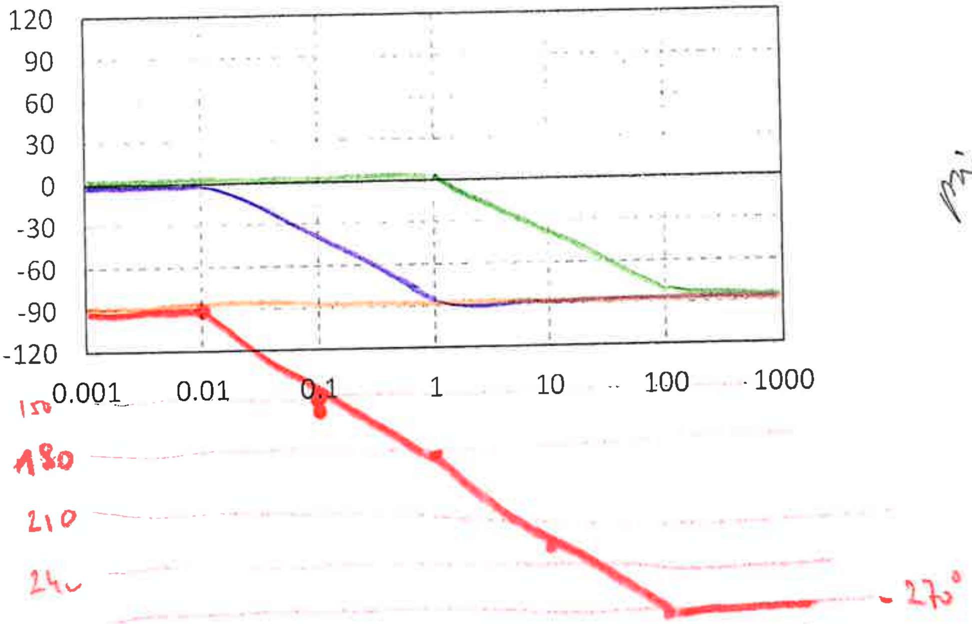
amplituda
(ojačanje)



$\tau_{21}=1$
$\tau_{11}=0,1$
$\tau_{22}=10$

bolje: $\tau_{21}=1$
$\tau_{11}=1$
$\tau_{22}=0,1$

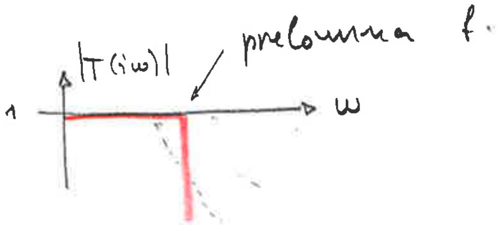
faza



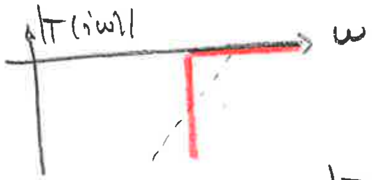
pripni mrežu dijagram

frekvenčni filtri:

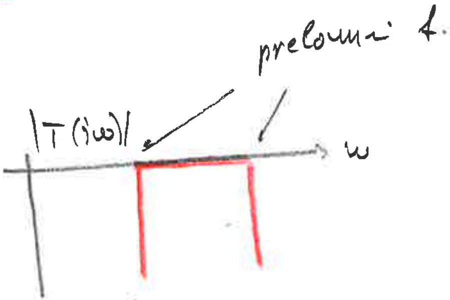
- nizkoprepustni : LPF



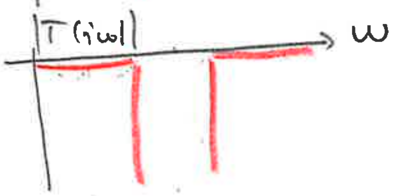
- visokoprepustni : HPF

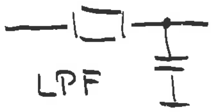


- pasovno prepustni : BPF

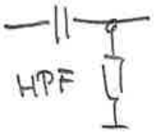


- pasovno zaporni : BSF





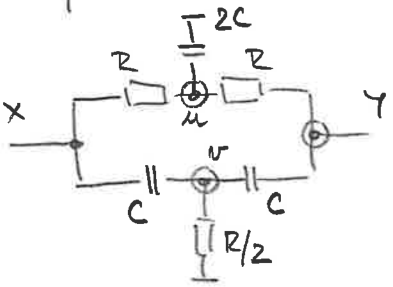
$$T(p) = \frac{1}{1+\tau p} \Rightarrow T(i\omega) = \frac{1}{1+i\omega\tau}$$



$$T(p) = \frac{\tau p}{1+\tau p} \Rightarrow T(i\omega) = \frac{i\omega\tau}{1+i\omega\tau}$$

---

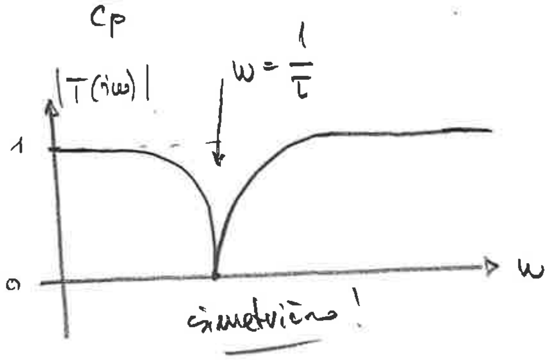
dvojni T filter



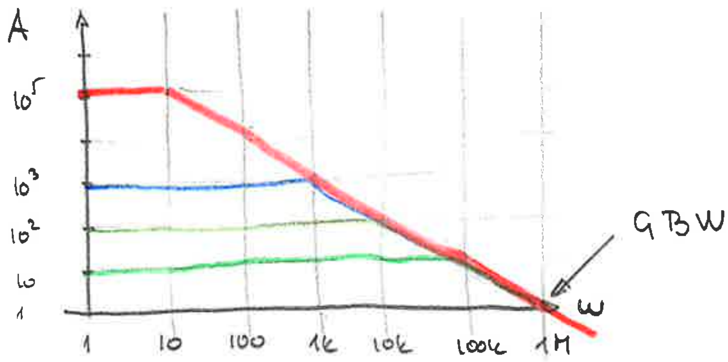
$$\left.\begin{array}{ll} u: & \frac{u-x}{R} + \frac{u-y}{R} + \frac{u}{\frac{1}{2Cp}} = 0 \\ v: & \frac{v-x}{\frac{1}{Cp}} + \frac{v-y}{\frac{1}{Cp}} + \frac{v}{\frac{R}{2}} = 0 \\ y: & \frac{u-y}{R} + \frac{v-y}{\frac{1}{Cp}} = 0 \end{array}\right\}$$

$$\Downarrow$$

$$T(i\omega) = \frac{1-\omega^2\tau^2}{1+4i\omega\tau-\omega^2\tau^2}$$



OP.01.



digitalna elektronika

npr. : 1/0 T/F H/L

bin. zapis

$$\underbrace{1/0}_{B_3} \quad \underbrace{1/0}_{B_2} \quad \underbrace{1/0}_{B_1} \quad \underbrace{1/0}_{B_0}$$

MSB ↗ B_3 B_0 ↖ LSB

$$\text{vrednost} = B_3 \cdot 2^3 + B_2 \cdot 2^2 + B_1 \cdot 2^1 + B_0 \cdot 2^0$$

$$1101_2 = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 =$$

$$= 8 + 4 + 0 + 1 = 13_{10}$$

$$\text{števila} = \begin{cases} 0 \\ 2^B - 1 \end{cases} \Rightarrow 2^B \text{ različnih vrednosti}$$

$$\underbrace{0110}\underbrace{1101}\ \underbrace{1000}\underbrace{0110}_2$$

hex. zapis $\quad 6 \quad D \quad 8 \quad 6_{16}$

10_{10}	$\equiv A_{16}$
11	B
12	C
13	D
14	E
15	F

$$6D86_{16} = 0110110110000110_2$$

$$FF38_{16} = 1111111100111000_2$$

$$6D86_{16} = 6 \cdot 16^3 + 13 \cdot 16^2 + 8 \cdot 16^1 + 6 \cdot 16^0$$

10 → 2

$169_{10} : 2 = 84 + 1$
$84 : 2 = 42 + 0$
$42 : 2 = 21 + 0$
$21 : 2 = 10 + 1$
$10 : 2 = 5 + 0$
$5 : 2 = 2 + 1$
$2 : 2 = 1 + 0$
$1 : 2 = 0 + 1$

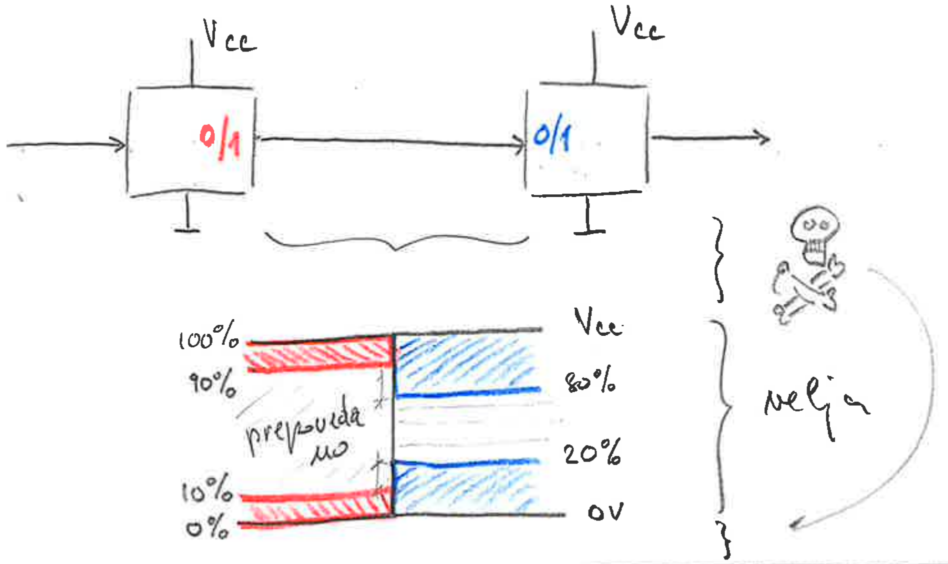
$$169_{10} = 10101001_2$$ ✓

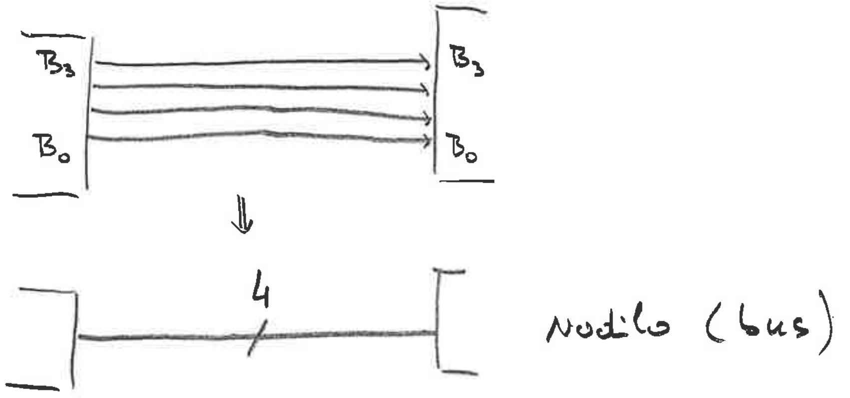
$169_{10} : 16 = 10 + 9$
$10 : 16 = 0 + 10$

$$169_{10} = A9_{16}$$ ✓

0/1 v vezju : napetost

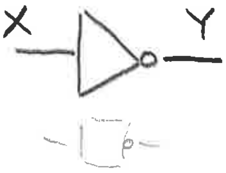
Vcc ↗ 5V
Vcc ↘ 3.3V





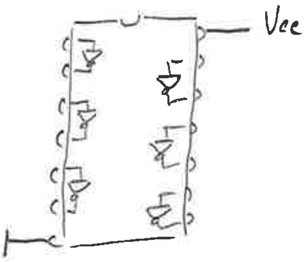
- osn. gradniki dig. vezij ⇒ log. vrata

a) negacija ≡ NOT, inverzija

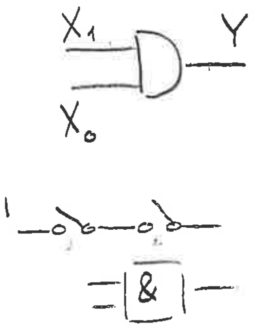


X	Y
0	1
1	0

$Y = \overline{X}$



b) konjunkcija ≡ AND

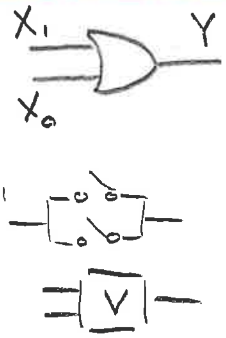


X_1	X_0	Y
0	0	0
0	1	0
1	0	0
1	1	1

$Y = X_1 \cdot X_0$

$= X_1 \wedge X_0$

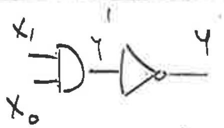
c) disjunkcija ≡ OR



X_1	X_0	Y
0	0	0
0	1	1
1	0	1
1	1	1

$Y = X_1 + X_2$

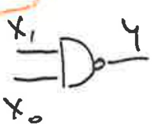
$= X_1 \vee X_2$

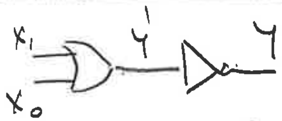


X_1	X_0	Y'	Y
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

$Y = \overline{X_1 \cdot X_0}$

NAND

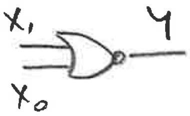




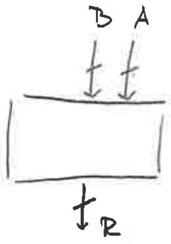
X_1	X_0	Y'	Y
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

$Y = \overline{X_1 + X_0}$

NOR

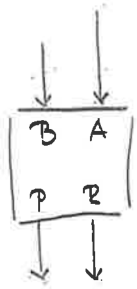


seštevalnik



$$\begin{array}{r} A \\ +\ B \\ \hline P\ R \end{array} \Rightarrow \begin{array}{r} {}^{1}\ {}^{1}\quad\ \ \\ 4\ 3\ 2\ 1 \\ 8\ 7\ 6\ 5 \\ \hline 1\ 3\ 0\ 8\ 6 \end{array} \Rightarrow \begin{array}{rrrrr} & A_3 & A_2 & A_1 & A_0 \\ + & B_3 & B_2 & B_1 & B_0 \\ & P_2 & P_1 & P_0 & \\ \hline P_3 & R_3 & R_2 & R_1 & R_0 \end{array}$$

delni seštevalnik



delni seštevalnik

$\downarrow$

$$\begin{array}{r} 0 \\ +\ 0 \\ \hline 0\ 0 \end{array} \quad \begin{array}{r} 0 \\ +\ 1 \\ \hline 0\ 1 \end{array} \quad \begin{array}{r} 1 \\ +\ 0 \\ \hline 0\ 1 \end{array} \quad \begin{array}{r} 1 \\ +\ 1 \\ \hline 1\ 0 \end{array} \Rightarrow$$

tabela

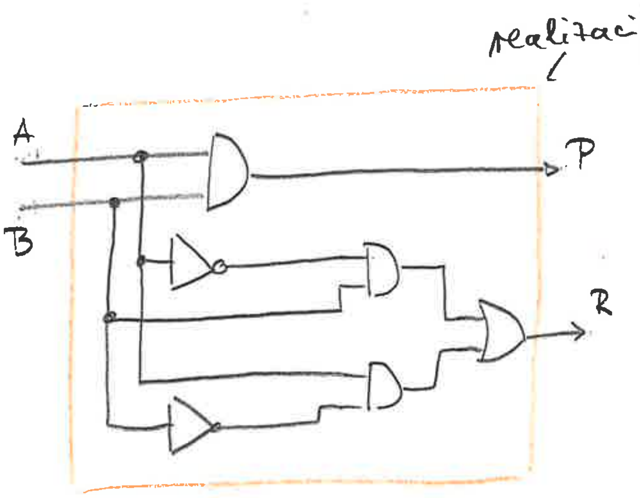
A	B	P	R
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

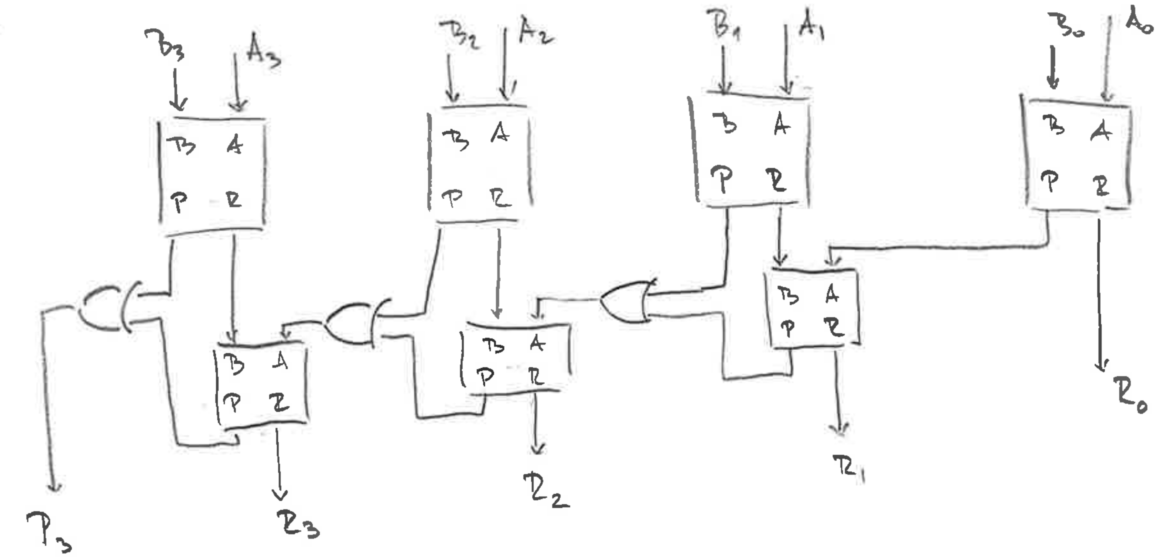
$$P = A \cdot B$$
$$R = \bar{A} \cdot B + A \cdot \bar{B}$$

← log. enačbe

$\Downarrow$

realizacija





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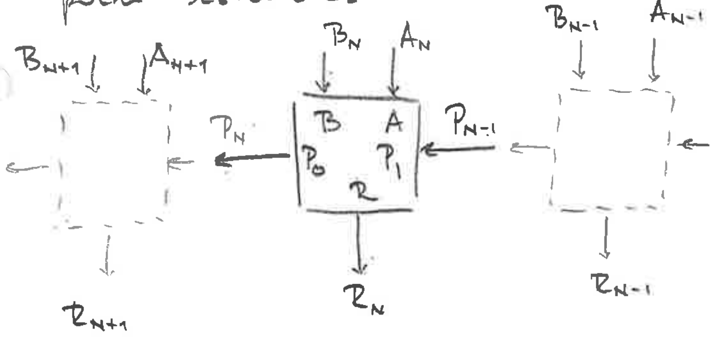
polni seštevalnik



⇓

P_1	A	B	P_0	R
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

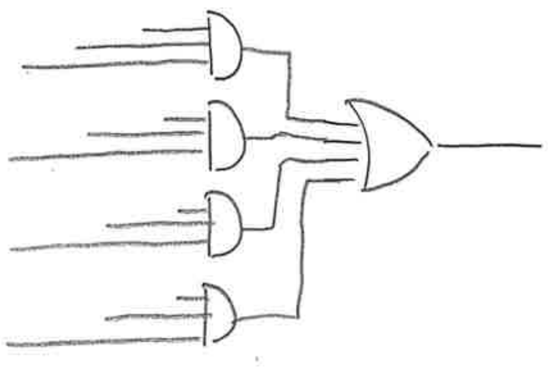
$$\overline{P_0} = \overline{P_1}\cdot\overline{A}\cdot\overline{B} + \overline{P_1}\cdot\overline{A}\cdot B + \overline{P_1}\cdot A\cdot\overline{B} + P_1\cdot\overline{A}\cdot\overline{B}$$

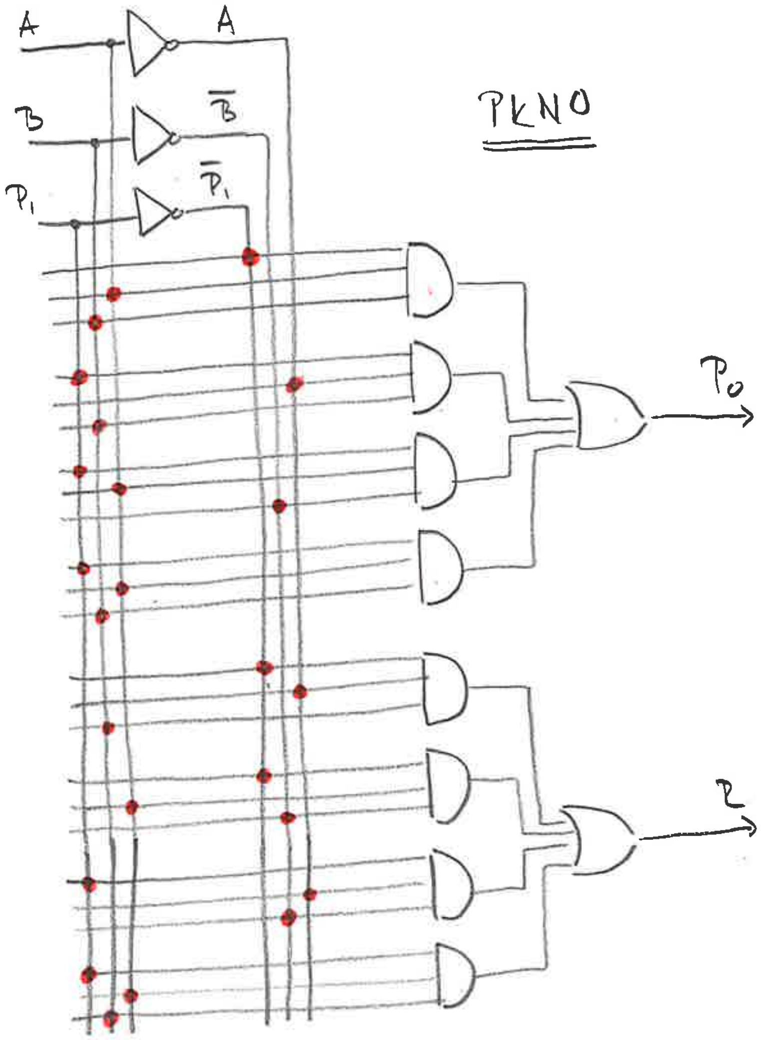
⇒

$$P_0 = \overline{P_1}\cdot A\cdot B + P_1\cdot\overline{A}\cdot B + P_1\cdot A\cdot\overline{B} + P_1\cdot A\cdot B$$

$$R = \overline{P_1}\cdot\overline{A}\cdot B + \overline{P_1}\cdot A\cdot\overline{B} + P_1\cdot\overline{A}\cdot\overline{B} + P_1\cdot A\cdot B$$

⇓





PKNO

PAL
GAL } tipično 16V8

pravila za poenostavljanje

① združljivost : $A\cdot B\cdot C = (A\cdot B)\cdot C = A\cdot(B\cdot C)$

$A+B+C = (A+B)+C = A+(B+C)$

② zamenljivost : $A\cdot B = B\cdot A$

$A+B = B+A$

③ prioriteta :

najprej INV
potem AND
na koncu OR

④ izpostavljanje : $F = A\cdot B + A\cdot C = A\cdot(B+C)$

$G = (A+B)\cdot(A+C) = A+(B\cdot C)$

⑤ AND

$A\cdot A = A$ $\qquad A\cdot 0 = 0$

$A\cdot 1 = A$ $\qquad A\cdot \overline{A} = 0$

⑥ OR

$A+A = A$ $\qquad A+0 = A$

$A+1 = 1$ $\qquad A+\overline{A} = 1$

⑦ inverzija $\overline{\overline{A}} = A$

⑧ De-Morganova teorema

$\overline{A+B} = \overline{A}\cdot\overline{B} \quad \leftarrow$

$\overline{A\cdot B} = \overline{A}+\overline{B}$

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

$$P_0 = \overline{P_1}\cdot A\cdot B + P_1\cdot \overline{A}\cdot B + P_1\cdot A\cdot \overline{B} + P_1\cdot A\cdot B =$$

$$= B\cdot\left(\overline{P_1}\cdot A + P_1\cdot \overline{A}\right) + P_1\cdot A\cdot\left(\underbrace{\overline{B}+B}_{1}\right) =$$

$$= P_1\cdot A + B\cdot\left(P_1\oplus A\right)$$

X_1	X_0	XOR
0	0	0
0	1	1
1	0	1
1	1	0

XOR

$X_1 \oplus X_0$

$$\underline{\underline{R}} = \overline{P_1}\cdot\overline{A}\cdot B + \overline{P_1}\cdot A\cdot\overline{B} + P_1\cdot\overline{A}\cdot\overline{B} + P_1\cdot A\cdot B =$$

$$= \overline{P_1}\cdot\left(A\oplus B\right) + P_1\left(\overline{A}\cdot\overline{B} + A\cdot B\right) =$$

$$= —\,''\,— \quad + P_1\,\overline{\overline{\left(\overline{A}\cdot\overline{B} + A\cdot B\right)}} =$$

$$= —\,''\,— \quad + P_1\,\overline{\left(\overline{\overline{A}\cdot\overline{B}}\ \bullet\ \overline{A\cdot B}\right)} =$$

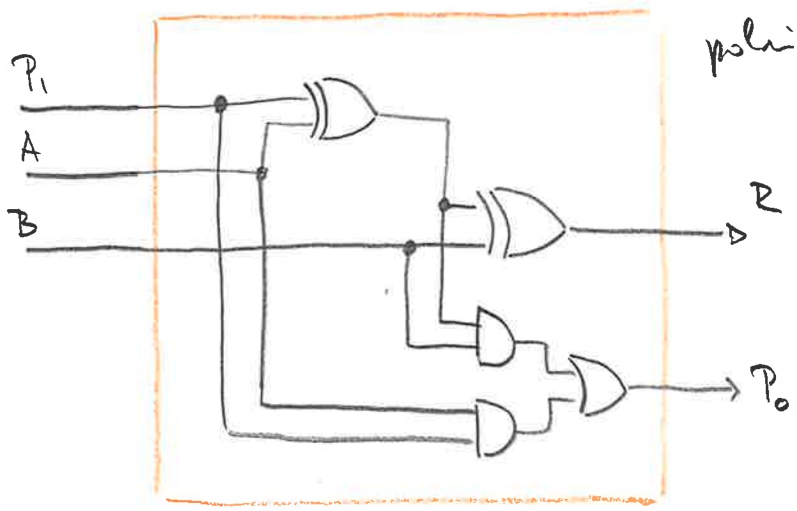
$$= —\,''\,— \quad + P_1\cdot\overline{\left(\left(\overline{\overline{A}}+\overline{\overline{B}}\right)\cdot\left(\overline{A}+\overline{B}\right)\right)} =$$

$$= —\,''\,— \quad + P_1\cdot\overline{\left(\left(A+B\right)\cdot\left(\overline{A}+\overline{B}\right)\right)} =$$

$$= —\,'\,— \quad + P_1\cdot\overline{\left(\underbrace{A\cdot\overline{A}}_{0} + \underbrace{A\cdot\overline{B} + \overline{A}\cdot B}_{XOR} + \underbrace{B\cdot\overline{B}}_{0}\right)} =$$

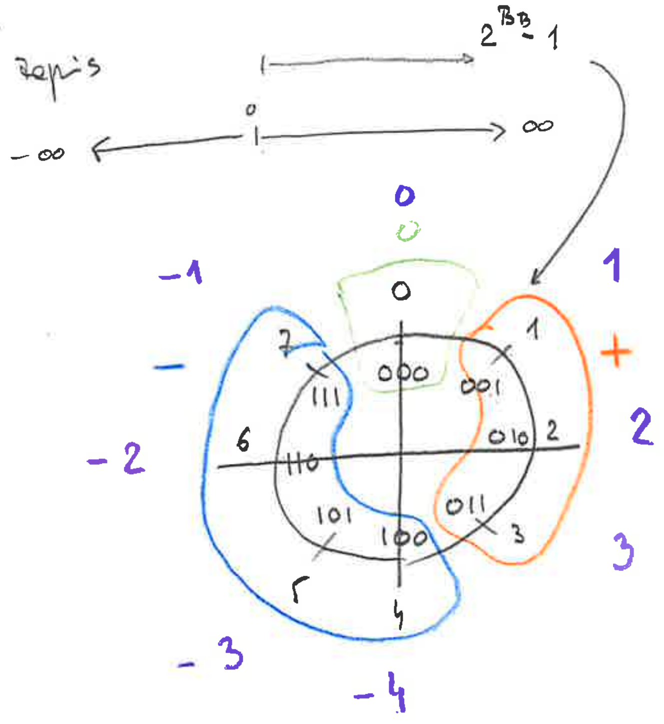
$$= \overline{P_1}\cdot\left(A\oplus B\right) + P_1\cdot\overline{\left(A\oplus B\right)} = \overline{P_1}\cdot F + P_1\cdot\overline{F} = P_1\oplus F =$$

$$= P_1\oplus\left(A\oplus B\right) = \underline{\underline{\left(P_1\oplus A\right)\oplus B}}$$



polni seštevalnik

Zapis



$$-\left(2^{B_B-1}\right) \dots 0 \dots \left(2^{B_B-1}-1\right)$$

$$R = A - B = A + \underline{(-B)}$$

$$= A + (-B) + 2^{B_B} = A + \left(2^{B_B} - B\right) \quad \text{za } B_B = 3$$

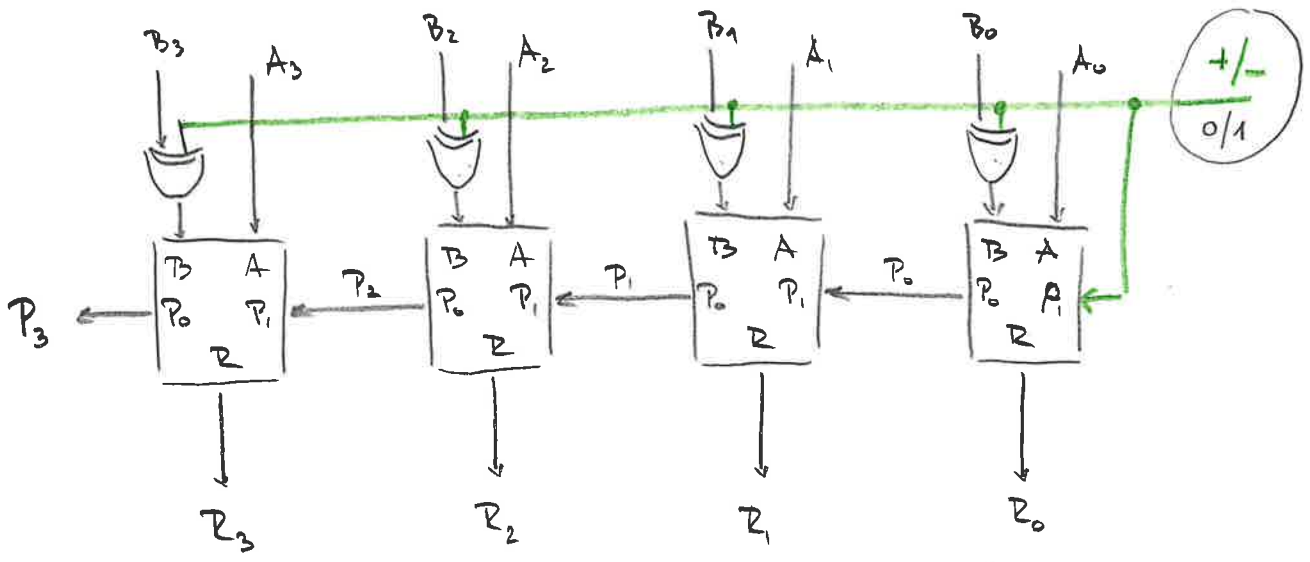
$$= A + (8 - B)$$

$$= A + (\underbrace{7 - B} + 1)$$

$$= A + \underline{(\overline{B} + 1)}$$

↑ dvojiški komplement

B	7 − B
000	111
001	110
010	101
011	100

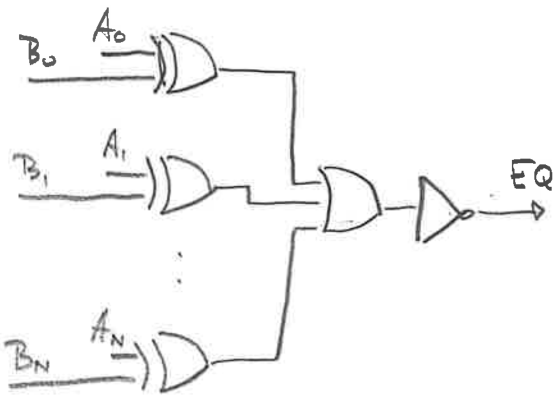


XOR

X_1	X_0	XOR
0	0	0
0	1	1
1	0	1
1	1	0

$A_3 A_2 A_1 A_0$

$B_3 B_2 B_1 B_0$



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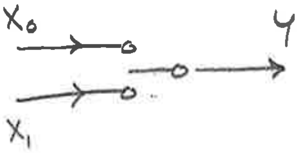
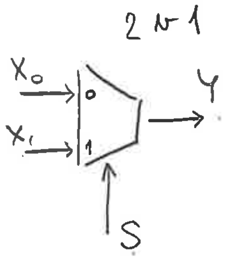
multiplexor



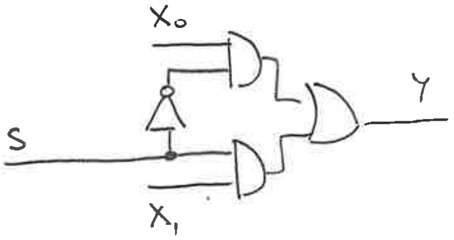
$\Rightarrow$

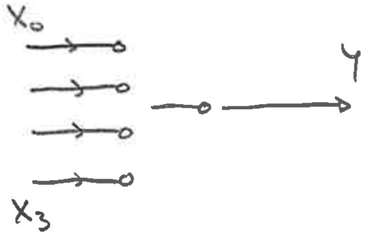


$$Y = \begin{cases} S = 0 \Rightarrow Y = X_0 \\ S = 1 \Rightarrow Y = X_1 \end{cases}$$

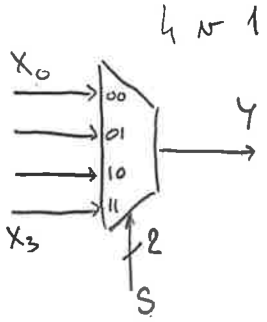
S	Y
0	X_0
1	X_1

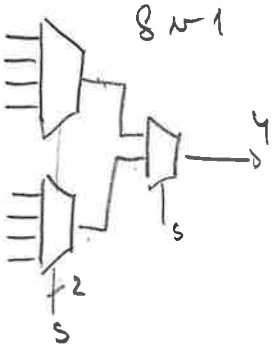
$Y = \overline{S} \cdot X_0 + S \cdot X_1 \Leftarrow$



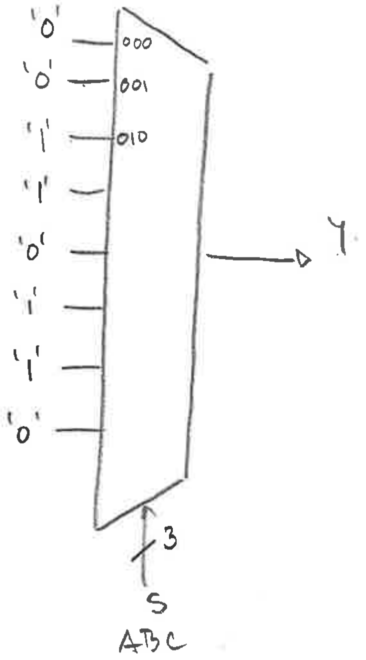


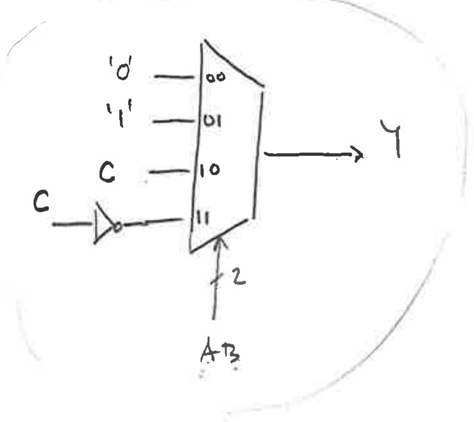
$\Rightarrow$





A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0





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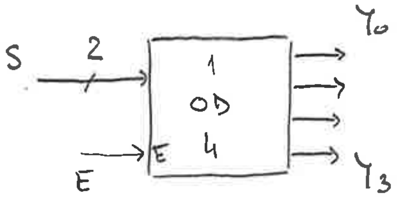
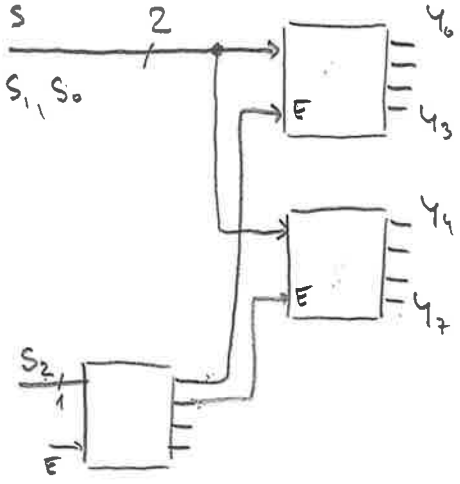
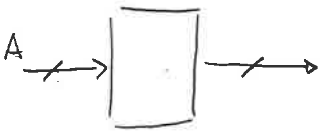
selektor



E	S_1	S_0	Y_3	Y_2	Y_1	Y_0
1	0	0	0	0	0	1
1	0	1	0	0	1	0
1	1	0	0	1	0	0
1	1	1	1	0	0	0
0	x	x	0	0	0	0

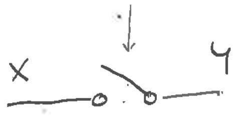


prekoder

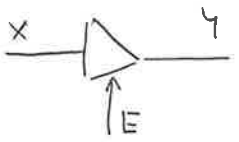


7-segmentni dekoder

A_3	A_2	A_1	A_0	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
		⋮					⋮			

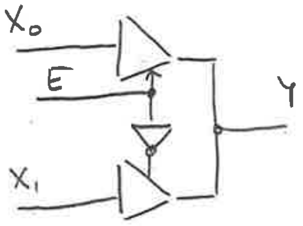


⇒



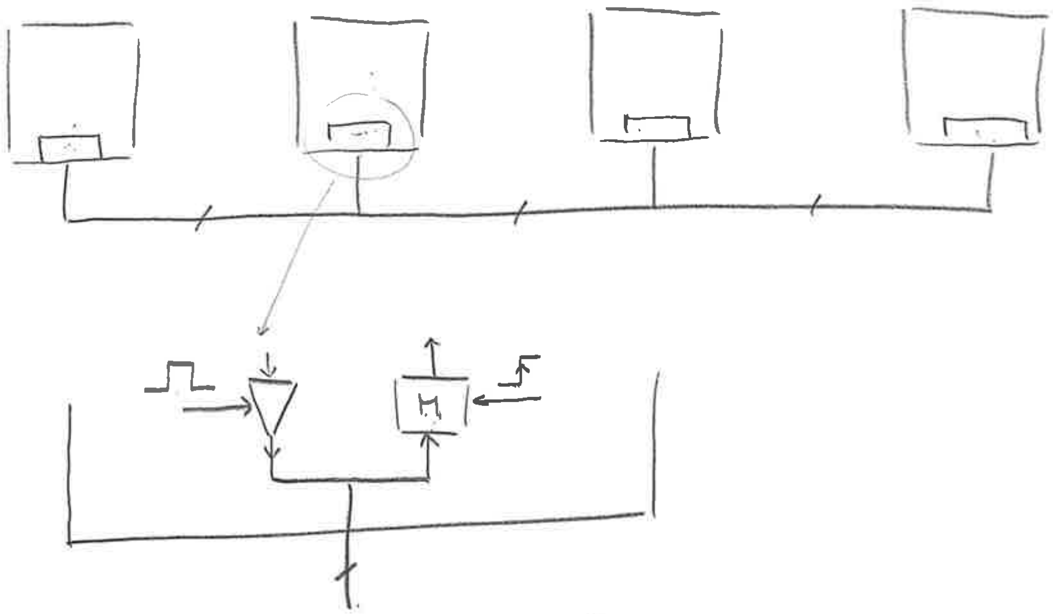
three-state buffer

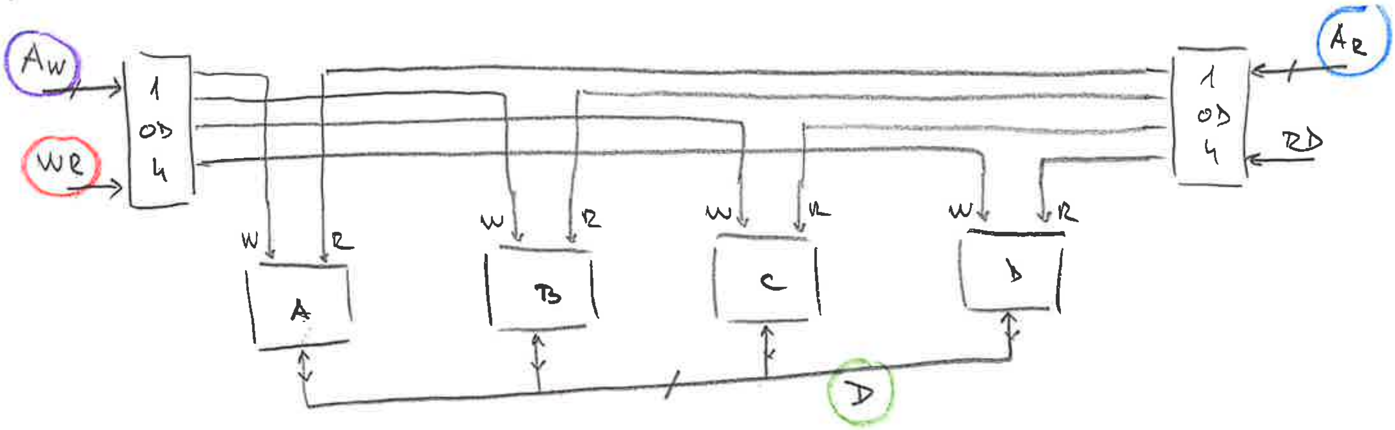
E	Y
1	X
0	Z

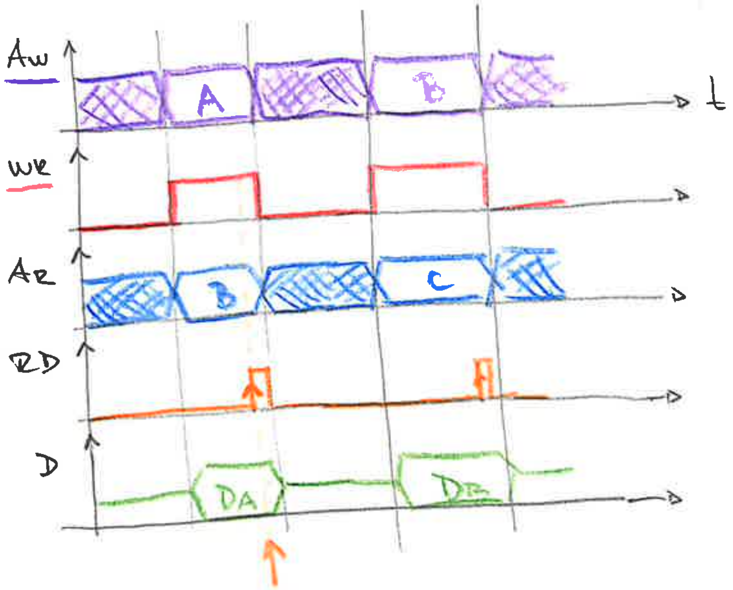


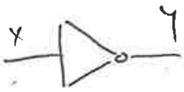
⇒

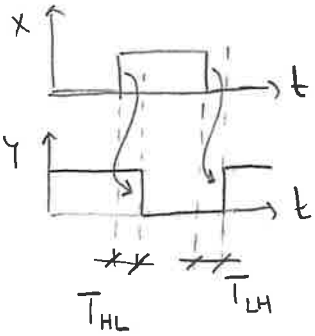
E	Y
1	X_0
0	X_1




AW
WE
1
OD
4
W
R
A
W
R
B
W
R
C
W
R
D
D
1
OD
4
AR
RD

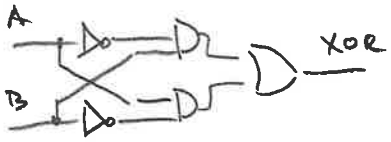

AW
A
B
t
WE
AR
B
C
RD
D
DA
DB





T_{HL} , T_{LH} ≈ ns

$$XOR = A\cdot\overline{B} + \overline{A}\cdot B$$



poti skozi veje so razlicno dolge

⇓

napak pri izmenicnu l.f.

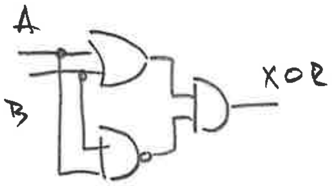
 spike, glitch

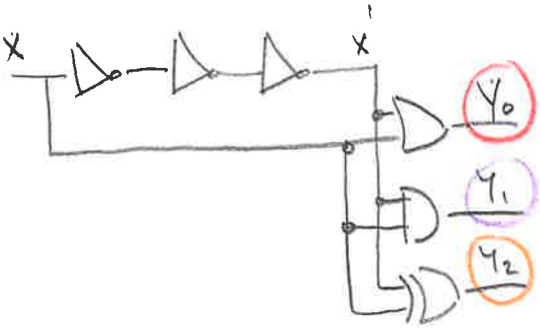
B \ A	0	1
0	0	1
1	1	0

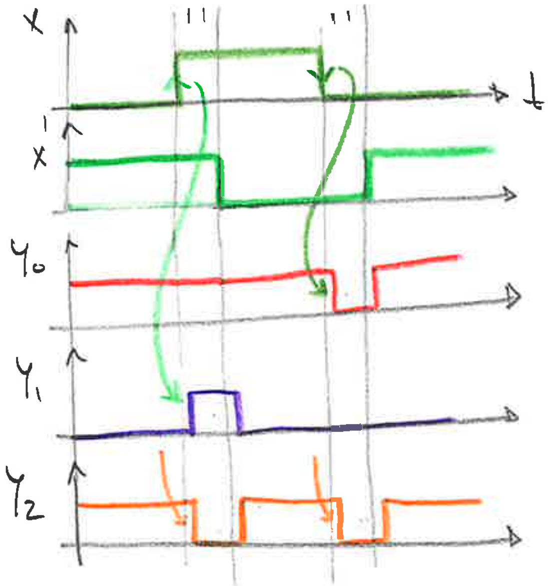
$$\boxed{XOR = (B + A)\cdot\overline{AB} =}$$

$$= (B + A)\cdot(\overline{A} + \overline{B}) =$$

$$= \underline{\overline{A}\cdot B} + \underbrace{A\cdot\overline{A}}_{0} + \underbrace{B\cdot\overline{B}}_{0} + \underline{A\cdot\overline{B}}$$

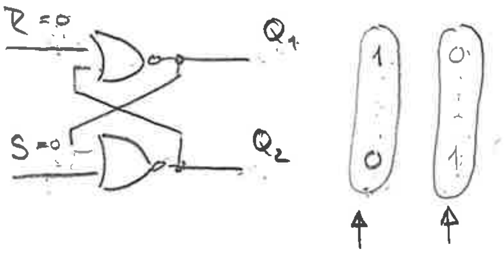






flip-flopi

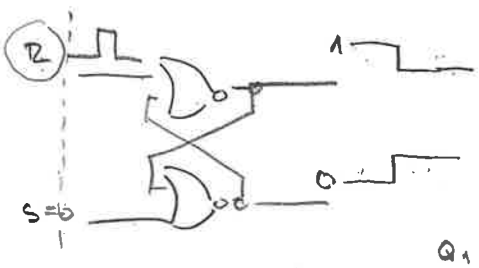
RS - FF

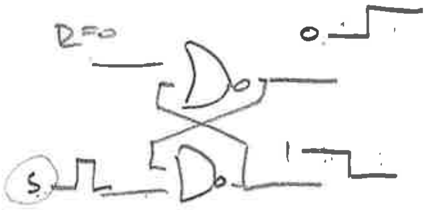


$$Q_1 = \overline{R + Q_2} = \overline{R + \overline{S + Q_1}}$$

$$= \overline{R} \cdot (S + Q_1)$$

$$= \overline{R} \cdot S + \overline{R} \cdot Q_1$$





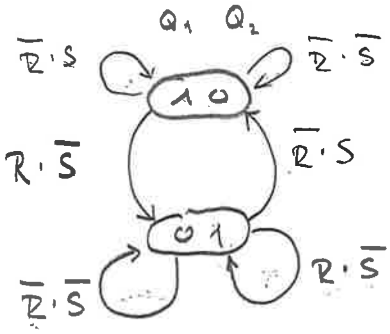
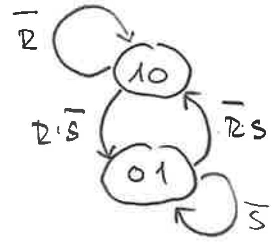
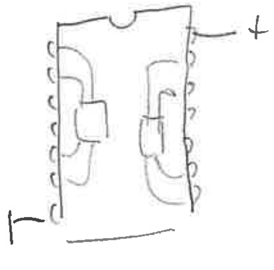


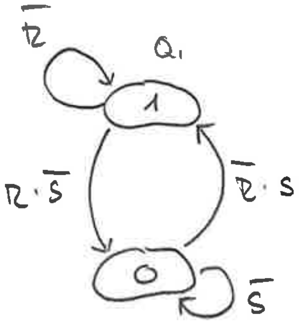
diagram prehodov

⇓

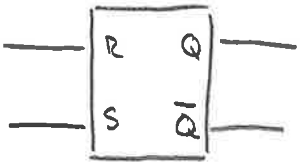




⇓

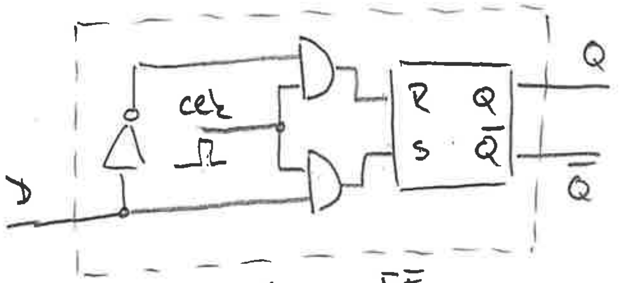


⟹

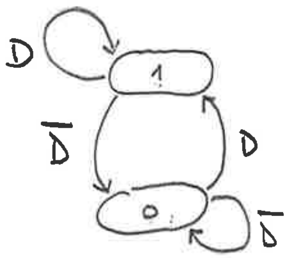


asinhroni FF RS

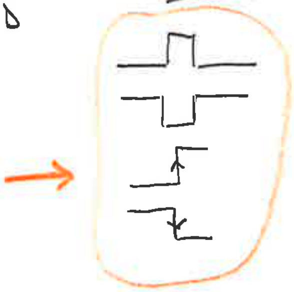
D-FF

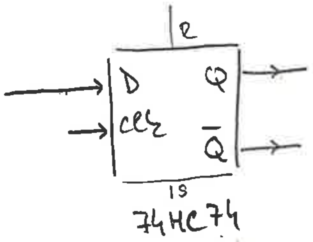


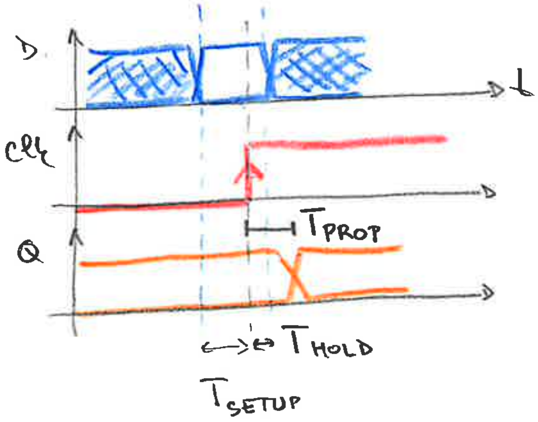
sinhroni FF



do prehoda pride le ob clk!



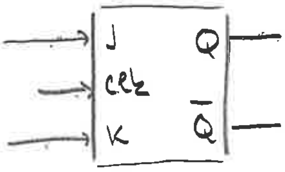




po prehodu ure

D	Q^+
0	0
1	1

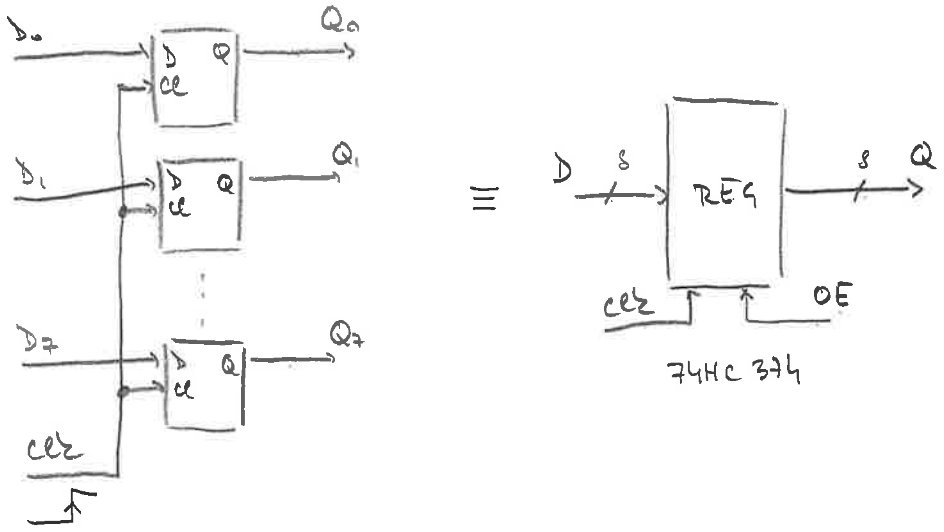
JK-FF



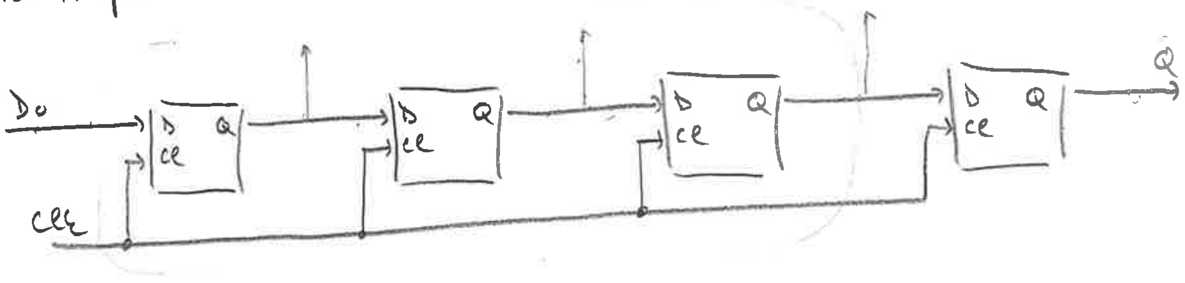
sinhroni FF

J	K	Q^+
0	0	Q
0	1	0
1	0	1
1	1	$\bar{Q}$

- register



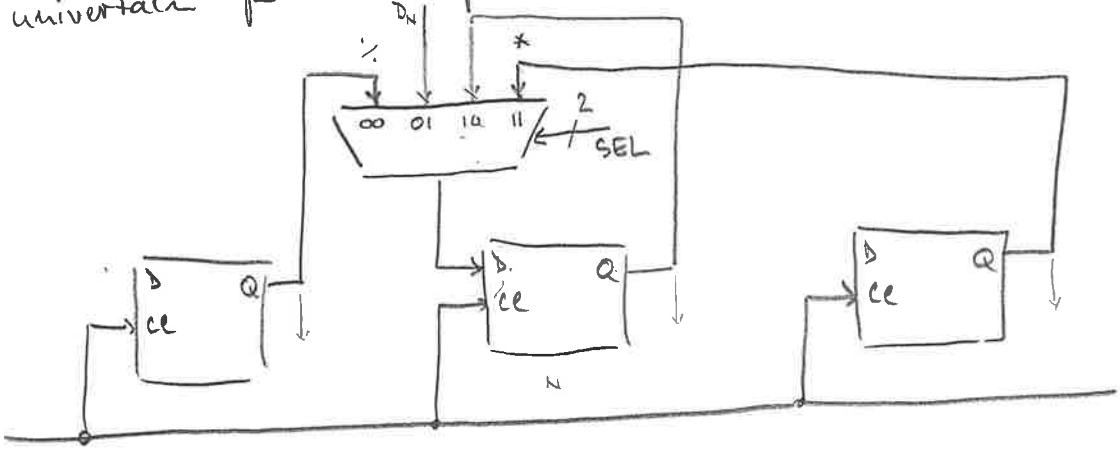
- pomični register



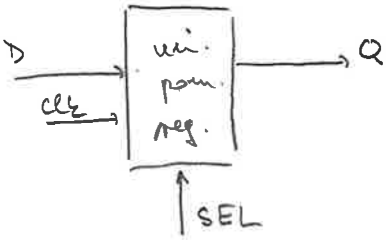
1010 ≡ 10_D

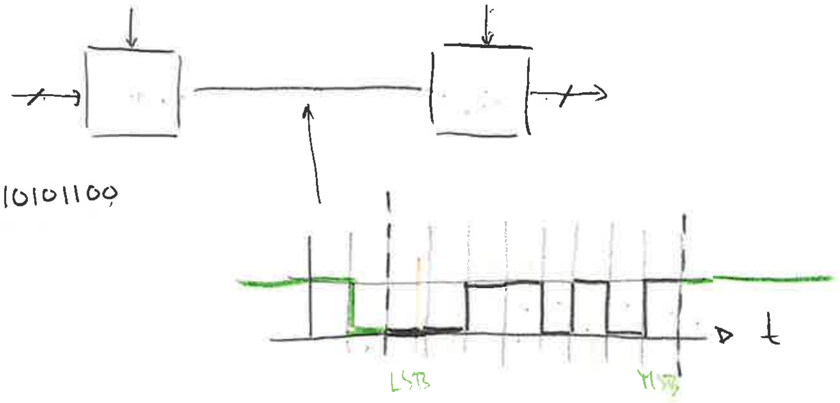
0101 ≡ 5_D

univerzalni pomični register

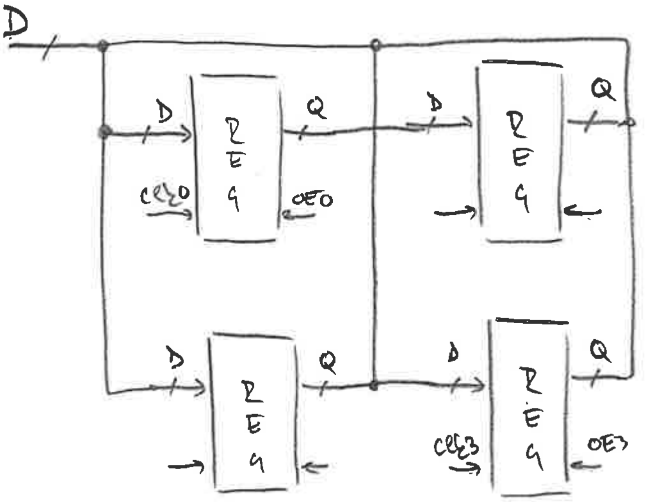


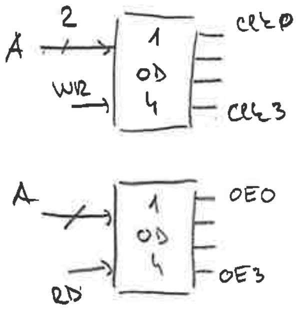
00 → deli z 2
01 → LOAD
10 → NO ACT
11 → MNOŽI z 2



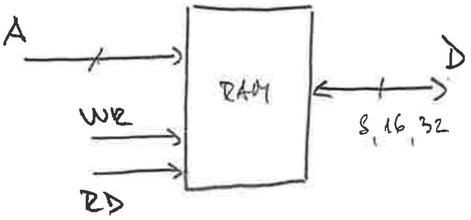

10101100
t
LSB
MSB

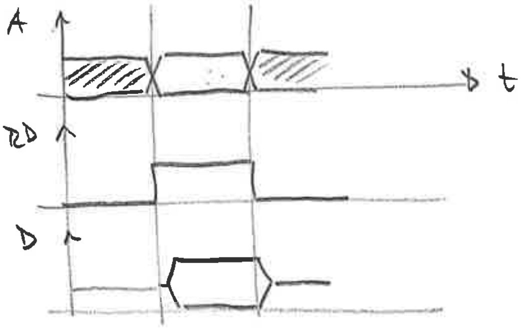
RAM


D
D
Q
REG
clk0
OE0
D
Q
REG
D
Q
REG
D
Q
REG
clk3
OE3

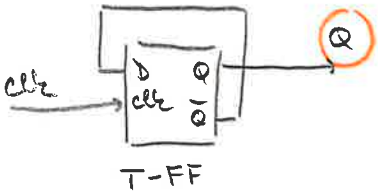

A
2
WR
1 od 4
clk0
clk3
A
RD
1 od 4
OE0
OE3

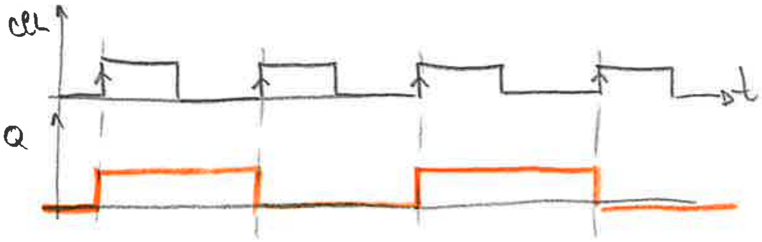
⇓

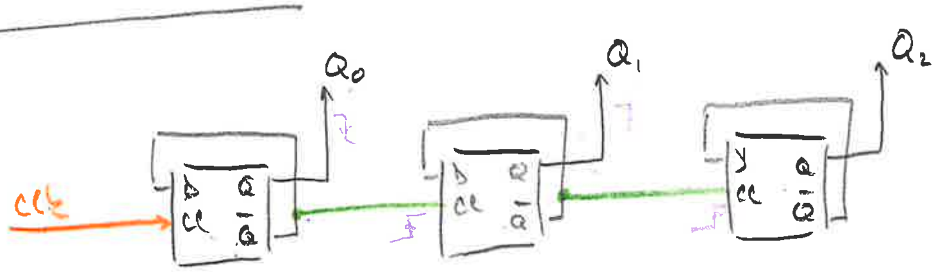

A
WR
RD
RAM
D
8, 16, 32


A
t
RD
D

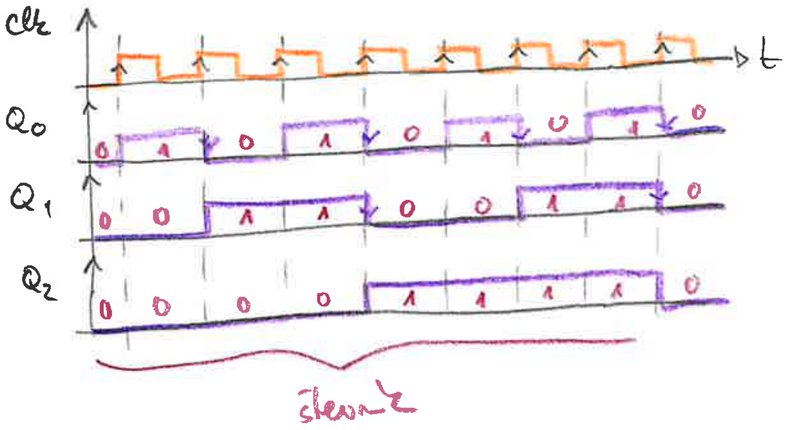
števniki

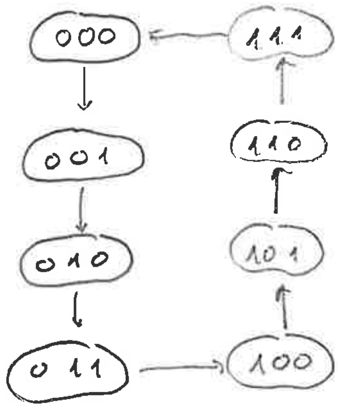




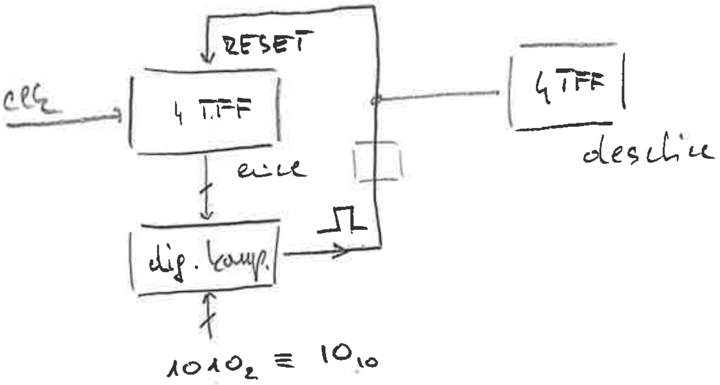


asinhroni števnik

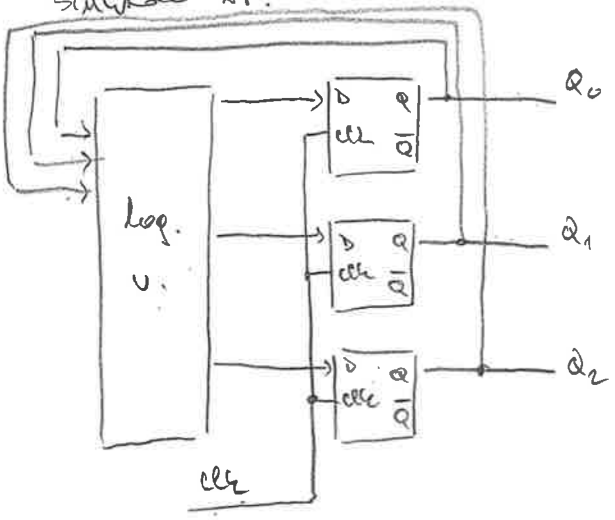




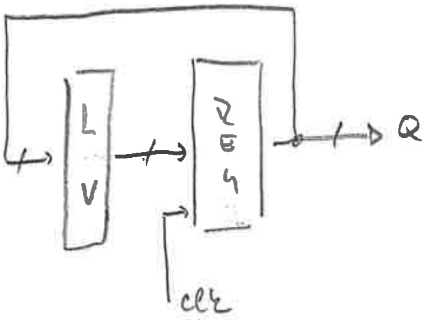
obseg: $0 \ldots 2^N - 1$; $N \equiv$ št. FF



sinhroni št.



⇒



② tabela prehodov

Q_2	Q_1	Q_0	D_2	D_1	D_0
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	0	1
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	0

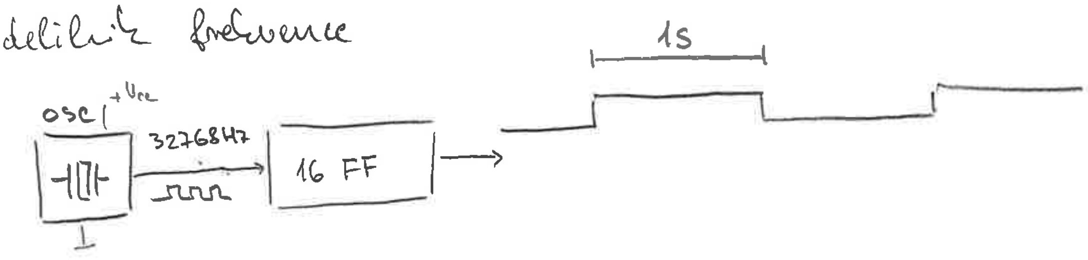
③ log. funkcije

$$D_0 = \overline{Q_0}$$

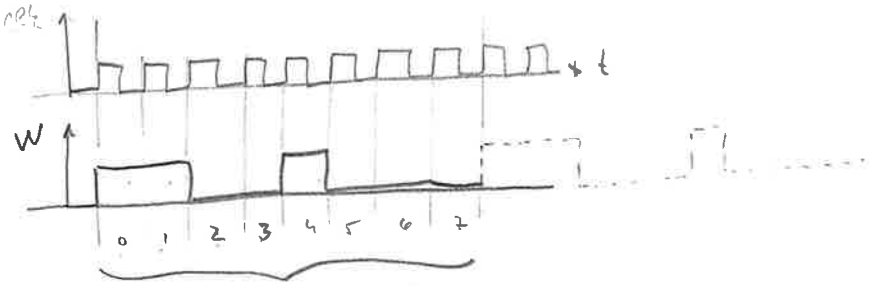
$$D_1 = Q_0 \oplus Q_1$$

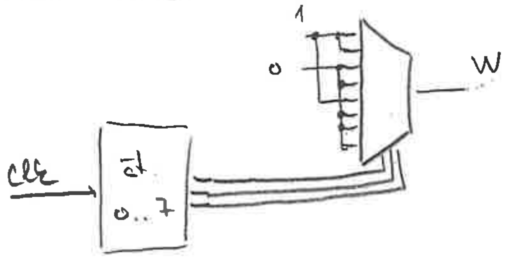
$$D_2 = Q_2 \cdot \overline{Q_1} + Q_2 \cdot \overline{Q_0} + \overline{Q_2} \cdot Q_1 \cdot Q_0$$

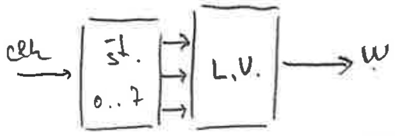
delilnik frekvence



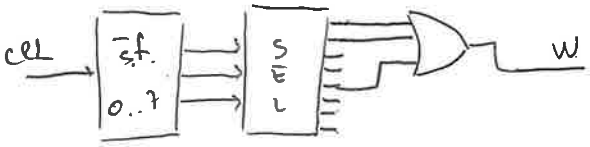
generiranje signalov - zaporedja

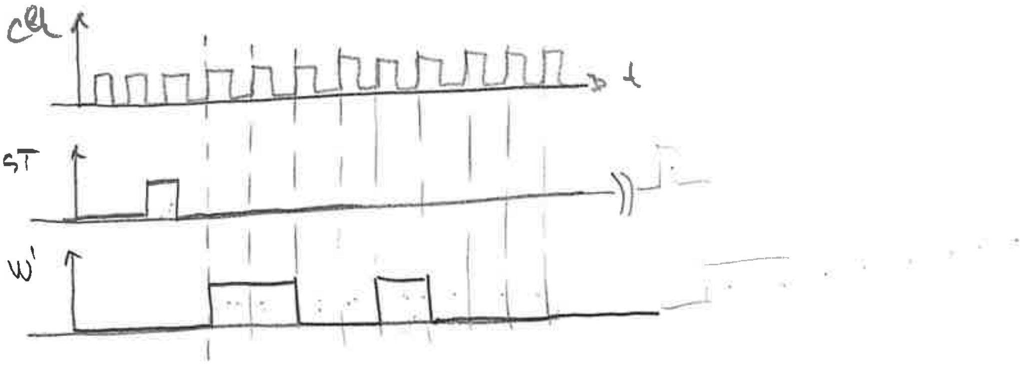


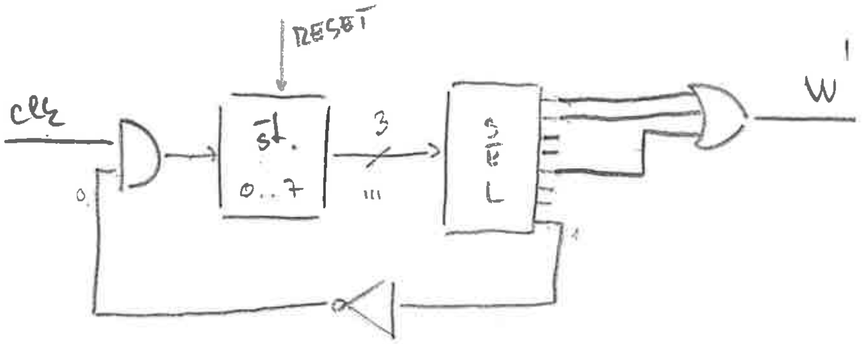




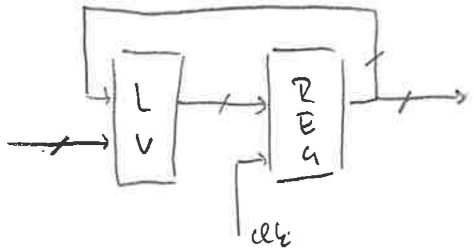
$$W = \overline{Q_2}\cdot\overline{Q_1}\cdot\overline{Q_0} + \overline{Q_2}\cdot\overline{Q_1}\cdot Q_0 + Q_2\cdot\overline{Q_1}\cdot\overline{Q_0}$$

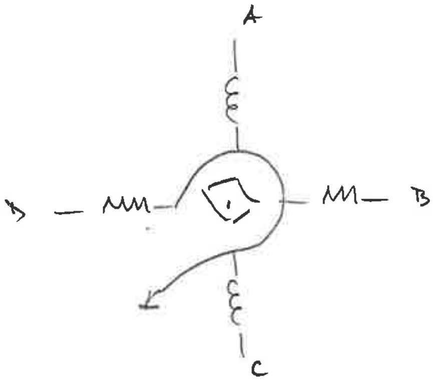






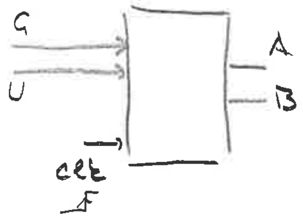
sinhroni avtomat



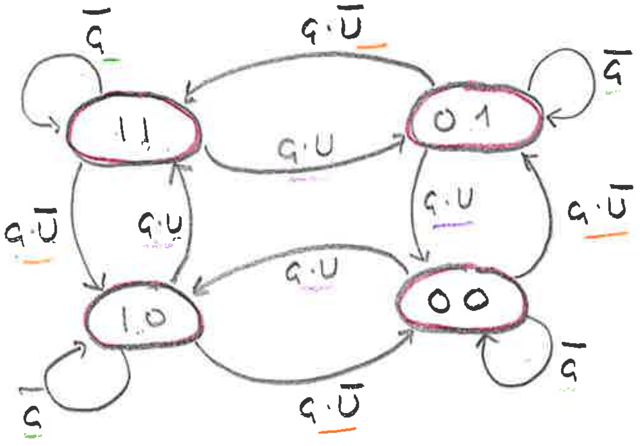


A	B	C	D
1	1	0	0
0	1	1	0
0	0	1	1
1	0	0	1

1: diagram prehodov
2: tabela -"-
3: log. enačbe
4: implementacija



①

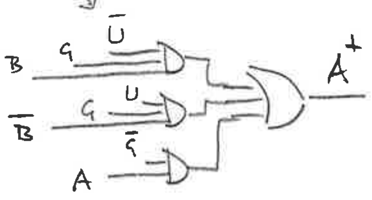


②

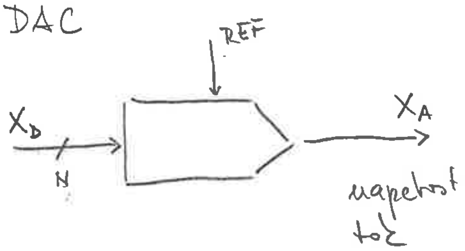
U	G	A	B	A^+	B^+
x	0	1	1	1	1
1	1	1	1	0	1
0	1	1	1	1	0
x	0	0	1	0	1
1	1	0	1	0	0
0	1	0	1	1	1
x	0	0	0	0	0
1	1	0	0	1	0
0	1	0	0	0	1
x	0	1	0	1	0
1	1	1	0	1	1
0	1	1	0	0	0

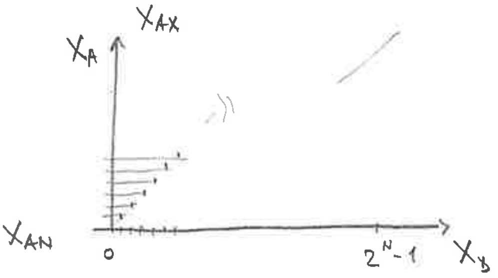
③ $A^+ = \bar{U}GB + UG\bar{B} +$
$+ \bar{G}A$

$B^+ =$ . . . . . .



DAC



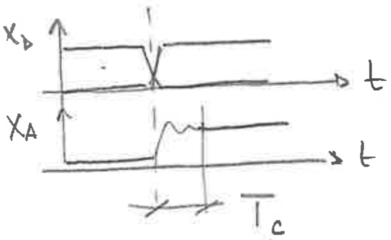


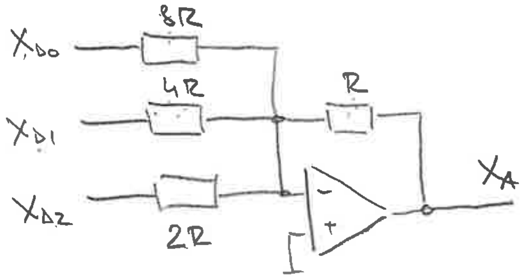
$$X_A = \frac{X_{AX} - X_{AN}}{2^N} \cdot X_D + X_{AN}$$

; $X_{AN} = 0V$, $X_{AX} = 1V$, $N = 8$

$$X_A = \frac{X_D}{2^N} = X_D \cdot \frac{1}{256}$$

N ≡ ločljivost



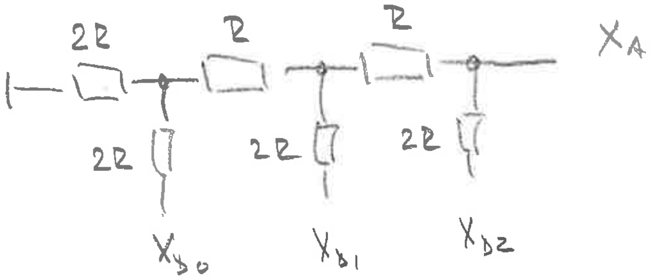


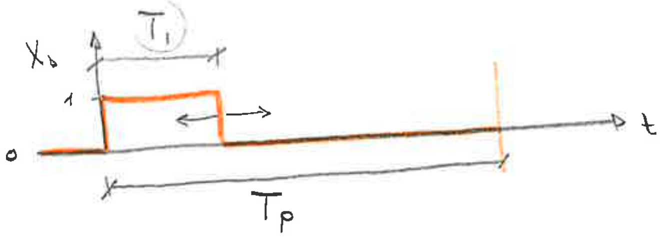
$$X_A = -\left[\frac{X_{D2}}{2} + \frac{X_{D1}}{4} + \frac{X_{D0}}{8} \right]$$

X_{D2}	X_{D1}	X_{D0}	X_A
0	0	0	0 V
0	0	1	- 0,125V
0	1	0	- 0,25V
0	1	1	- 0,375V
1	0	0	- 0,5V
⋮			⋮

✓

R/2R lestvica





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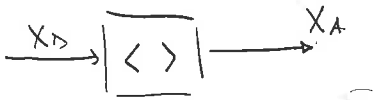
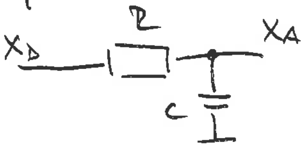
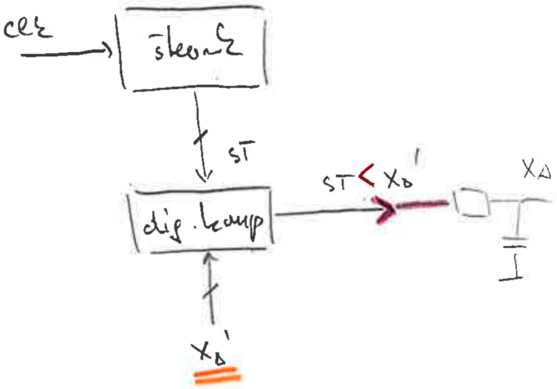


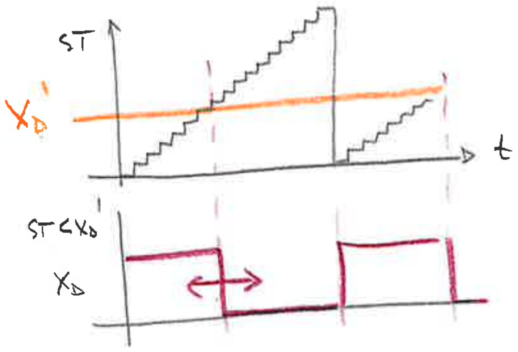
$$\underline{\underline{X_A}} = \underset{\Downarrow \atop 1}{X_D} \cdot \frac{T_1}{T_p}$$

povprečenje dosežemo z RC

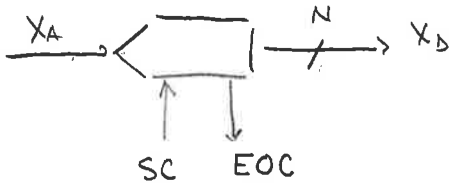


; $R \cdot C = \tau \gg T_p$

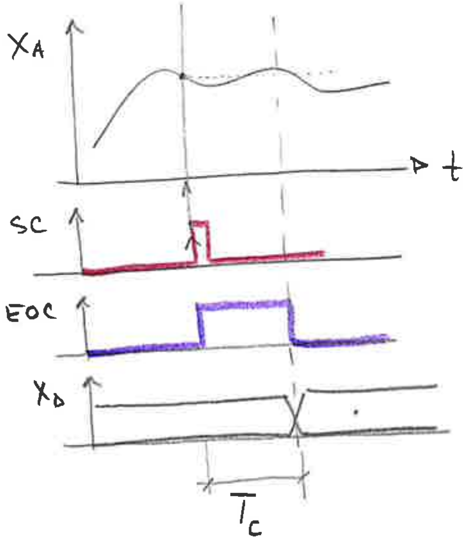


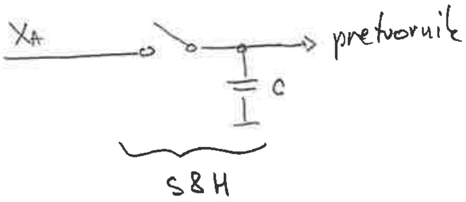


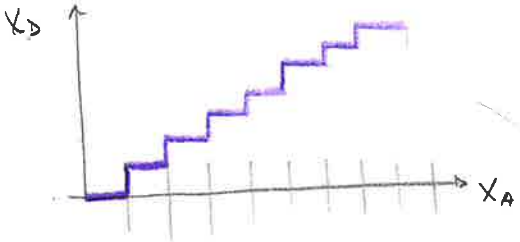
ADC



ločljivost ≡ N



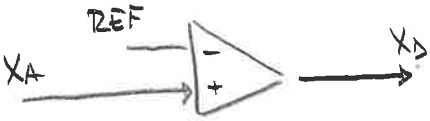




$$X_D = X_A \cdot \frac{2^N}{X_{AMAX}}$$

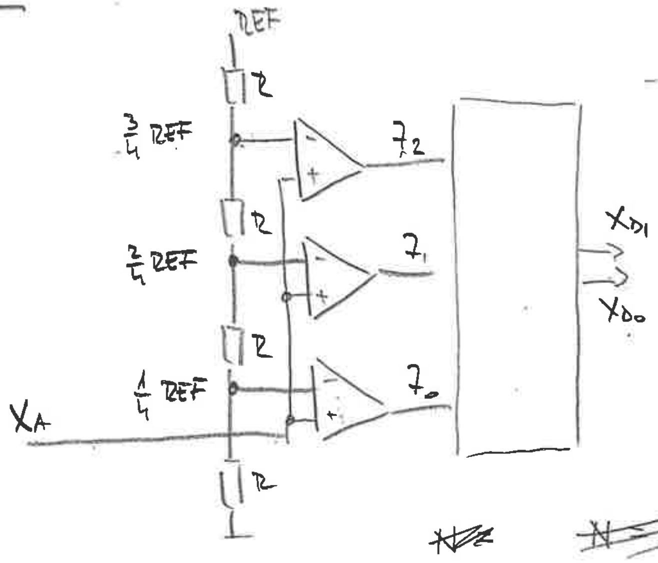
---

komparator



$$X_D = \begin{cases} 0 & \text{za} \quad X_A < REF \\ 1 & \text{za} \quad X_A > REF \end{cases}$$

flash ADC

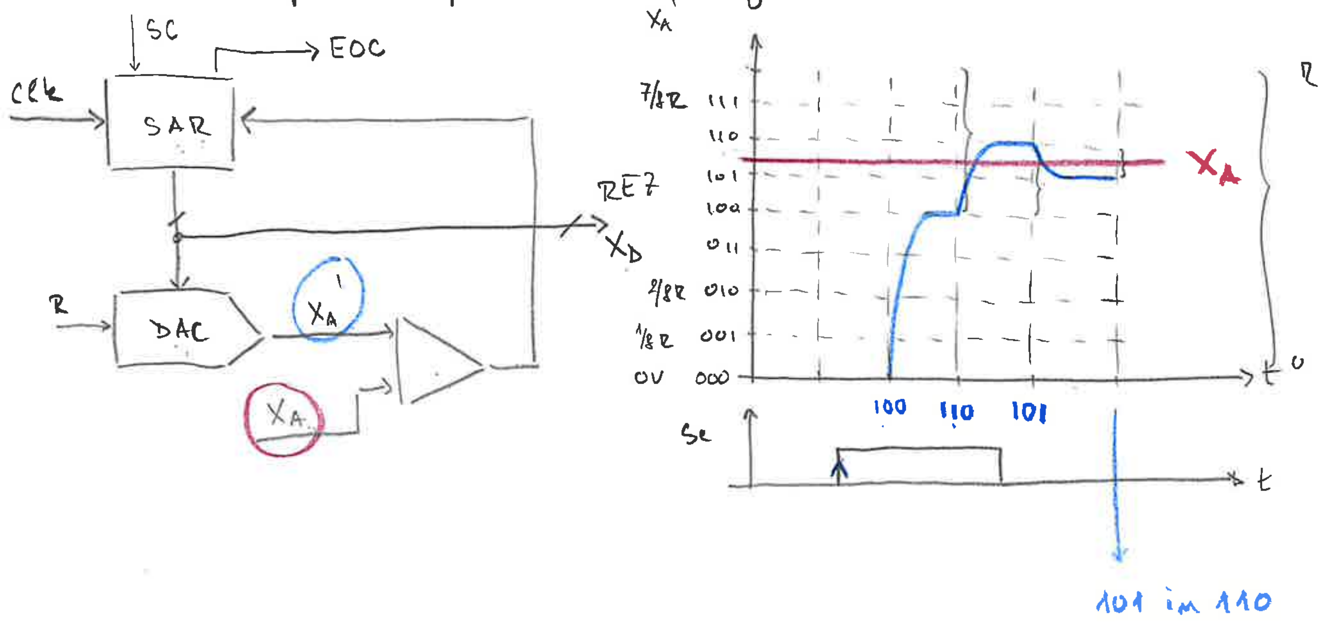


X_{D1}, X_{D0}	X_A	Z_2	Z_1	Z_0
00	$X_A < \frac{1}{4} REF$	0	0	0
01	$\frac{1}{4} REF < X_A < \frac{2}{4} REF$	0	0	1
10	$\frac{2}{4} REF < X_A < \frac{3}{4} REF$	0	1	1
11	$\frac{3}{4} REF < X$	1	1	1

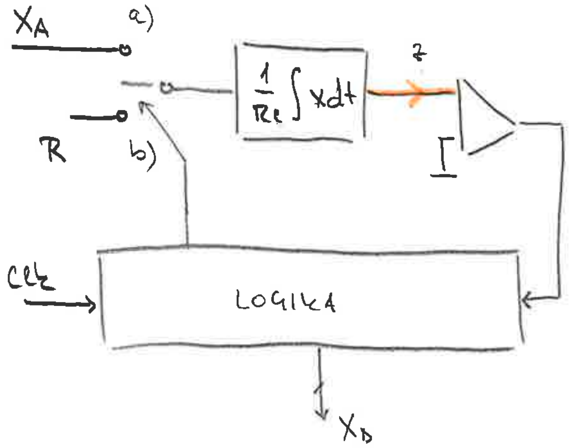
~~$N = 2^{\text{št. komp}}$~~

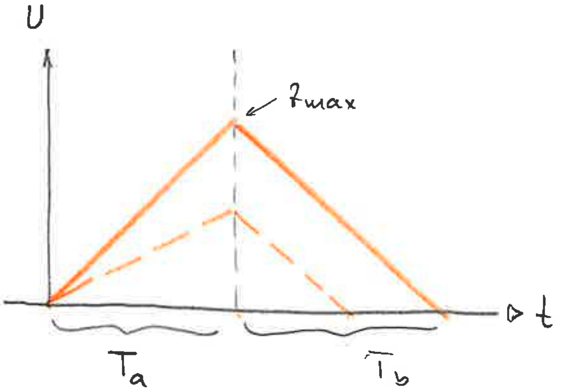
$$\text{št. komp.} = 2^N - 1$$

ADC : sukcesivna aproksimacija



ADC z dvojno strmino





a) integriramo X_A znan čas T_a

b) integriramo R ne-znan čas

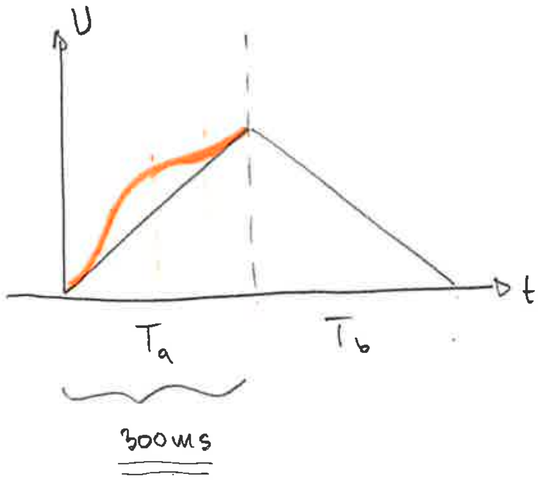
$$z = \frac{1}{RC}\int x\,dt$$

$$\text{a)}\quad \underline{z_{max}} = \frac{1}{RC}\int_{T_a} X_A\,dt = \frac{X_A\cdot T_a}{RC} = \frac{X_A\cdot T_a}{\tau}$$

$$\text{b)}\quad \underline{z_{max}} = \frac{1}{RC}\int_{T_b} R\,dt = \frac{R\,T_b}{RC} = \frac{R\cdot T_b}{\tau}$$

$$\frac{X_A\cdot T_a}{\cancel{\tau}} = \frac{R\cdot T_b}{\cancel{\tau}} \Rightarrow T_b = T_a\cdot\frac{\boxed{X_A}}{R}$$

$$\underline{\underline{\underline{N_b}}}\cdot\cancel{T_{clk}} = \underline{\underline{\underline{N_a}}}\cdot\cancel{T_{clk}}\cdot\frac{X_A}{R}$$



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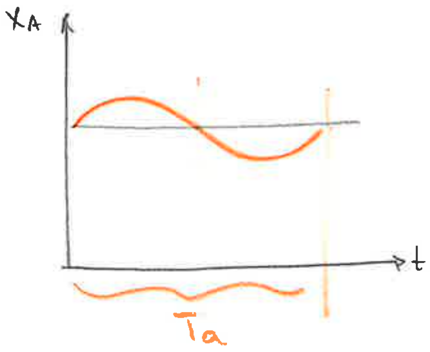
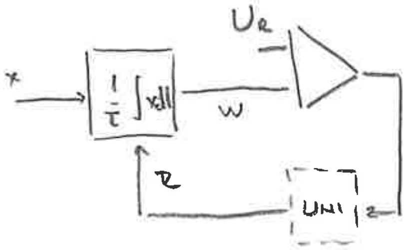


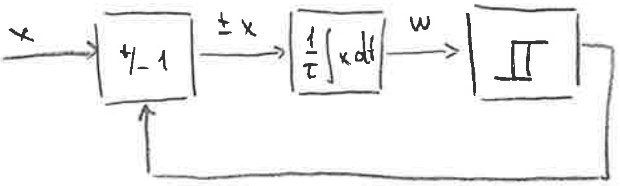
V → f

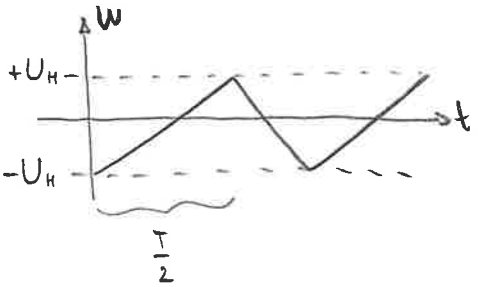


$$w = \frac{1}{\tau}\int_0^T x\,dt = x\frac{T}{\tau} = U_e \Rightarrow T = \frac{U_e}{x}\tau$$

$$\underline{\underline{f = \frac{1}{T} = \frac{x}{U_e}\cdot\frac{1}{\tau}}}$$

oppure UNI

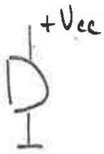


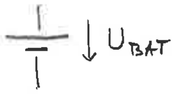


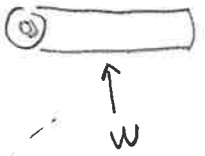
$$w = \frac{1}{\tau}\int_0^{T/2} x\,dt = \underbrace{\frac{T}{2\tau}x}_{} = 2U_H$$

$$f = x\,\frac{1}{4U_H\tau}$$

napajanja





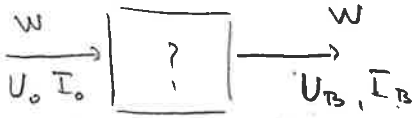


AA $\underbrace{2000\,mAh \cdot 1{,}2V}$

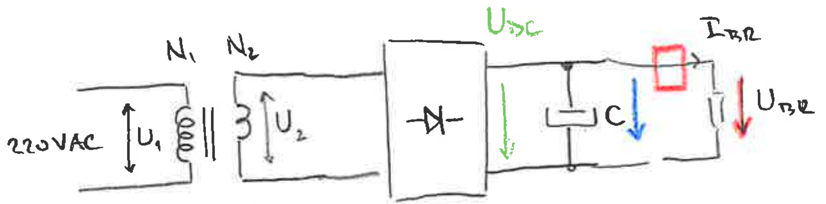
$W = 2Ah \cdot 1{,}2V = \underline{\underline{2{,}4Wh}}$

⇓

1€



aunverje

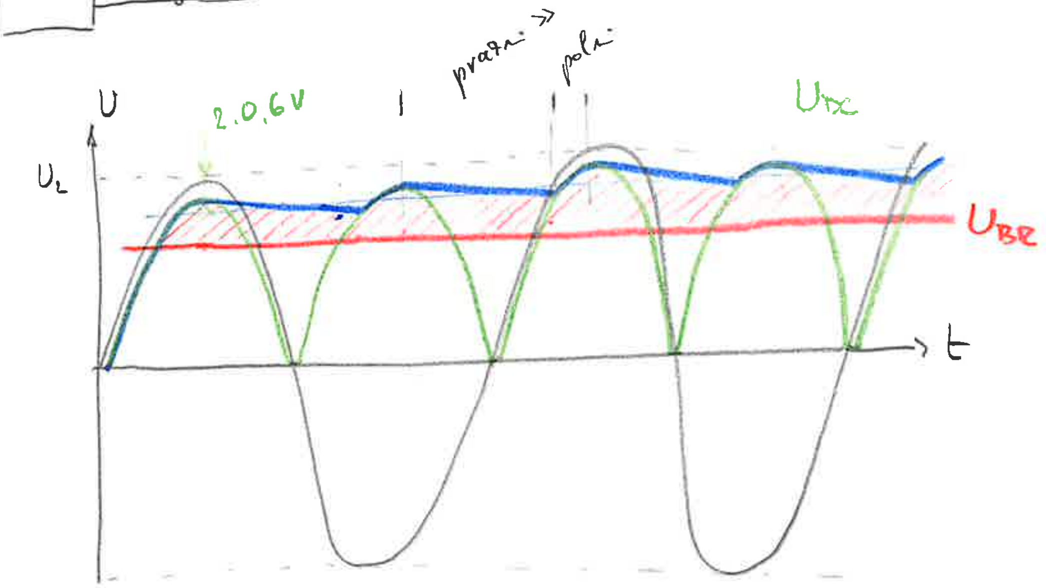


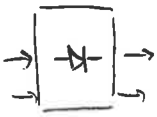
$$U_2 = U_1 \cdot \frac{N_2}{N_1}$$

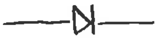
$$I_2 = I \cdot \frac{N_1}{N_2}$$

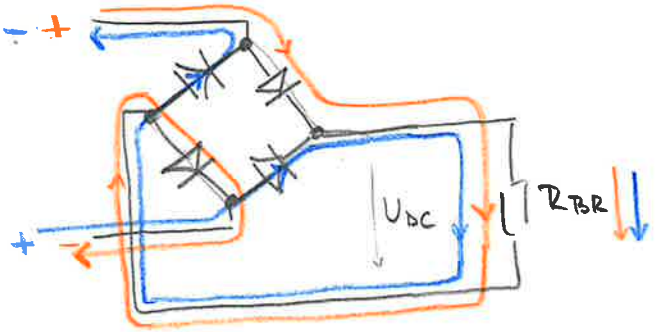
$$P_1 = P_2$$

$$U_2 \cdot I_2 = U_1 \cdot I_1$$





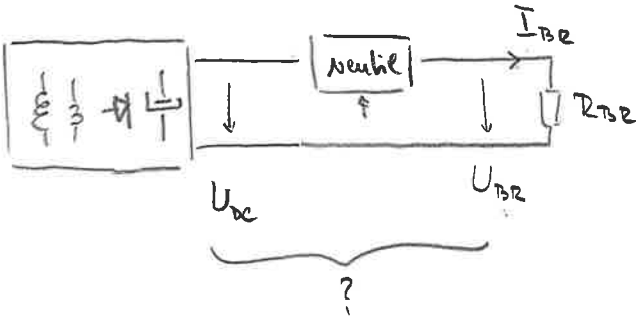


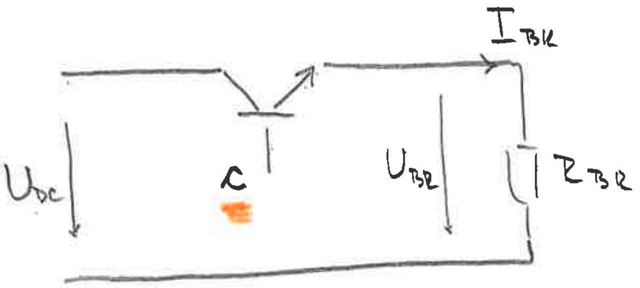


$$\Delta Q = I \cdot \Delta T = C \cdot \Delta U$$

$$\Rightarrow I_{BR} \cdot \Delta T = C \cdot \Delta U_{BR} \Rightarrow C = \frac{\overset{1A}{I_{BR}} \cdot \overset{10ms}{\Delta T}}{\underset{1V}{\Delta U_{BR}}} = \frac{1 \cdot 0{,}01}{1} = \underline{\underline{10000 \mu F}}$$

∢ Zener





TR: $I_c = \beta \cdot I_B$

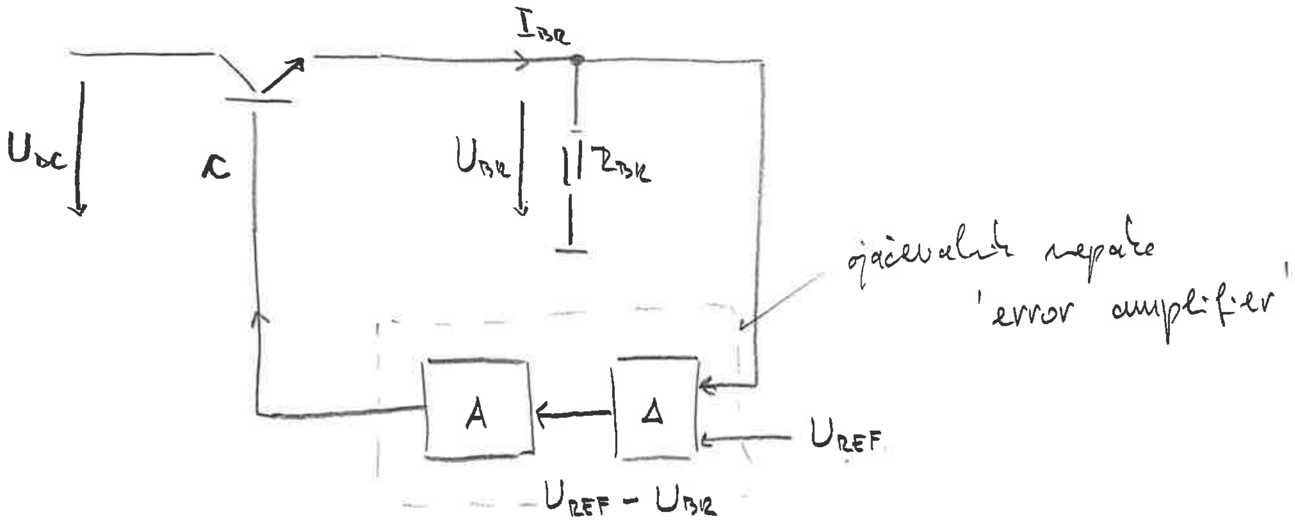
$$I_{BR} = I_E = I_B + I_c = I_B \cdot (\beta + 1) \doteq \beta I_B$$

$$I_B = I_{B0} \cdot \left(e^{U_{BE}/U_T} - 1 \right) \doteq I_{B0} \cdot e^{\frac{c - U_{BE}}{U_T}}$$

$$I_{BR} = \underbrace{\beta \cdot I_{B0} \cdot e^{\frac{c - U_{BR}}{U_T}} + (U_{DC} - U_{BR}) \cdot \gamma = \frac{U_{BR}}{R_{BR}}}$$

$$U_T \ln \frac{\frac{U_{BR}}{R_{BR}} - \gamma (U_{DC} - U_{BR})}{\beta I_{B0}} = \underset{\uparrow}{c} - U_{BR}$$

$$\underline{\underline{c}} = U_{BR} + U_T \cdot \ln \frac{\overset{I_{BR}}{\frac{U_{BR}}{R_{BR}}} - \gamma (U_{DC} - U_{BR})}{\beta I_{B0}}$$



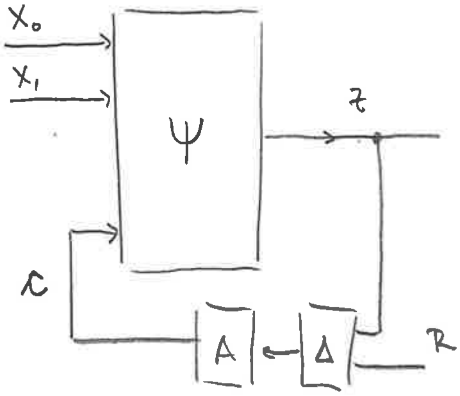
$$\underbrace{c = A \cdot \left(U_{REF} - U_{BR} \right) = U_{BR} + U_T \cdot \ln \frac{\cdots}{\cdots}}$$

$$U_{REF} - U_{BR} = \frac{U_{BR} + U_T \cdot \ln \underline{\qquad}}{A}$$

$$\boxed{U_{BR} = U_{REF} - \frac{U_{BR} + U_T \cdot \ln \underline{\qquad}}{A}}$$

$$\Downarrow \quad A \longrightarrow \infty$$

$$\boxed{U_{BR} = U_{REF}}$$



$$z = \Psi(x_0, x_1, \dots, c)$$

$$\Downarrow$$

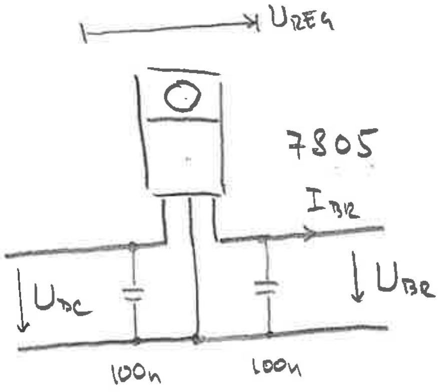
$$c = \Psi^{-1}(x_0, x_1, \dots, z)$$

$$c = A \cdot (R - z)$$

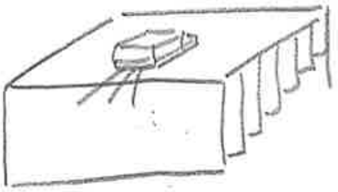
$$\Downarrow$$

$$\frac{\Psi^{-1}(x_0, x_1, \dots, z)}{A} = R - z$$

$$\boxed{\underline{\underline{z = R}} \quad \text{za} \quad A \Rightarrow \infty}$$



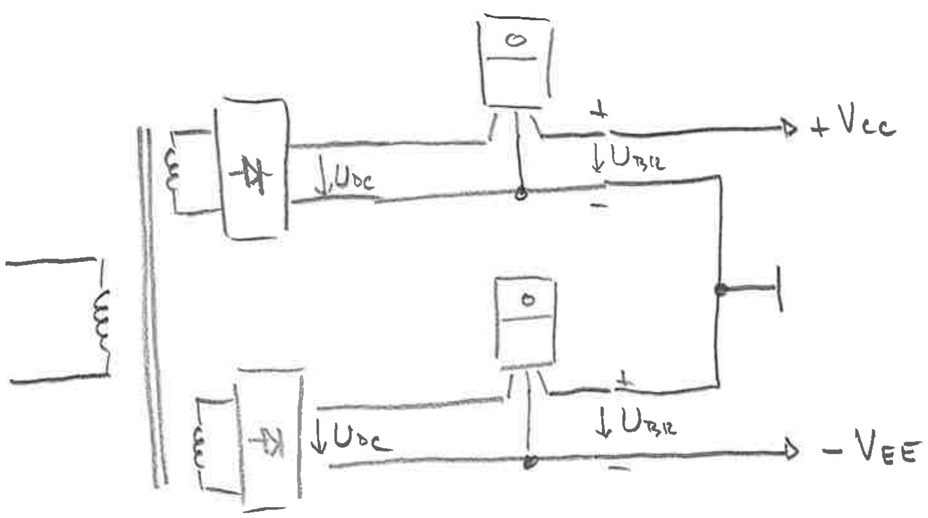
$$P_{REG} = U_{REG} \cdot I_{BR}$$

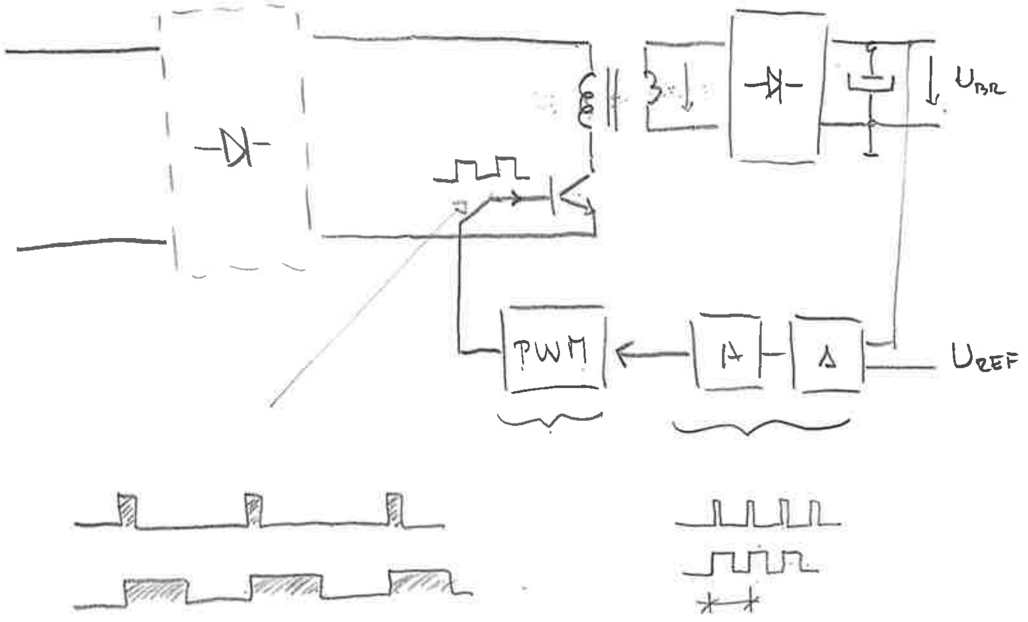


termička upornost

$[°/W]$

1 K/W





U_BR
PWM
U_REF

Regulacije

1. Regulacijska zanka

Pogosto želimo izbrano fizikalno veličino obdržati pri izbrani vrednosti navkljub vplivom okolice, ki želijo to fizikalno veličino prikrojiti po svoje. Za zgled naj služi temperatura, ki jo moramo med peko kruha v peči obdržati na 200°C. Najlažje gre tako, da temperaturo v peči merimo in po potrebi povečamo ali zmanjšamo moč gretja tako, da je temperature ravno prava.

Seveda želimo postopek avtomatizirati. Namesto osebe, ki opazuje termometer in vrti gumbe na peči, naj to delo opravi elektronsko vezje, imenujemo ga regulator. Regulator sprejema signal iz senzorja fizikalne veličine in ga primerja z željeno vrednostjo te veličine ter na podlagi razlike med njima krmili fizikalni sistem tako, da bo imela željena fizikalna veličina ravno pravo vrednost.

Tak regulacijski sistem ponazarja bločna shema na sliki 1. Na fizikalni sistem *F* vpliva več dejavnikov iz okolice, imenujmo jih A_1, A_2, A_3, poleg teh pa lahko na fizikalni sistem vplivamo še preko dejavnika *c*, ki ga neodvisno od vplivov okolice krmilimo sami. Izhodna veličina fizikalnega sistema je odvisna od vseh vhodnih dejavnikov in je označena z *Y*; to je hkrati tudi veličina, ki jo merimo s primernim senzorjem. V regulatorju izmerjeno vrednost odštejemo od željene vrednosti, ki je označena z Y_G, rezultat odštevanja pa je napaka regulacije *err*. Napako v regulatorju dodatno matematično obdelamo tako, da dobimo primeren signal za poseganje v sistem preko dejavnika *c*. Najenostavneje gre, če posegamo v sistem sorazmerno napaki *err*; za majhne razlike med željeno in dejansko vrednostjo regulirane fizikalne veličine posegamo v sistem le malo, za velike razlike pa močneje. Pravimo, da v fizikalni sistem posegamo proporcionalno. Regulator naj zato poleg vezja za računanje napake vsebuje še ojačevalnik, ki napako *err* poveča na primerno vrednost za poseganje v fizikalni sistem. Ojačevalniku pripišemo ojačenje A. Kako veliko pa naj bi to ojačenje bilo?

Zapišimo enačbo. Dejavnik *c* izračunamo kot:

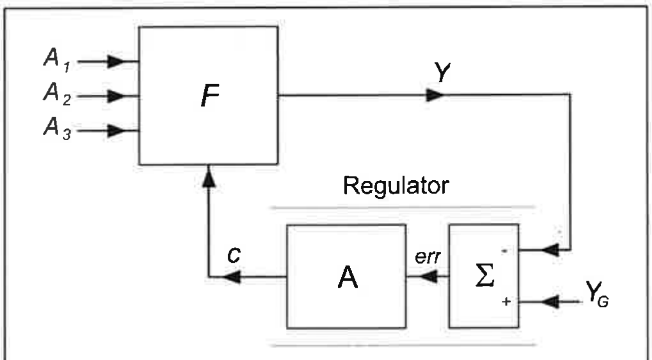
$$c = A(Y_G - Y)$$

Fizikalnemu sistemu pripišemo lastnost *F*, ki povezuje izhodno vrednost *Y* in vplivne veličine:

$$Y = F(A_1, A_2, A_3, c)$$

Z nekaj truda in matematične sreče lahko zgornjo formulo preuredimo v:

$$c = F^{-1}(A_1, A_2, A_3, Y)$$



Slika 1: Fizikalni sistem in regulator tvorita regulacijsko zanko

Ter tako zvemo kakšen dejavnik *c* je potreben, da fizikalni sistem da od sebe veličino z vrednostjo *Y* kljub vplivom ostalih dejavnikov A_1 do A_3. Ta dejavnik *c* dobimo iz regulatorja, zato:

$$A(Y_G - Y) = F^{-1}(A_1, A_2, A_3, Y)$$

Po preureditvi enačbe dobimo:

$$Y = Y_G - \frac{F^{-1}(A_1, A_2, A_3, Y)}{A}$$

Uporabili smo operatorski zapis za integriranje: operator *p* predstavlja odvajanje, recipročna vrednost *p* pa integriranje. Združitev zgornjih treh enačb da:

$$T_{H2O} = T_G \cdot \frac{1}{1+\frac{\lambda}{\beta A}} \cdot \frac{1}{1+\frac{\tau_{H2O}}{\beta A+\lambda}p} \quad + \quad T_E \cdot \frac{1}{1+\frac{\beta A}{\lambda}} \cdot \frac{1}{1+\frac{\tau_{H2O}}{\beta A+\lambda}p}$$

Preverimo skrajne rešitve te enačbe:

- Brez povratne regulacijske zanke ($A = 0$) je temperatura vode T_{H2O} odvisna le od temperature okolice T_E. Tej temperaturi se približuje eksponentno, kot smo že spoznali pri elektroniki ob analizi operatorskega zapisa prenosne funkcije oblike ${}^{1}/_{(1+\tau p)}$. Časovna konstanta približevanja τ je podana z lastnostmi vode τ_{H2O} in sten bazena λ.
- Za neskončno veliko ojačenje ($A \to \infty$) temperatura okolice T_E ne vpliva na temperaturo vode T_{H2O} v bazenu. Matematika kaže, da je takrat temperatura vode T_{H2O} natanko enaka željeni temperaturi T_G. To velja tudi v primeru, ko željeno temperaturo T_G hipoma spremenimo. Seveda bi bila za hipno spremembo temperature večje količine vode potrebna zelo velika moč P_C, delujoča kratek čas. Ker grelnik tolikšne moči ne obstaja lahko ugibamo, da pride do prehodnega pojava, med katerem se temperatura vode bolj ali manj hitro, pač glede na lastnosti vode in grelnika, približuje željeni temperaturi. Ko je približevanje končano, je temperatura vode enaka željeni temperaturi ($T_{H2O} = T_G$).

Za končno velika ojačenja *A* je približevanje temperature vode željeni temperaturi eksponentno (zaradi člena $\frac{1}{1+\frac{\tau_{H2O}}{\beta A+\lambda}p}$), časovno konstanto približevanja pa diktirajo lastnosti vode, grelnika in ojačenja. Večje, ko je ojačenje, hitrejše je približevanje. Seveda je za hitrejše približevanje potrebna večja moč grelnika P_C. če grelnik potrebne moči ne zmore, bo približevanje enakomerno do trenutka, ko grelnik zahtevano moč zmore, od takrat naprej pa eksponentno.

Zgornje enačbe napovedujejo še eno, ne tako lepo lastnost sistema. Običajno je temperatura okolice drugačna od željene temperature, zato iz bazena vez čas odteka toplota v okolico. Ker želimo temperaturo bazena obdržati na željeni vrednosti, mora grelnik ves čas enako količino toplote uvajati v bazen, torej P_C ni nič. Zato tudi signal *c* ni nič in dejanska temperatura vode T_{H2O}ne more biti enaka željeni temperaturi T_G. V realnem primeru, ko je ojačenje v povratni zanki končno, opisani regulacijski sistem ne more zagotavljati enakosti temperatur; enakosti se lahko le čim bolj približamo s povečevanjem ojačenja *A*. Enako velja za katerikoli regulirani sistem: ojačenje v povratni zanki želimo čim bolj povečati, saj bo po prehodnem pojavu regulirana veličina bolj podoba željeni, pa še hitreje bo dosežena, če le zmoremo močno poseganje v sistem.

3. Reguliranje sistema drugega reda

3.1. Proporcionalna regulacija

V pravkar analiziranem sistemu, ki je bil sicer precej idealiziran, smo upoštevali le časovno akumuliranje toplote v bazenu in prišli do operatorsko zapisane enačbe prvega reda za ta sistem. V njej je operator *p* nastopal v prvi potenci in ugotovitve se nanašajo na vse regulacijske sisteme, ki jih lahko popišemo z enačbo istega reda. Obstajajo pa tudi sistemi, v katerih več elementov vnaša kasnitve. Kako se obnašajo taki regulirani sistemi?

Za primer uporabimo isti bazen, a tokrat pripišimo kasnitev še termometru. Znano je, da termometer potrebuje nekaj časa preden sporoči pravo temperaturo. Ta čas je odvisen od toplotne kapacitete termometra, prevodnosti njegovega ohišja in podobno. Za pravilen odčitek medicinskega termometra

- Če je ojačenje v povratni regulacijski zanki končno veliko, je temperatura vode nekje med temperaturo okolice in željeno temperaturo, odvisno od temperatur in lastnosti bazena ter povratne zanke.

Je pa to stacionarno stanje treba doseči. Po vsaki spremembi željene ali okoliške temperature je treba počakati čas prehodnega pojava, da sistem doseže stacionarno stanje. Raziščimo obnašanje tako reguliranega sistema, ki ga opisuje operatorsko zapisana enačba drugega reda, med prehodnim pojavom. Iz zgoraj zapisane operatorske formule je videti, da sta odziva na željeno in okoliško temperaturo po obliki sorodna, zato bomo tule obravnavali le obliko odziva na spremembo željene temperature; oblika odziva na spremembo druge temperature je po obliki enaka. Obravnavamo torej enačbo:

$$T_{H2O} = \frac{T_G \beta A(1+\tau_M p)}{ap^2 + bp + c} \qquad \Leftrightarrow \qquad a \cdot \ddot{T}_{H2O} + b \cdot \dot{T}_{H2O} + c = T_G \cdot \beta A + \dot{T}_G \cdot \beta A \tau_M$$

Pri tem: $a = \tau_{H2O}\, \tau_M$, $b = \tau_{H2O} + \lambda \tau_M$, $c = \lambda + \beta A$

Obnašanje te enačbe med približevanjem stacionarnemu stanju določajo rešitve homogene verzije iste enačbe, torej :

$$a \cdot \ddot{T}_{H2O} + b \cdot \dot{T}_{H2O} + c = 0$$

Rešitve te enačbe iščemo v obliki:

$$T_{H2O} = T_{H2Os}(1 - e^{\gamma_{1,2} t}) \qquad \text{kjer:} \quad \gamma_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Za majhno razliko med dejansko in željeno temperaturo vode v stacionarnem stanju potrebujemo veliko ojačenje *A*; žal je ojačenje skrito v parametru *c*. Če ojačenje preveč povečamo, postane parameter *c* dovolj velik, da je vrednost pod korenom negativna. Koreni karakteristične enačbe so zaradi tega kompleksni, kar implicira rešitve v obliki:

$$T_{H2O} = T_{H2Os}\left(1 - e^{RE(\gamma_{1,2})t} * e^{IM(\gamma_{1,2})t}\right) = T_{H2Os}\left(1 - e^{-\frac{b}{2a}t} * \cos\left(\frac{\sqrt{b^2 - 4ac}}{2a}\, t\right)\right)$$

Taka oblika rešitve pa pomeni nihanje okoli stacionarne vrednosti T_{H2Os}; nihanja si ne želimo. Nihanje bo po zgornji enačbi počasi izzvenevalo; časovna konstanta izzvenevanja je podana z $^{2a}/_{b}$. Ker nihanja ne želimo, je največje možno ojačenje (optimalno ojačenje A_{opt}) v povratni zanki omejeno z:

$$b^2 - 4ac = 0 \qquad \rightarrow \qquad A_{opt} = \frac{(\tau_{H2O} + \lambda \tau_M)^2}{4\, \beta\, \tau_{H2O}\, \tau_M} - \frac{\lambda}{\beta}$$

Če izberemo ojačenje manjše od A_{opt}, je približevanje željeni vrednosti počasnejše. Taka izbira namreč povzroči dva realna korena karakteristične enačbe, od katerih je en manjši od optimalnega in torej povzroči še daljšo časovno konstanto.

Temperatura vode v stacionarnem stanju ne more biti enaka željeni, ker ne želimo dušenega nihanja okoli željene temperature in torej ne smemo izbrati neskončno velikega ojačenja *A*. Proporcionalna regulacija takrat, ko imamo opravka z reguliranim sistemom drugega reda, ne daje dovolj dobrih rezultatov.

3.2. Proporcionalno diferencialna regulacija

Ker so fizikalne lastnosti reguliranega sistema marsikdaj nespremenljive, se v tem zapisu osredotočamo na regulator, ki je v domeni elektronike. Namesto ojačevalnika v povratni regulacijski zanki ali vzporedno ojačevalniku lahko vežemo dodatne elektronske module in se nadejamo izboljšav regulacije. Prvi pomislek bi morda bil sledeč: če se temperatura vode hitro približuje željeni vrednosti in je hkrati razlika temperatur majhna,

$$P_c = \beta c = \beta \frac{1}{\tau_I p}(T_G - T_M) = \beta \frac{1}{\tau_I p}(T_G - T_{H2O})$$

In ponovno lahko z uporabo na začetku podanih formul izpeljemo izraz za temperaturo vode T_{H2O}:

$$T_{H2O} = \frac{T_G\,\beta + T_E\,\lambda\,\tau_I\,p}{\tau_{H2O}\tau_I p^2 + \lambda\,\tau_I\,p + \beta} = \frac{T_G\,\beta + T_E\,\lambda\,\tau_I\,p}{ap^2 + bp + c}$$

Pri tem so: $a = \tau_{H2O}\tau_I$, $b = \lambda\,\tau_I$, $c = \beta$

Tokrat v povratni vezavi nismo uporabili ojačevalnika, zato je vredno posebej preveriti temperaturo vode v stacionarnem stanju, ko so vsi odvodi enaki nič. Če iz zgornjega izraza odstranimo vse člene z operatorjem *p*, dobimo:

$$T_{H2Os} = T_G$$

Torej je v stacionarnem stanju temperatura vode enaka željeni temperaturi tudi v primeru, ko v povratni zanki ni ojačevalnika. Poglejmo še obnašanje reguliranega sistema med prehodnim obdobjem. Ob enakem razmišljanju tudi tokrat iščemo korene karakteristične enačbe ustrezne homogene diferencialne enačbe za imenovalec:

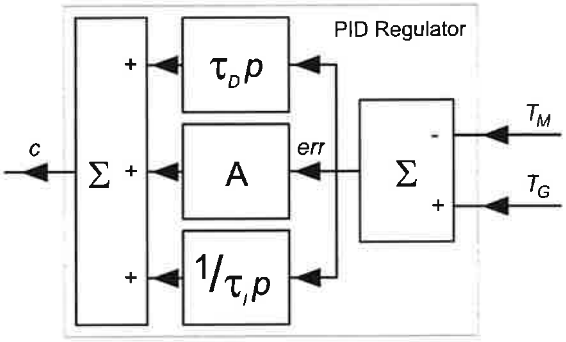
$$\gamma_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-\lambda\,\tau_I \pm \sqrt{\lambda^2\tau_I{}^2 - 4\,\tau_{H2O}\,\beta}}{2\,\tau_{H2O}\,\tau_I}$$

V optimalnem primeru mora biti izraz pod korenom enak nič, takrat bo približevanje temperatur najhitrejše. Kaže, da mora imeti uporabljeni integrator dolgo časovno konstanto (zelo približno rečeno: primerljivo s časovno konstanto bazena vode popravljeno za nekaj parametrov). Če bo izraz pod korenom negativen, bo temperature vode dušeno nihala okoli željene vrednosti. Če bo časovna konstanta uporabljenega integratorja predolga, bo izraz pod korenom pozitiven in bo zaradi tega približevanje temperature vode željeni vrednosti eksponentno in počasnejše.

Z integralno regulacijo torej dosežemo, da je regulirana vrednost enaka željeni, a moramo zaradi dolgih časovnih konstant na to čakati dolgo časa. Poleg tega sprememba vpliva okolice, v našem primeru je to temperatura okolice T_E, močno vpliva na regulirano veličino, saj najdemo v formuli odvod okoliške temperature. Tudi integralna regulacija ima torej slabe lastnosti. Izboljšanje lahko pričakujemo s kombinacijo vseh treh načinov regulacije.

3.4. Proporcionalno integralna diferencialna regulacija – PID

Tokrat v regulatorju uporabimo vse tri do sedaj obravnavane module: ojačevalnik, diferenciator in integrator. Njihove izhodne signale seštejemo v skupni izhodni signal *c*, slika 6.

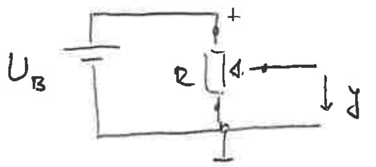


Slika 6: Proporcionalno integralno diferencialni (PID) regulator

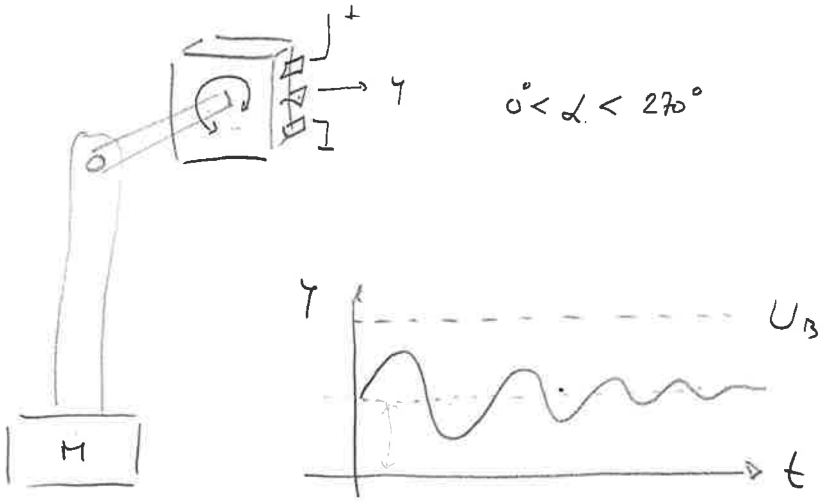
Izhodni signal *c* takega regulatorja v našem primeru gretja vode je:

$$c = \left(A + \tau_D p + \frac{1}{\tau_I p}\right)(T_G - T_M)$$

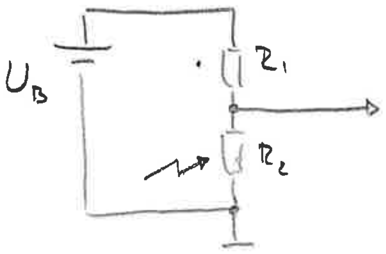
Zdaj imamo na razpolago tri parametre, s katerimi lahko optimiramo kakovost regulacije. Z velikim ojačenjem *A* dosežemo hiter odziv sistema na spremembe referenčne vrednosti ali vplivov okolja, z časovno konstanto diferenciatorja τ_D preprečimo nihanje okoli stacionarne vrednosti regulirane veličine. S



$$y = \frac{\alpha}{270} \cdot U_B$$



$0° < \alpha < 270°$



$$y = U_B \frac{R_2}{R_1 + R_2}$$

; $R_1 >> R_2 \Rightarrow \underline{\underline{y = U_B \frac{R_2}{R_1}}}$

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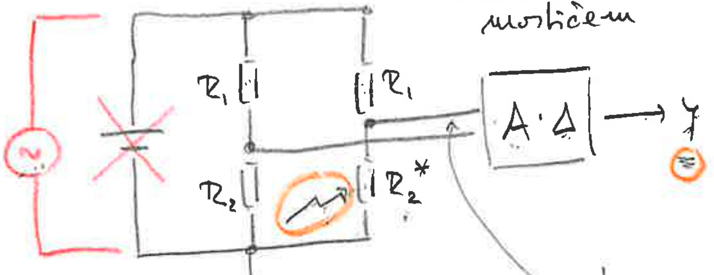
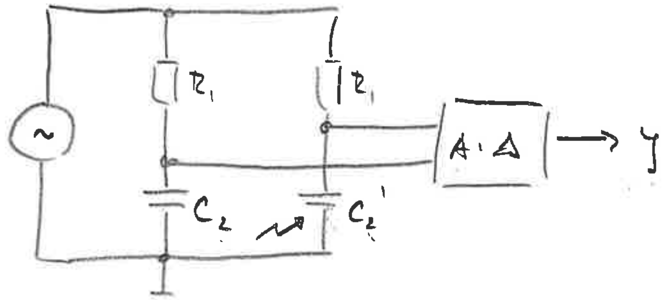
Wheatston mostičem



spremeni le velikost

$$y' = A_x \cdot \cos \omega_x t \cdot \frac{R_2^*}{R_1}$$

$x = A_x \cdot \cos \omega_x t$



spremeni se velikost in faza

Wheatstonov mostiček : samo R ⟶ amplituda

R, C, L ⟶ amplituda ali/in faza

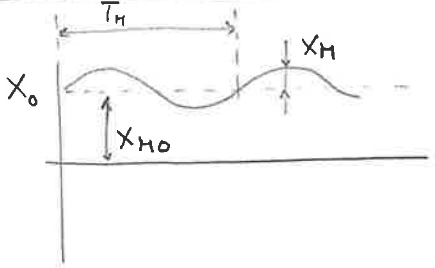
frekvenčna modulacija ⟶ oscilator

$$x = A_x \cdot \cos(\omega_x t + \varphi_x)$$

fazna modulacija

amplitudna modulacija

Amplitudna modulacija

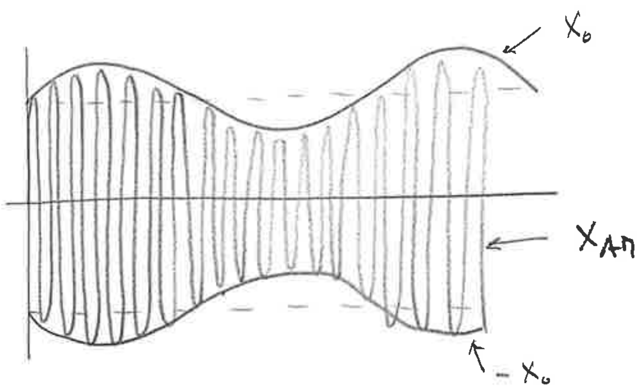


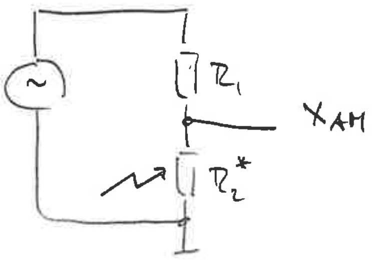
$\omega_M = 2\pi/T_M$

$X_M < X_{M0}$

$$x_{AM} = (\underbrace{X_{M0} + X_M \cdot \cos\omega_M t}_{X_0 \equiv A_x}) \cdot \cos\omega_x t =$$

$$= X_{M0} \cdot \cos\omega_x t + \frac{X_M}{2}\left[\cos(\omega_M + \omega_x)t + \cos(\omega_M - \omega_x)t\right]$$





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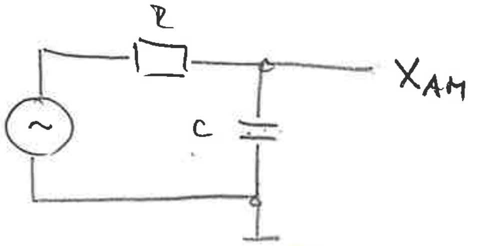
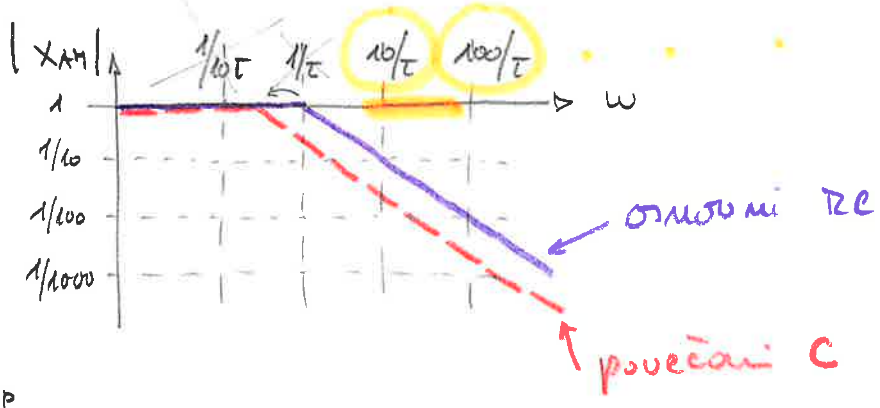
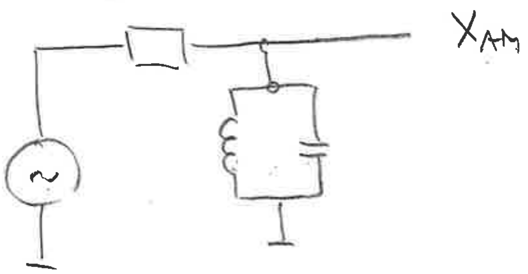
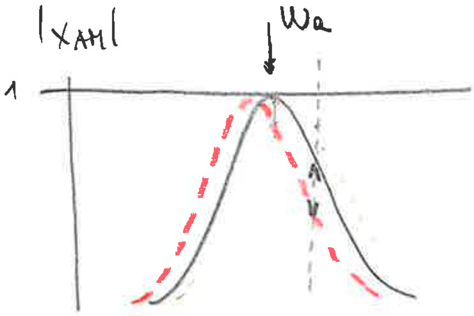


$\frac{1}{1+Tp}$



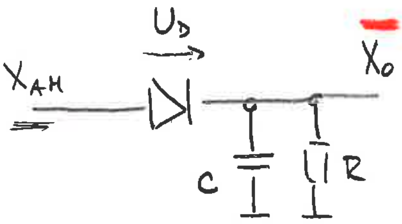
$3\omega_p < \omega_x < 30\omega_p$





$\omega_x > \omega_R$ ali $\omega_x < \omega_R$

$X_{AM} = X_0 \cdot \cos\omega_x t$



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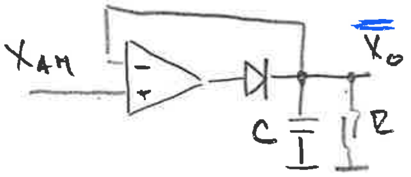
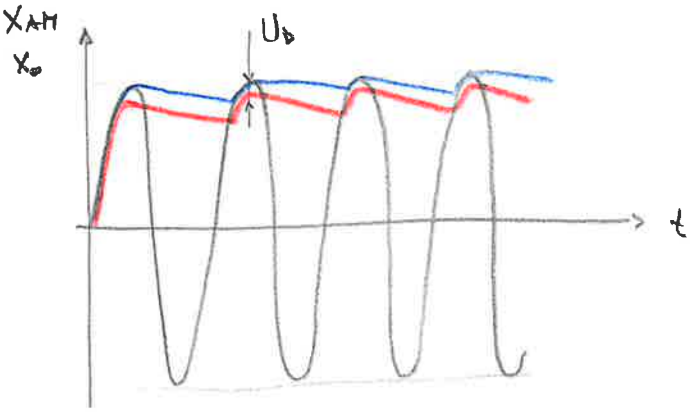
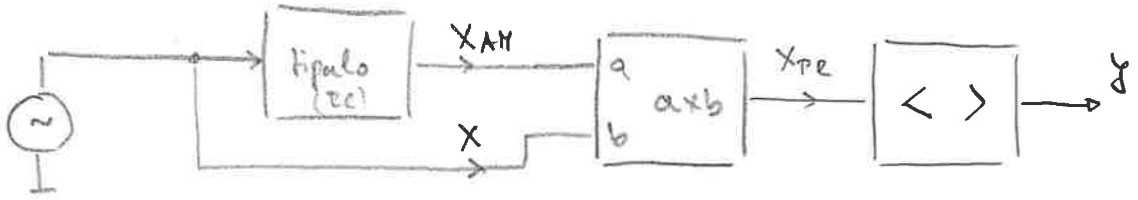






$x = A_x \cdot \cos \omega_x t$ ✓

$X_{AM} = X_{OU} \cdot \cos \omega_x t \quad + X_{MOT} \cdot \cos \omega_M t$

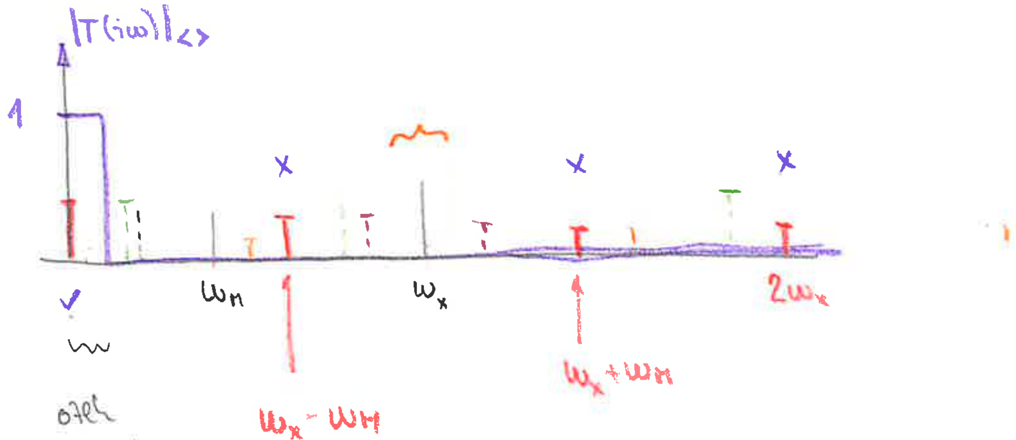
$$X_{PR} = A_x \cdot \cos\omega_x t \cdot X_{OU} \cdot \cos\omega_x t \quad + A_x \cdot \cos\omega_x t \cdot X_{MOT} \cdot \cos\omega_M t$$

$$= \frac{A_x \cdot X_{OU}}{2}\left[\cos 2\omega_x t + \cos 0 \cdot t\right] \quad + \frac{A_x \cdot X_{MOT}}{2}\left[\cos(\omega_x+\omega_M)t + \cos(\omega_x-\omega_M)t\right]$$

$$= \cancel{\frac{A_x \cdot X_{OU}}{2} \cdot \cos 2\omega_x t} + \frac{A_x \cdot X_{OU}}{2} \quad + \quad - \quad '' \quad -$$

$$\boxed{y = \frac{A_x \cdot X_{OU}}{2}}$$

sinhrona demodulacija



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množilnik ≡ težko narediti v analogni tehniki

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goljufamo

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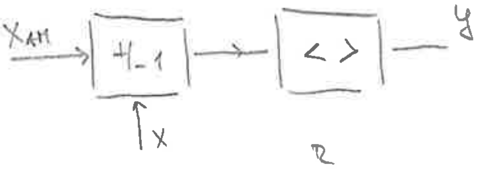
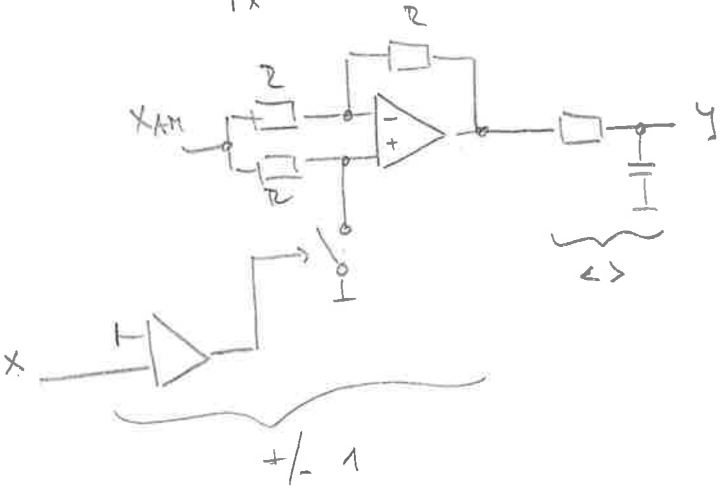
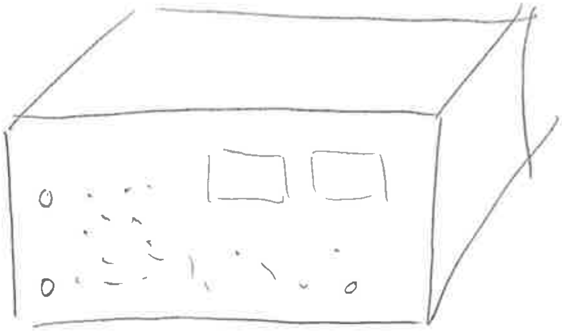




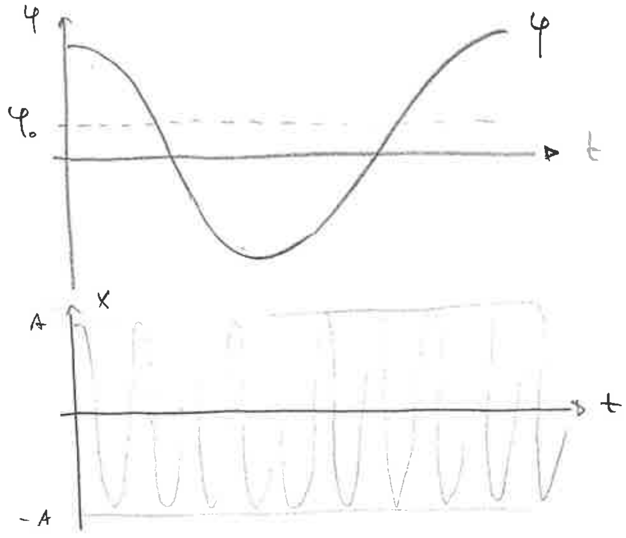
lock-in detektor



Fazna modulacija

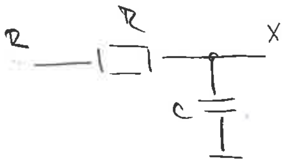
$$x = A \cdot \cos(\omega t + \varphi) \qquad \varphi = \varphi_0 + \varphi_M \cdot \cos(\omega_M t)$$

$$= A \cdot \cos(\omega t + \varphi_0 + \varphi_M \cdot \cos\omega_M t)$$



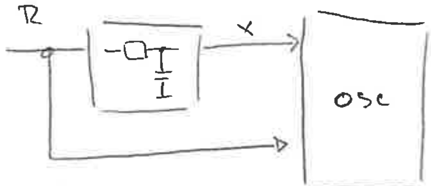
$2 \cos \cdot \cos \rightarrow \cos\Sigma + \cos\Delta$

$2 \sin \cdot \cos \rightarrow \sin\Sigma + \sin\Delta$



$R = \cos \omega t \quad ; \quad \omega = \dfrac{1}{RC}$

$x = \cos(\omega t + \varphi)$



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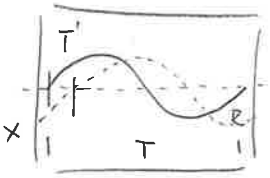
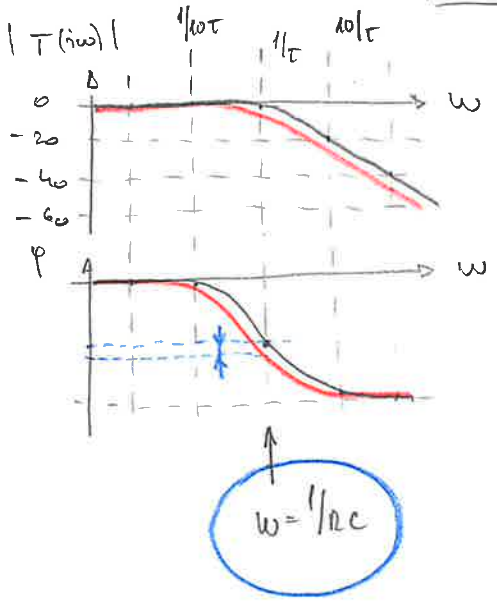
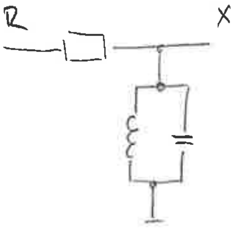
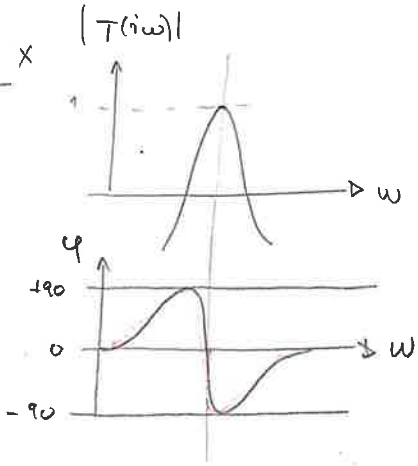


$$\varphi = \frac{T'}{T} \cdot 2\pi$$





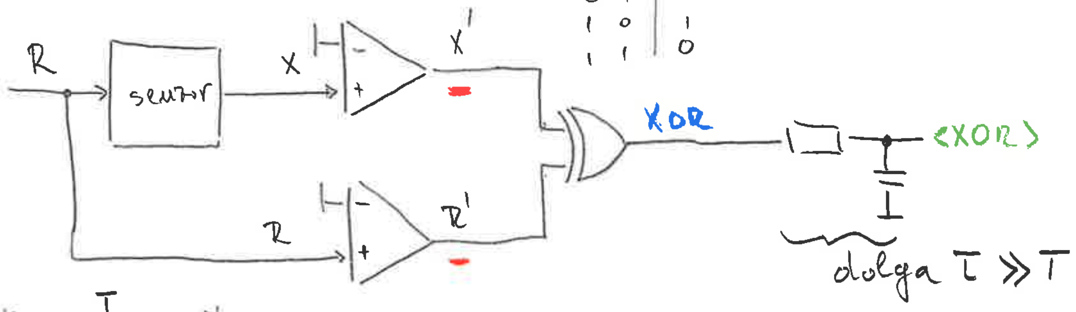


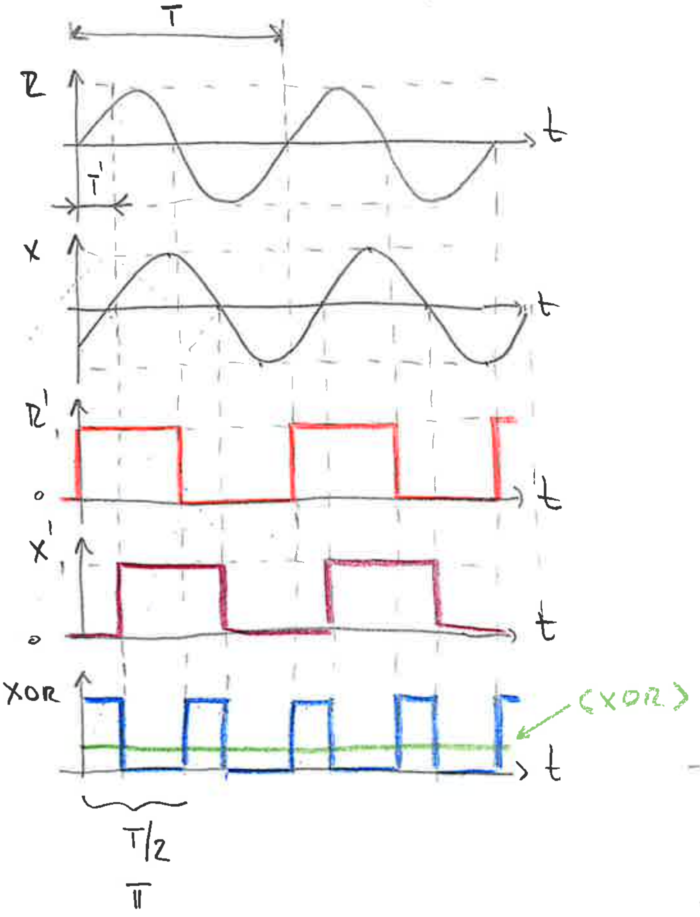
demodulacija

$$R = \cos \omega t$$

$$x = \cos(\omega t + \varphi)$$

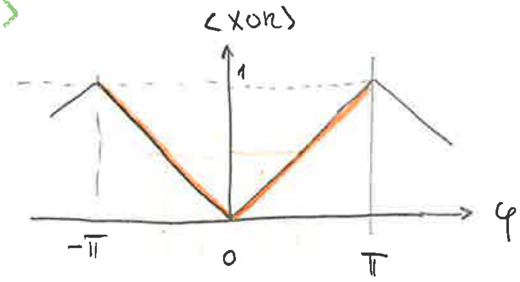
a	b	XOR
0	0	0
0	1	1
1	0	1
1	1	0

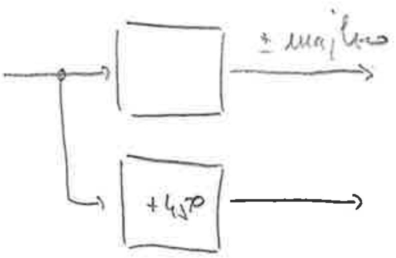


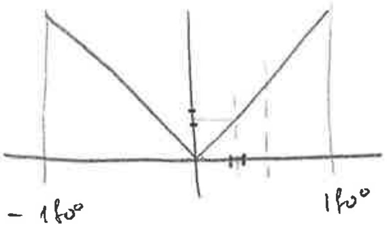


$$\varphi = \frac{T'}{T} \cdot 2\pi$$

$$\underline{\underline{\langle XOR \rangle}} = \frac{T'}{\frac{T}{2}} = \underline{\underline{2\frac{T'}{T}}}$$

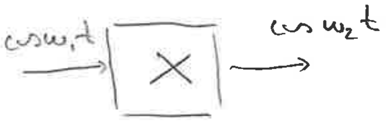






frekvenčna modulacija

$$x = A \cdot \cos \underset{\uparrow}{\omega} t \qquad \omega = \underbrace{\omega_0 + \Delta_\omega \cdot \cos(2\pi f_M t)}$$



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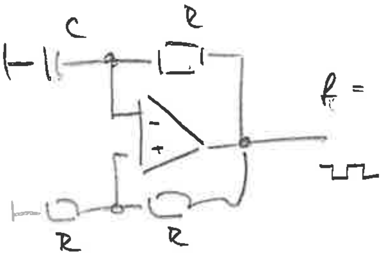
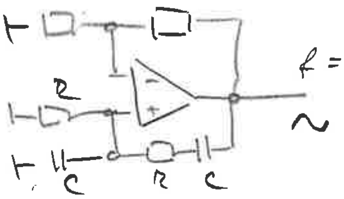
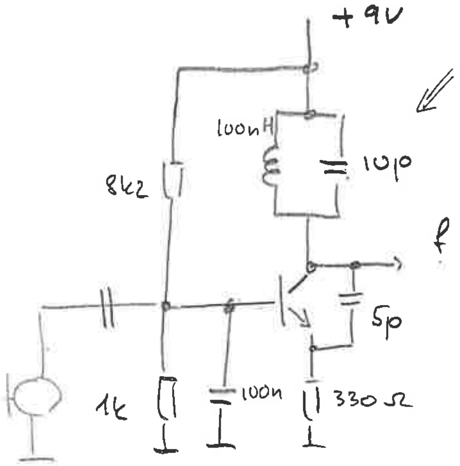


$$f = \frac{1}{2RC \ln 3}$$

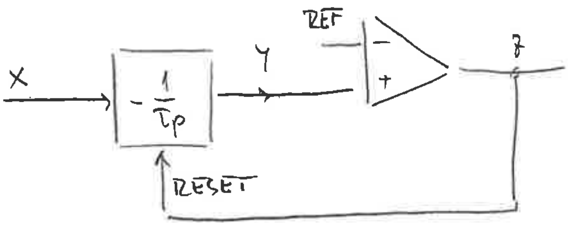


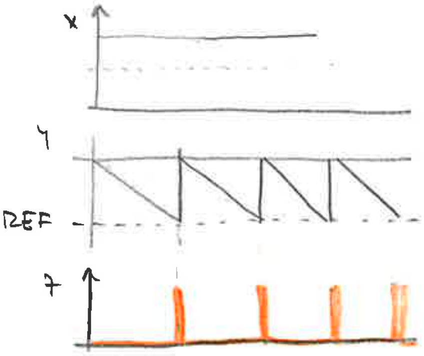
$$f = \frac{1}{2\pi RC}$$

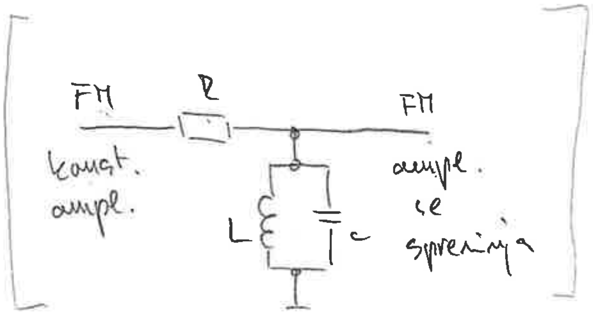


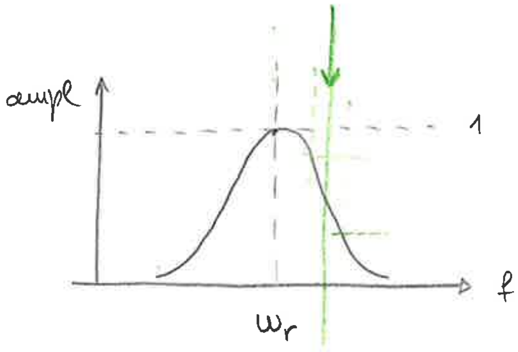
$$f = \frac{1}{2\pi\sqrt{LC}}$$

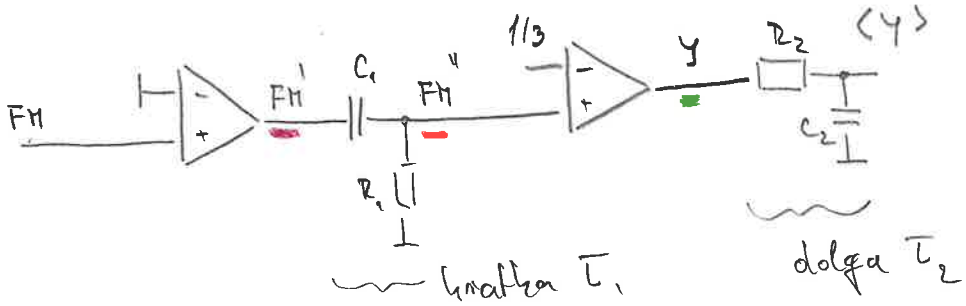
↓

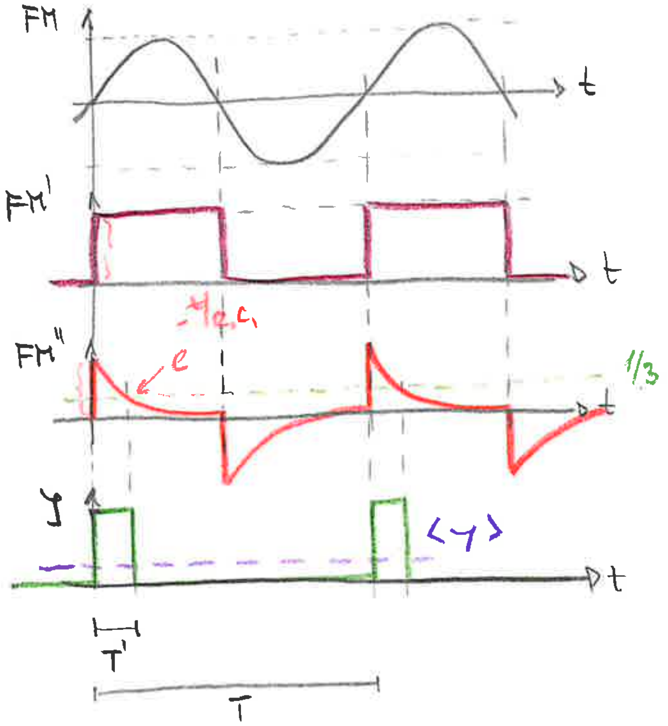












$$T = \frac{1}{f_x} \qquad T' = R_1 C_1$$

$$\boxed{\underline{\underline{\langle y \rangle}} = \frac{T'}{T} = \underline{\underline{R_1 C_1 \cdot f_x}}}$$

$$\underbrace{R_2 C_2 \gg T_{max} = \frac{1}{f_{max}}}$$

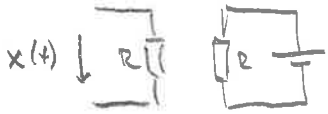
šum

- naključno gibanje nosilcev naboja rezultira v naključni napetosti na sponkah elementov

- šum moti določanje napetosti, toke ⇒ fiz. veličin
  - loči : motnje , naključni šum

- vrednotenje
  - po velikosti = po učinku ≡ enako greje : $P(x(t)) = P(x_{EF})$

šum

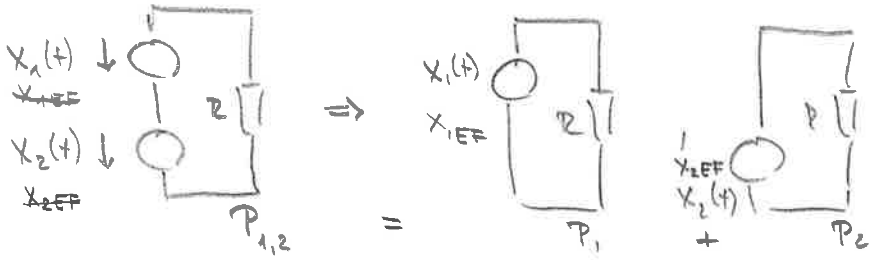


X_{EF} , X_{RMS} DC

- po spektru : naključni šum lahko razstavimo na harmonske komponente } Fourier

  $e_n(\nu)$
  - beli šum : pri vseh frekvencah enako velik
  - barvasti šum : različen

- sestavljanje šumov :



$$\underbrace{P_{12} = \frac{X_{12EF}^2}{R} \qquad P_1 = X_{1EF}^2/R \qquad P_2 = X_{2EF}^2/R}$$

$$X_{12EF}^2 = X_{1EF}^2 + X_{2EF}^2$$

⇓

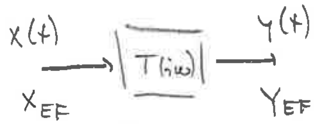
$$X_{skEF} = \sqrt{\sum_n X_{nEF}^2}$$

in

$e_n(\nu) = 1mV$, $\Delta\nu = 10k$

$X_{skEF} = 0{,}1V$ ← $X_{skEF} = \sqrt{\int_0^\infty e_n^2(\nu)\, d\nu}$

- šumi skozi elektronska vezja



$e_{xn}(\omega)$ $e_{yn}(\omega)$ ⟶ $\underline{e_{yn}(\omega) = e_{xn}(\omega) \cdot |T(i\omega)|}$

$$\Downarrow$$

$$y_{skEF}^2 = \int_0^\infty e_{xn}^2(\omega) \cdot |T(i\omega)|^2 d\nu$$

- šum za upor : $e_{nR}(\omega) = \sqrt{4kTR}$

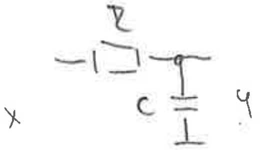
$$X_{skEF} = \sqrt{\int_0^\infty e_{nR}^2(\omega) \cdot d\nu} = \infty$$

bolj realno: območje frekvenc je omejeno za $0 < \nu < 1MHz$ in $R = 1k$

$$X_{skEF} = \sqrt{4kTR \cdot \Delta\nu}$$

$$= \sqrt{4 \cdot 1.38 \cdot 10^{-23} \cdot 293 \cdot 10^3 \cdot 10^6} \doteq 4 \mu V$$

- šum skozi RC ; $x \equiv$ beli šum $\equiv e_{xn}(\omega) = konst$



$$y_{skEF}^2 = \int_0^\infty e_{xn}^2(\omega) \cdot \left| \frac{1}{1 + i\omega CR} \right|^2 d\nu$$

$$= e_{xn}^2 \frac{1}{2\pi} \int \frac{d\omega}{1 + \omega^2 R^2 C^2} = e_{xn}^2 \frac{1}{4RC}$$