

Cyclical connectivity can be defined in terms of vertices, too. A graph G is *cyclically k -connected* if it cannot be separated into two components, each containing a cycle, by removing fewer than k vertices. The following property of fullerene graphs is a consequence of their 3-regularity and cyclical 4-edge connectivity.

Corollary: Every fullerene graph G is cyclically 4-connected.

Proof: Take a fullerene graph G and a cut-set $C = \{v_1, v_2, v_3\}$ such that both components, G' and G'' of $G - C$, contain a cycle. There are nine edges emanating from C toward the rest of G . At least three of them must connect C and G' . If there are exactly 3 such edges, we get a contradiction, since G is cyclically 4-edge connected. From the same reason, no component of $G - C$ can be connected with C by more than 5 edges. So, the only remaining possibility is 4 edges from C to one component, say G' , and 5 edges to the other component. From there it follows that two vertices of C , say v_1 and v_2 , issue one edge each toward G' , and the third vertex, v_3 , issues two edges toward G' . But now, the two edges between $\{v_1, v_2\}$ and G' and the edge between v_3 and G'' form a set of three edges, whose removal leaves two components, each containing a cycle. we have arrived at a contradiction again, hence G must be at least cyclically 4-connected.