Theorem 8. Every fullerene graph is cyclically 4-edge-connected.

Proof. Suppose, to the contrary, that G is not cyclically 4-edge-connected. This means that G can be separated into two components, each containing a cycle, by deleting exactly three edges. (Less than three edges would not suffice because of the 3-connectedness of G.) Let us denote these 3 edges by $e_i, i = 1, 2, 3$, and their endpoints by $v', v'', i = 1, 2, 3$, respectively. Let us consider the Schlegel diagram of G. Because of 3-connectedness and 3-regularity of G, there are two cycles, C' and C", such that every edge e_i has one endpoint, say v_i' on C', the other endpoint, v', on C", and no other edge connects C' with C". If there were no additional points on C', then C' would be a triangle, contrary to the fact that G is a fullerene graph. The same is valid for C". Let us denote the number of additional points on C' and C"by k' and k", respectively. Because of 3-regularity and 3-connectedness of G, k' (and k") must be at least 3. (It is possible to retain these properties even with k' = 2, but in this case the two additional points must be connected by an edge, forming a triangle in G. So the case k' = 2 must be dismissed.) Thus, $k' + k" \geq 6$. On the other hand, $k' + k" \leq 6$, because it is not possible to place more than 6 additional points on C' and C"placing at least 3 on each of them, without forming a face of G with more than 6 sides. So, $k' = k" = 3$, i.e., each one of the cycles C', C"is a hexagon. Let C' be the inner one. Then each one of 3 additional points on C' is an endpoint of an edge pointing toward the interior of C'. Let us denote these edges by $e_i', i = 1, 2, 3$. If any two of them have a common endpoint, v_i, then the 3-connectedness and the 3-regularity of G imply that the third edge has to have v' as its endpoint, too. In this case, G would contain at least one triangular or quadrangular face (depending on the placement of additional points on C'), contrary to our assumptions about G. Then the same reasoning as above implies that endpoints of e_i' must be on a cycle, C_1', with exactly 3 additional points on C_1'. We can proceed further in applying the above reasoning on C_1'. Because of the finiteness of G, after a finite number of steps, say n, we will obtain a cycle C_n' which will be either a triangle, or a hexagon with exactly one point inside: Both of these outcomes are in contradiction with our initial assumptions about G.