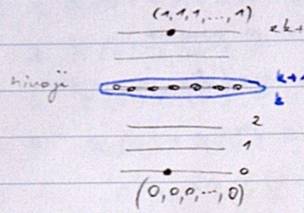
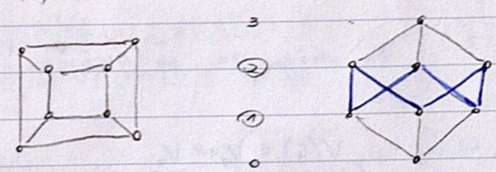
MIDDLE LEVEL CONJECTURE (Ivan Havel)



Q_{2k+1}

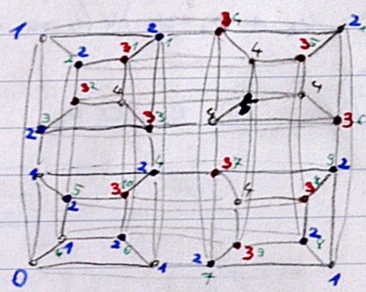
Točke v pasovih k, k+1 inducirajo Hamiltonov graf

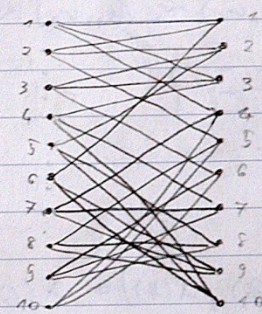
Q_3:

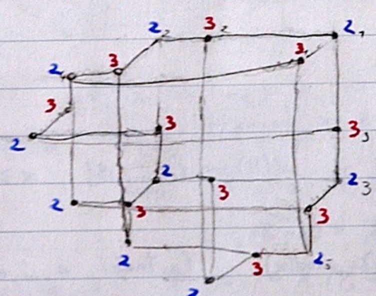


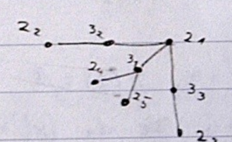
k=1

← Ta graf je hamiltonski

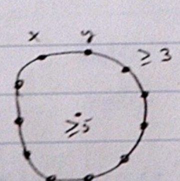




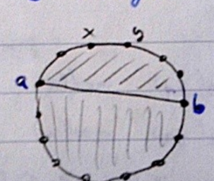




Vsak ravninski graf je 5-izbirljiv. ⇒ 5-obarljiv 22.2.07



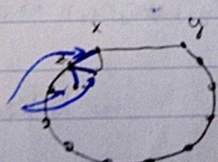
1.



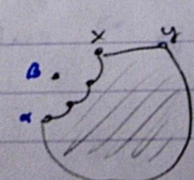
diagonala

2.

odstr. α, β.



α, β



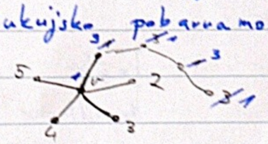
indukcija

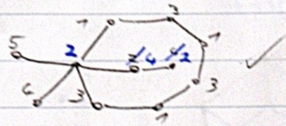
Vsak ravninski graf je 5-obarvljiv.

$\exists$ točka stopnje ≤ 5.

- < 5: odstranimo, pobarvamo z indukcijo in jo vrnemo.
- $= 5$: odstranimo in indukcijsko pobarvamo graf ter jo vrnemo.

Lahko se zgodi

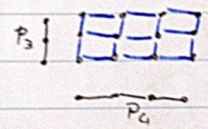


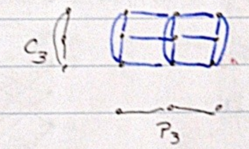


Ta veriga pa je omejena!

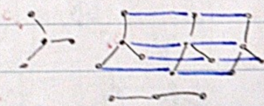
Kartezični produkt $G_1 \square G_2$: $V(G) = V_1 \times V_2$

$(u_1, v_1) \sim_G (u_2, v_2) \Leftrightarrow (u_1 \sim_{G_1} u_2 \wedge v_1 = v_2) \vee (u_1 = u_2 \wedge v_1 \sim_{G_2} v_2)$

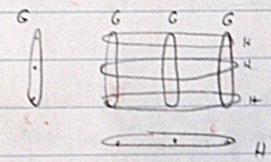




? $K_{1,3} \square P_3$



$\max(\chi(G), \chi(H)) \le \chi(G \square H)$



$\chi(H) = \alpha$, $\chi(G) = \beta$, $\alpha \ge \beta$

$\forall x \in G$ se da $\{x\} \times H$ pobarvati z α barvami.

$c_1 : V(H) \to \{1, \dots, \alpha\}$, $c_2 : V(H) \to \{1, \dots, \beta\}$, $\alpha \ge \beta$

$c : V(H) \times V(G)$

$c(x, y) = c_1(x) + c_2(y) \mod(\alpha)$ (ostanki $1, \dots, \alpha$)

$(x_1, y_1) \sim_{G \square H} (x_2, y_2)$

$\underline{c(x_1, y_1) \ne c(x_2, y_2)}$

$c(x_1, y_1) = c_1(x_1) + c_2(y_1) \mod(\alpha)$

$c(x_2, y_2) = c_1(x_2) + c_2(y_2) \mod(\alpha)$

$x_1 = x_2 \Rightarrow y_1 \sim y_2 \Rightarrow c_2(y_1) \ne c_2(y_2) \Rightarrow c(x_1, y_1) \ne c(x_2, y_2)$

$y_1 = y_2 \Rightarrow$ enako

res barvanje

1. $\chi(G) + \chi(\bar{G}) \leq n+1$ ← velja $\forall G$ na n vozliščih.
2. $\chi(G)\chi(\bar{G}) \leq \frac{(n+1)^2}{4}$
3. $\chi(G)\chi(\bar{G}) \geq n$
4. $\chi(G) + \chi(\bar{G}) \geq 2\sqrt{n}$

2.: $\chi(G) + \chi(\bar{G}) \leq n+1 \quad /^2$

$$\chi^2(G) + \chi^2(\bar{G}) + 2\chi(G)\chi(\bar{G}) \leq (n+1)^2 \qquad \chi^2(G) + \chi^2(\bar{G}) \geq 2\chi(G)\chi(\bar{G})$$

$$4\chi(G)\chi(\bar{G}) \leq (n+1)^2 \Rightarrow 2.$$

$a+b \geq 2\sqrt{ab}$ 4.: $n+1 \geq \chi(G) + \chi(\bar{G}) \geq 2\sqrt{\chi(G)\cdot\chi(\bar{G})} \overset{3.}{\geq} 2\sqrt{n} \Rightarrow 4.$

3.: $\chi(G) = k$ Največji barvni razred v $\bar{G}$: $\lceil \frac{n}{k} \rceil \leq \chi(\bar{G}) \Rightarrow 3.$

k neodvisnih razredov $\Rightarrow$ klika velikosti $\lceil \frac{n}{k} \rceil$ v $\bar{G}$

Dokaz 1.:

Indukcija: $n=1$: $\chi(G) = 1$, $\chi(\bar{G}) = 1$ $\quad G = K_1 \quad \bar{G} = K_1$

$1+1 \leq 1+1$ ✓

$n=2$ ✓

$n \to n+1$: Odstranimo $x \in G \Rightarrow \frac{G-x}{\overline{G-x}}$ na n točkah. $\chi(G-x) + \chi(\overline{G-x}) \leq n+1$

Recimo: $\chi(G-x) + \chi(\overline{G-x}) \leq n$

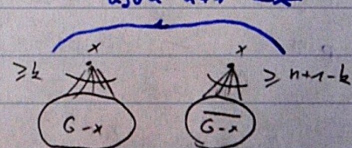
vemo $\chi(G-x) \leq \chi(G) - 1 \Rightarrow \chi(G) + \chi(\bar{G}) \leq n+2$

$\chi(\overline{G-x}) \leq \chi(\bar{G}) - 1$

vsota $n+1$ → ×

Vzemimo še: $\chi(G-x) + \chi(\overline{G-x}) = n+1$

$\chi(G-x) = k$, $\chi(\overline{G-x}) = n+1-k$



bodisi: $\chi(G-x) = \chi(G)$ in $\chi(\overline{G-x}) \leq \chi(\bar{G}) - 1$

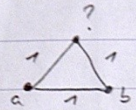
bodisi: $\chi(\overline{G-x}) = \chi(\bar{G})$ in $\chi(G-x) \leq \chi(G) - 1$

unit distance graph.

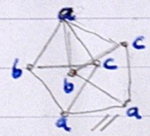
$V(G) = \mathbb{R}^2$, povezana med u,v, če $dist(u,v) = 1$

Točki v $\mathbb{R}^2$ sta različno pobarvani, če $dist(u,v) = 1$.

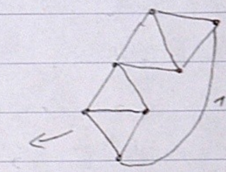
$4 \le \chi(G) \le 7$

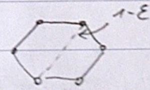
$\chi(G) = 2$: 

se ne da

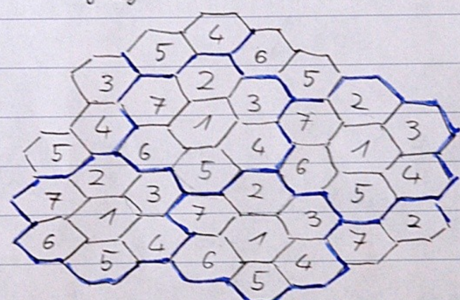
$\chi(G) = 3$: 

se ne da



$\chi(G) \le 7$: 

Pokrijemo ravnino s takimi 6-kotniki.

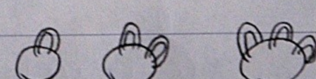


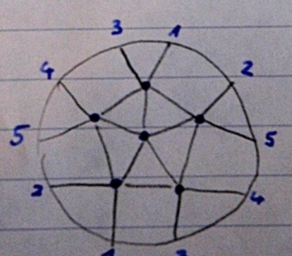
IZREK

Naj bo S ploskev z Eulerjevim rodom $g > 0$ in naj G leži na ploskvi S. Potem velja $\chi(G) \le \left\lfloor \frac{7+\sqrt{1+24g}}{2} \right\rfloor$.

Eulerjev rod

torus	2	$\mathbb{RP}$	1
2 torus	4	KS	2
3 torus	6		





K_6 vložen v $\mathbb{RP}$